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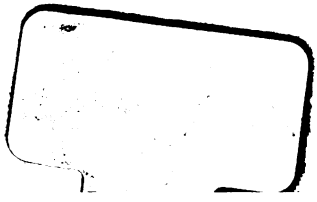

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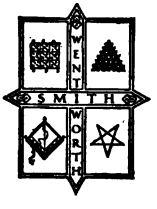
WENTWORTH-SMITH MATHEMATICAL SERIES

SCHOOL ARITHMETICS

BOOK TWO

BY

GEORGE WENTWORTH
AND
DAVID EUGENE SMITH



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PREFACE

This work has been prepared to meet a demand for a book which shall present the great essentials of arithmetic as required in Grades V and VI of our schools. The authors have not attempted to make a book of methods for teachers or to provide for the technical education of pupils in any special vocation; but they have furnished the teacher with the labor-saving material needed in a thorough course in arithmetic, they have motivated every process, they have suggested fields for the development of interesting local problems, they have suggested a large number of appropriate projects, and they have given to the pupils those fundamental elements of the subject which assure the command of numbers that will be needed in after life.

For the convenience of teachers the book has been divided into four chapters, each chapter containing sufficient work for a half year. Teachers may omit such exercises as are necessary for carrying out this plan. Indeed, in general, a teacher should learn the importance of omitting exercises whenever those exercises do not relate to the experiences or probable needs of the pupils. In a rural community all problems relating to the farm may properly be given, while many of those relating to the city or to manufacturing should be omitted. Similarly, in a city school many problems relating to agriculture will concern the pupils only remotely, and hence only a few exercises of this kind need be given.

The arrangement of the book is topical, so that a pupil stays long enough with a subject at one time to acquire that feeling of mastery which is his right and privilege. Along with this sequence of topics, however, there are several features which are noteworthy. One is the Little Examinations, a brief series of tests covering each chapter

in turn; a second feature is the Review and Drill section, also placed near the end of each chapter and furnishing a cumulative review of all preceding work; and a third is the sets of Problems without Numbers and the Problems for Completion, each of which requires new lines of independent thought on the part of the pupils. By the aid of these features a teacher may be assured that the pupil is kept refreshed upon those essentials of computation without which he cannot hope to succeed and that he is trained in independent thinking in arithmetic. There have also been inserted several elementary psychological tests relating to arithmetic, and these may be used or not in the discretion of the teacher.

In the theory of the work the authors have no more sympathy with the idea that a pupil should be told to do a thing in a certain way with no knowledge of why this way is the right one, than they have with the notion that he must explain every operation with all the care that a textbook writer would show. They believe that every process should be presented with a definite motive, that it should be learned with an appeal to the pupil's understanding, and that thereafter it should become entirely mechanical; and in this way each operation has been presented in this book.

The applications are more numerous and more real to the pupil than is usually the case in textbooks, but these applications are not permitted to exclude the abstract drill work without which no pupil has ever become a good computer. To balance adequately the abstract and the concrete, the drill work and the applied problem, the review work and the new material, has been one of the earnest endeavors of the authors in the preparation of this series.

The authors hope that their effort to prepare a perfectly usable textbook, free from those eccentricities which, while attracting momentary attention, fail to give the pupils the power they need, will prove helpful to the schools of the country.

GEORGE WENTWORTH
DAVID EUGENE SMITH

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SCHOOL ARITHMETICS

BOOK TWO

CHAPTER I

I. FUNDAMENTAL OPERATIONS

What you have Learned of Arithmetic. You have already learned much about arithmetic. You have learned how to count and to write numbers, at least to a million; how to add, subtract, multiply, and divide whole numbers or integers; how to solve many problems; how to keep simple accounts; how to do some easy work with fractions; how to write United States money; how to measure lines and rectangles; and how to use the ordinary tables needed in making these measurements.

What there is still to Learn. There is, however, much more to learn about the uses of arithmetic and more to learn about how to work with numbers. Every one of us is interested in what the world is doing and is interested in our country and its products. We shall therefore review some of the most important things about arithmetic and shall then study some new problems relating to business and to our everyday life.

Notes and explanations in this type are intended primarily for the teacher, to be used as needed.

Numerals. Symbols used to represent numbers are called *numerals*. The *common numerals* 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are also called *figures*, *Arabic numerals*, or *Hindu-Arabic numerals*. The symbol 0 is called *zero*, *naught*, *cipher*, or *o*.

Little attention is now given to the distinction between *notation* (the writing of numbers) and *numeration* (the reading of numbers) or to the memorizing of such definitions as those given on this page.

Schools should recognize the modern reading of numbers in telephoning, as "three–nine–six–o" for 3960, and in entries in bookkeeping, as "nineteen seventy-five point fifty" for $1975.50.

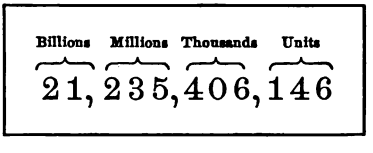
The nine figures, 1, 2, 3, 4, 5, 6, 7, 8, and 9, are called *digits*.

Period. For ease in reading we separate the figures of a large number into groups of three, beginning at the right. Each of these groups is called a *period*. We indicate the periods by commas or, in print, by spaces, thus: 14,375,286 or $756 283.75.

In an automobile number a hyphen is sometimes used, as in 45-784. In separating the figures of a number into periods, the pupil should begin at the right (or at the decimal point if there is one) and count toward the left. The left-hand period may have one, two, or three figures. Periods are not commonly indicated in numbers of four figures, and in this book we shall use commas to indicate periods only in numbers of five figures or more.

Place Value. The value of a period or figure depends upon the place it occupies. Therefore each period has a *place value*.

In the number 21,235,406,146 we have four periods as here shown. In each period there are three orders: units, tens, and hundreds.



The number is read "twenty-one billion, two hundred thirty-five million, four hundred six thousand, one hundred forty-six."

A thousand million is called a *billion*.

READING AND WRITING NUMBERS

Numbers 1 to 16, oral

1. Looking at your fingers, do you see why people came to count by tens? Why was it?

Give the proper name to each of the following:

2. 3 tens. 3. 10 tens. 4. 1000 thousand. 5. 1000 million.

Read the following numbers:

6. $26.06. 8. 427,000. 10. 5,000,000. 12. 3,125,000,000.
7. $37.50. 9. 400,027. 11. 7,275,060. 13. 4,000,125,000.

14. What advantage has our method of writing nine (9) over the method the Romans used, as shown on a clock face (IX)?

15. In writing 738 which place has the 8? the 3? the 7?

16. In the number 34,628,059,483 what name has the period 483? the period 059? the period 628? the period 34?

Write the following in words:

17. 35,850. 19. 330,330. 21. 7,642,858.
18. 64,907. 20. 600,006. 22. 42,429,845.

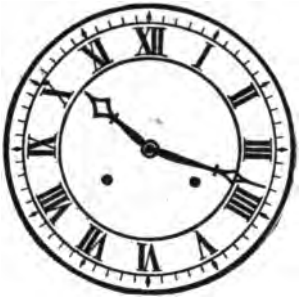
Write the following in figures:

23. Seven million, twenty-five thousand, fifty-nine.
24. Eighty-seven thousand, six hundred eighty-seven.
25. Six hundred four thousand, eight hundred twenty.
26. One billion, two million, one hundred sixty.

Exercises are for written work unless the contrary is stated. The motive for this work having been well established, the exercises in the present review are largely abstract.

Roman Numerals. Many years ago the Romans had a system of numerals which we still see used on clocks and for numbering the chapters of books. Sometimes we see them used in dates, as in the number of the year.

Everyone knows the first twelve numerals as they appear on a clock face. They are as follows:



I (1)	V (5)	IX (9)
II (2)	VI (6)	X (10)
III (3)	VII (7)	XI (11)
IV or IIII (4)	VIII (8)	XII (12)

The Romans usually wrote IIII for four, and VIIII for nine.

At present in writing Roman numerals we make use of seven letters, usually capitals, as follows:

Letters,	I	V	X	L	C	D	M
Values,	1	5	10	50	100	500	1000

These letters are combined according to the following rules:

1. *If a letter of less value is written after a letter of greater value, the sum of the values is to be taken.*

Thus, V = 5, I = 1, and so VI = 5 + 1 = 6.

2. *If a letter of less value is written before a letter of greater value, the difference of the values is to be taken.*

Thus, X = 10, I = 1, and so IX = 10 − 1 = 9.

3. *Repeating a letter repeats its value.*

Thus, C = 100, and so CC = 100 + 100 = 200. Usually letters are taken not more than three times, but, as with IIII, this is not always the case. In ancient times CCCC was commonly used instead of CD for 400.

Sometimes a dash is placed over a letter to increase its value a thousandfold, as in $\overline{\text{V}}$ for 5000, but this is too rarely used to be learned.

Writing Roman Numerals. In writing a number in Roman numerals we begin at the left and write the highest order, then the next highest order, and so on. Thus MDCCLXXVI is written for 1776, the letters having the values shown at the right.

M	= 1000
DCC	= 700
LXX	= 70
VI	= 6
MDCCLXXVI	= 1776

In writing 1924 we may write MDCCCCXXIV, which is substantially the way the Romans would have written it, or we may write MCMXXIV, which is allowed by modern custom.

Thus we see that the tens are written thus:

X, XX, XXX, XL, L, LX, LXX, LXXX, XC.

The hundreds are written thus:

C, CC, CCC, CD, D, DC, DCC, DCCC, DCCCC, or **CM.**

The numbers from eleven to nineteen are written thus:

XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, XIX.

ROMAN NUMERALS

Write the following in common numerals:

1. XIV.	4. XXV.	7. LXXI.	10. LXXXVII.
2. LXI.	5. XLI.	8. XCVII.	11. CCXXXIV.
3. LXX.	6. DCX.	9. MDCX.	12. MDCCCCXXVI.

Write the following in Roman numerals:

13. 27.	15. 46.	17. 78.	19. 1775.	21. 1928.
14. 32.	16. 54.	18. 87.	20. 1920.	22. 1930.

Addition. The process of finding the number which is equal to two or more given numbers together is called *addition*. The numbers added are called *addends*, and the result is called the *sum*.

Unit. A *unit* is any standard used in measuring or counting. Thus, in speaking of \$8 the unit is \$1; in speaking of 7 eggs the unit is 1 egg, but in speaking of 14 dozen eggs the unit is 1 dozen eggs; in speaking of 80 thousand bricks the unit is a thousand bricks; in counting 1, 2, 3, 4, and so on, the unit is 1; in counting by thousands the unit is 1000; and in measuring long distances the unit may be 1 mile.

Like Numbers. Numbers which are applied to the same unit are called *like numbers*. Thus, \$9 and \$16 are like numbers, and 57 and 95 are also like numbers.

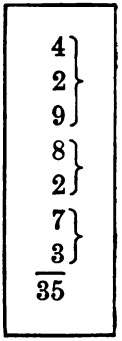
Only like numbers can be added, and the sum is like the addends.

That is, we cannot add \$4 and 7 yd. or 8 lb. and 3 books. If we add \$15 and \$28 the sum must be like the addends; that is, it must be \$43, not 43 feet. It is true that we can put 3 books with 9 pencils, but we have to think of them only as *things* and say that we have 12 things.

How to Add. In adding a column of figures try to read the column like a word. Try especially to see the 10's, the 5's, or the 15's.

In this column, for example, you should see the two 10's and the 15 and write the answer, 35. However, it is not wise to skip numbers in order to group by 10's, because you may forget the numbers you have skipped and thus make a serious error.

Check. As a *check*, or *proof*, of the work we add the column downwards as well as upwards.



WRITTEN SPEED TEST

Copy the numbers in lines 1 to 8 inclusive, and add; then copy the numbers in columns 9 to 17 inclusive, and add:

	9.	10.	11.	12.	13.	14.	15.	16.	17.
1.	5	6	7	3	7	6	5	7	2
2.	3	3	2	2	9	7	4	3	8
3.	7	8	5	3	4	8	8	6	7
4.	9	7	6	9	8	3	2	8	8
5.	8	2	3	4	5	4	3	5	5
6.	6	2	8	1	9	7	3	6	6
7.	9	5	4	7	8	8	7	4	4
8.	4	5	7	8	2	7	9	3	8

Copy the numbers in lines 18 to 25 inclusive, and add; then copy the numbers in columns 26 to 34 inclusive, and add:

	26.	27.	28.	29.	30.	31.	32.	33.	34.
18.	7	3	6	2	5	5	4	6	7
19.	9	5	7	8	8	5	7	6	7
20.	9	8	7	5	5	6	3	7	6
21.	8	6	4	6	7	4	4	2	5
22.	7	7	5	7	9	8	9	4	8
23.	1	5	6	6	6	5	4	7	7
24.	2	4	6	3	9	5	9	8	4
25.	5	9	6	5	7	6	8	7	9

In such tests the teacher should keep a record for future reviews. This record should show the number of correct results and also of incorrect results secured by each pupil in 4 min., not including the time of copying.

WRITTEN SPEED TEST

Copy the following and then, at a signal from the teacher, see how many columns you can add in 2 min.:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
1	2	8	5	7	4	6	9	2	6
5	7	9	2	9	2	7	2	1	4
6	3	4	7	6	7	8	3	9	9
2	6	3	3	2	3	2	4	8	8
4	4	6	6	3	6	3	6	7	3
8	8	2	8	8	1	4	8	6	5
3	9	0	9	4	9	0	6	2	2
7	7	1	4	1	8	1	4	3	7
9	6	7	3	7	7	7	7	8	8
2	2	2	1	6	2	9	9	4	1
1	7	9	2	5	5	6	7	5	9
7	3	8	7	0	4	2	6	4	6
3	4	6	4	8	6	8	3	0	7
4	8	4	2	9	8	4	4	6	8
6	1	3	6	3	3	7	2	8	4
2	6	3	3	2	5	1	7	1	9
4	3	2	3	3	7	0	3	2	7
4	1	5	4	4	2	8	1	3	6

As on page 7, the teacher should keep, for use in future reviews of this work, the record of the number of correct results and also the number of incorrect results secured by each pupil. In speed contests it is convenient to separate the class into three groups, about equally matched, changing these groups from time to time. Seek to ascertain the pupil's difficulty. It may be that he does not write the figures distinctly before the test begins, or that he does not know certain combinations. In the latter case, drill with "flash cards" is very helpful.

ORAL SPEED TEST

Look at each example and state only the sum:

1. 14	9. 75	17. 80	25. 71	33. 75	41. 44
3	13	20	35	32	53
2. 24	10. 61	18. 83	26. 32	34. 31	42. 43
3	12	20	70	92	34
3. 36	11. 72	19. 42	27. 41	35. 41	43. 55
23	14	63	88	81	24
4. 67	12. 96	20. 81	28. 30	36. 80	44. 84
32	4	20	73	30	57
5. 77	13. 94	21. 76	29. 81	37. 62	45. 92
32	10	30	40	40	40
6. 98	14. 94	22. 85	30. 71	38. 50	46. 92
10	11	15	31	60	49
7. 98	15. 94	23. 87	31. 40	39. 51	47. 85
30	40	20	60	62	45
8. 41	16. 23	24. 20	32. 21	40. 20	48. 30
62	40	42	42	96	40
10	10	20	20	10	50

From three to five minutes of such rapid work in each recitation will do much to remove the charge that pupils are usually slow and inaccurate. Particular attention should be given to oral drill in addition by endings. The pupil should think of 15 + 13 as 15 + 10 + 3, or 25 + 3, and of 25 + 17 as 25 + 10 + 7, or 35 + 7.

ORAL SPEED TEST

Add the following:

1.	2.	3.	4.	5.	6.
26	42	28	36	57	29
30	21	15	14	18	19
14	13	10	22	16	24

The pupil should add each of these in one operation, not adding the units first and then the tens. For example, in Ex. 1 he should think: "14, 44, 64, 70," which is merely a short way of thinking "14, 14 + 30, 44 + 20, 64 + 6 = 70." Only the result should be stated.

Tests of this kind, the numbers being changed, should be written on the blackboard for speed contests of class groups, as suggested on page 8.

Add the following:

7.	9.	11.	13.	15.	17.
10	38	83	37	56	36
14	22	9	42	49	48
23	14	40	61	78	73
42	26	66	88	37	96

8.	10.	12.	14.	16.	18.
21	53	75	29	29	85
32	41	25	38	66	37
41	66	32	67	75	44
60	78	59	48	88	68

In Exs. 7–18 the pupil should add in one operation as in Exs. 1–6.

Add the following:

19.	20.	21.	22.	23.	24.
26	33	18	67	48	33
32	47	17	48	67	57
44	68	25	39	35	68
25	42	32	56	29	79
15	30	48	75	16	66

USING WHAT YOU HAVE LEARNED

1. A gas meter shows the following consumption of gas per month for three months: November, 3700 cu. ft.; December, 4200 cu. ft.; January, 4350 cu. ft. Find the total amount consumed in the three months.

2. In a certain week Mr. Russell's meat bill was $5.83; his grocery bill, $12.66; and his milk bill, $1.49. Find the sum of these three items.

3. The Dixie Highway connects Chicago and Jacksonville. An automobile party traveled along this road 168 mi. the first day, 157 mi. the second, 164 mi. the third, and 154 mi. the fourth. What was the total distance thus far traveled?

4. A street-car conductor collected 681 fares on Thursday, 719 on Friday, 934 on Saturday, and 1067 on Sunday. How many fares did he collect during the four days?

5. The Lincoln Highway extends from New York to San Francisco, passing through Chicago and Cheyenne. The approximate distances are as follows: New York to Chicago, 863 mi.; Chicago to Cheyenne, 1085 mi.; Cheyenne to San Francisco, 1386 mi. Find the total distance.

6. A wholesale fruit dealer bought the following items: apples, $368.25; oranges, $78.45; California grapes, $82.64. Find the total amount.

7. A contractor paid $3200 for a building lot, and on this he built a house which cost him $9640. At what price must he sell the property in order to make a profit of $2500?

8. An Iowa farmer marketed four carloads of cattle. The first carload weighed 30,680 lb.; the second, 29,725 lb.; the third, 26,467 lb.; and the fourth, 20,590 lb. Find the total weight of the cattle.

ADDING NUMBERS

1. The following table shows the number of games won and lost by each one of the eight baseball teams of the National League in a recent year. Adding vertically for the games lost and across for the games won, find the totals. Check by showing that the total number of games won is equal to the total number of games lost.

CLUBS	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	WON
A		12	14	16	15	12	16	15	
B	10		10	9	13	13	17	18	
C	8	12		12	11	15	14	16	
D	6	12	10		11	13	14	12	
E	7	9	11	11		13	12	11	
F	10	9	7	9	8		13	15	
G	6	5	8	8	10	9		11	
H	7	3	6	10	11	7	11		
Lost									

See how long it takes to copy and add these columns and to check the results, writing the time required:

2.	**3.**	**4.**	**5.**
$3640.45	$4667.48	$3987.96	$7240.68
2139.70	3208.46	2197.75	3280.95
863.45	400.73	3299.80	3557.75
4096.37	382.64	2763.86	263.44
2800.42	2529.93	1889.95	2986.75
7436.00	4743.64	2263.45	3945.50
2517.33	3082.37	426.48	2095.06
6318.26	1098.02	2029.00	3711.40
5872.20	4380.98	2706.50	5874.24

Subtraction. The process of taking one number from another is called *subtraction.*

Subtraction may also be defined as the operation of finding the number which when added to the smaller of two given numbers makes the larger. Since a great deal of subtraction is "making change," where the work is based on addition, the latter definition is often preferred.

Subtrahend. The number which is subtracted is called the *subtrahend.*

Minuend. The number from which the subtrahend is subtracted is called the *minuend.*

Difference. The number found in subtraction is called the *difference* between the two given numbers; thus, the difference between 873 and 237 is 636. The difference is sometimes called the *remainder.*

873 minuend
237 subtrahend
636 difference

Check. To check the work in subtraction we add the difference and the subtrahend. If the work is correct, this sum should be equal to the minuend.

But little time should be spent on definitions like these, since the terms are rarely used in business. The business man says, "*Deduct* \$10 and pay me the *balance,*" or "*Take out* \$10 and pay me the *rest,*" instead of "*Subtract* \$10 and pay me the *difference.*" The teacher should lead the pupil to see that 100 − 37 is the sum of 90 − 30 and 10 − 7, or 63.

ORAL DRILL IN SUBTRACTION

Subtract the following numbers:

1. 37 − 4	3. 26 − 3	5. 33 − 1	7. 25 − 5	9. 46 − 15	11. 57 − 29	13. 43 − 35
2. 47 − 4	4. 56 − 2	6. 73 − 1	8. 46 − 10	10. 57 − 27	12. 43 − 33	14. 65 − 45

How to Subtract. If one village has 2743 inhabitants and another has 867, and we wish to find how many more persons live in the first village than in the second, we need to subtract 867 from 2743. We cannot subtract 7 from 3, but we can subtract 7 from 13, for $13-7=6$. We have now used 10 of the 40, leaving 30. We cannot subtract 60 from 30, but $130-60=70$. We have now used 100 of the 2700, and $2600-800=1800$. Therefore the result is 1876.

$$\begin{array}{r} 2743 \\ 867 \\ \hline 1876 \end{array}$$

Check. Since $1876+867=2743$, the work is evidently correct.

Since methods of subtraction vary so much, the teacher should let each pupil in the upper grades follow the method he has already learned.

SUBTRACTING NUMBERS

1. Mr. Johnson, the grocer, found that in one week he paid \$7.75 for light, \$96 for help, \$50 for rent, and \$18.85 for printing and other overhead charges. In the same week he sold goods for \$860 which cost him \$550. Find his profit for that week.

Teachers should explain the meaning of *overhead charges* whenever the term is given, or until it is fully understood. It covers such items as insurance, repairs, rent, and wages.

Subtract as indicated:

2. 2990 − 1794	5. 3890 − 2994	8. 4095 − 2396	11. 7040 − 4398	14. 8016 − 3589
3. 6230 − 3878	6. 5230 − 3915	9. 8230 − 2846	12. 7231 − 2806	15. 8000 − 3928
4. 8234 − 5689	7. 9207 − 4653	10. 8170 − 7692	13. 7500 − 5286	16. 8111 − 5792

Making Change. If you owe 58¢ to a merchant and give him $1, he says, "58 and 2 are 60, and 10 more are 70, and 5 are 75, and 25 makes a dollar," at the same time laying down 2¢, 10¢, 5¢, and 25¢. You thus see that $1 − $0.58 = $0.42.

MAKING CHANGE

All work oral

Make change for 25¢ when you owe:

1. 10¢.	3. 9¢.	5. 5¢.	7. 8¢.	9. 22¢.
2. 13¢.	4. 19¢.	6. 15¢.	8. 18¢.	10. 17¢.

Make change for 50¢ when you owe:

11. 38¢.	13. 32¢.	15. 26¢.	17. 16¢.	19. 23¢.
12. 28¢.	14. 42¢.	16. 24¢.	18. 15¢.	20. 29¢.

Make change for $1 when you owe:

21. 28¢.	23. 50¢.	25. 86¢.	27. 75¢.	29. 93¢.
22. 36¢.	24. 56¢.	26. 84¢.	28. 77¢.	30. 97¢.

Make change for $2 when you owe:

31. $1.25.	33. $1.35.	35. $1.50.	37. $1.48.
32. $1.70.	34. $1.75.	36. $1.83.	38. $1.87.

Imagine yourself buying goods at a store and handing the clerk the first amount given, the goods costing the second amount. State the amount of change due you in each case:

39. $1; 68¢.	43. $4; $3.52.	47. $5; $2.85.
40. $2; $1.46.	44. $5; $2.28.	48. $5; $2.65.
41. $2; $1.72.	45. $5; $3.60.	49. $10; $8.08.
42. $3; $2.38.	46. $10; $7.75.	50. $10; $7.37.

DRILL WORK IN SUBTRACTION

See how long it takes to copy each column of 10 exercises and to subtract and check, writing the time with the last result:

1. 373,250 151,175	11. 684,560 328,989	21. 975,438 821,795	31. 580,000 538,876
2. 572,527 221,962	12. 347,892 129,999	22. 900,000 328,965	32. 990,900 666,907
3. $675.62 181.98	13. $829.80 167.98	23. $708.07 298.78	33. $800.10 469.86
4. $570.06 379.99	14. $970.45 291.66	24. $709.16 387.68	34. $504.76 362.97
5. $802.68 469.81	15. $700.00 293.76	25. $508.76 235.92	35. $972.93 598.98
6. $906.97 423.49	16. $982.75 793.89	26. $700.00 387.50	36. $820.36 428.92
7. $729.81 249.93	17. $408.72 329.86	27. $898.05 509.79	37. $706.00 328.85
8. $824.75 568.45	18. $711.20 667.58	28. $652.04 227.45	38. $950.00 652.90
9. $402.25 387.56	19. $884.75 615.28	29. $712.00 479.63	39. $527.63 343.56
10. $926.00 547.62	20. $824.70 652.35	30. $506.50 309.52	40. $725.82 537.94

Teachers will do well to return to these efficiency tests occasionally.

USING WHAT YOU HAVE LEARNED

1. At a sale a man bought a suit of clothes for $44.50 and an overcoat for $39.75. How much more did he pay for the suit than for the overcoat?

2. I have $72.95 in the bank and I owe $67.47. If I pay this debt from the money in the bank, how much shall I have left?

3. Out of a regiment of 2985 men, 2247 returned from an engagement. How many failed to return?

4. Fred spent $767.30 the first year that he was in college, and $845.65 the second year. How much more did he spend the second year than the first?

5. Mr. Wheeler bought an automobile for $1388 and was allowed $297 for his old car. He gave a check for the balance. Find the amount of the check.

It is desirable to mention informally such matters as checks when occasion arises. If the word is not familiar to the pupils, it may be explained briefly as a kind of letter to a bank to pay the money from what Mr. Wheeler has there.

6. Colfax County built 976 rods of concrete road in one year, 440 rods the second year, and 826 rods the third year. Find the gain or the loss in the number of rods the third year as compared with the number in each of the other years.

7. The owner of a residence rents it to Mr. Brown for $950 a year and pays $224.90 for taxes and improvements. Find his profit over the amount spent for taxes and improvements.

8. Mrs. Morgan bought a crate of eggs in April for $15.60 and preserved them in water glass for which she paid 85¢. By doing this how much did she save on a crate of eggs which would cost $20.40 if bought in December?

Cash Account. In keeping a cash account you should write the receipts on the left-hand side, called the *debit* side, and the payments on the right-hand side, called the *credit* side, thus:

	RECEIPTS						PAYMENTS					
	19—						19—					
a	May	6	Cash on hand	2	75		May	7	Paid for ball		30	*f*
b		7	From father		20			8	Ball game		25	*g*
c		8	From mother		10			8	Marbles		15	*h*
d		8	Earned, errands		25			9	Balance	2	60	*i*
e				3	30					3	30	*j*
k	May	9	Balance	2	60							

It may be explained that account-book paper is usually ruled as above, which makes it unnecessary to write the dollar sign or decimal point.

Here *a* shows cash on hand when this page of the account is begun; *b*, *c*, and *d* are receipts; *e* is the sum of these items on the day the account is *balanced*. On the right-hand side *f*, *g*, and *h* are amounts paid. To find the amount on hand we subtract the sum of *f*, *g*, and *h* from *e*, and find that the *balance* is $2.60, which we write on the right, at *i*, and on the left, at *k*. In order to check the work, *f*, *g*, *h*, and *i* are added, and the sums at *e* and *j* must agree.

KEEPING A CASH ACCOUNT

1. Make out a cash account, using the following items, and balance it on Dec. 9 as above. Receipts: Dec. 6, cash on hand, $145.80; Dec. 6, M. P. Jones, $85.70; Dec. 7, A. B. Smith, $68.90; Dec. 8, P. H. Haynes, $38.25. Expenses: Dec. 6, rent, $85; Dec. 7, salaries, $75; Dec. 8, coal, $28.60; Dec. 8, account book, $2.50; Dec. 8, telephone, $5.80; Dec. 8, gas bill, $4.75; Dec. 8, stationery, $3.50; Dec. 9, book, $1.25.

2. William is writing up his cash account. He received a present of \$2 on May 15, 35¢ on May 16 for helping his father in the store, 15¢ on May 18 for running errands, and 35¢ on May 19 for cutting the lawn. He paid 45¢ on May 16 for a book, 15¢ on May 17 for marbles, 20¢ on May 18 for paper and ink, 50¢ on May 20 for a knife, and on May 21 he put the balance in the bank, where he already had \$16.80. Make out the cash account and find how much he then had in the bank.

3. William made out a statement of his father's grocery business. He arranged the following items as in a cash account: value of building, \$14,750; groceries, \$2275.50; motor truck, \$875; due from customers, \$735.60. His father owed \$5850 on the building, and bills amounting to \$850.75. Make the statement showing the value of the business after deducting what was owed.

Simple problems of this kind should be made up by the class. The pupils should also be urged to keep their personal accounts.

WRITTEN SPEED TEST

See how long it takes to copy these numbers, to subtract, and to check each result:

1. 46,701 29,684	4. 30,123 19,468	7. 71,247 56,493	10. 60,000 19,877
2. 51,206 36,819	5. 67,329 30,538	8. 91,426 48,790	11. 82,573 49,696
3. 62,763 41,929	6. 80,230 57,629	9. 78,123 49,206	12. 93,042 28,768

Group contests as suggested on page 8 may be used to advantage. The time should be kept by the teacher, with a record of the correct results and also of the incorrect ones. This is often more satisfactory than to see how many subtractions can be made in a given time.

See how long it takes to copy the following, to add the receipt columns, and to insert the balance at the foot of each of the payment columns:

13.

Receipts		Payments	
1476	81	1087	20
298	30	176	39
3400	00	2173	48
1287	64	976	92
536	98	1042	38
2176	92	1106	54
384	08	*Bal.*	

14.

Receipts		Payments	
3629	42	2986	29
2173	60	2432	30
984	38	874	60
1268	72	2230	09
2500	00	1687	52
3078	06	943	70
984	37	*Bal.*	

USING WHAT YOU HAVE LEARNED

1. A farmer has 100 acres of corn in three fields. The first field contains 27 acres and the second 34 acres. How many acres are there in the third field? This is how many more acres than there are in the first field?

2. Of 100 acres planted in corn, 67 were harvested with a corn harvester, the corn on 18 was used for silage, and the corn on the remainder was husked standing on the hill. How many acres were husked standing on the hill?

3. In a year when a farmer's account showed his income to be $8750.78 he received $2682.65 from the sale of hogs and $2972.87 from the sale of cattle. What was his income from other sources besides hogs and cattle?

4. A farmer sold his farm for $40,000 at a public sale and then sold his stock and implements for $16,950. After buying a house and lot in town for $12,450, he invested the remainder in Victory Bonds. How much did he invest in the bonds?

5. On an automobile trip of 532 mi. a man and his family went 139 mi. the first day and 147 mi. the second day. How many more miles have they to go?

6. The distance from New York to Boston is 229 mi.; from New York to New Haven, 72 mi.; from New London to Boston, 106 mi. These places being all on the same line between New York and Boston, find the number of miles from New Haven to New London.

7. The distance from New York to Binghamton is 193 mi.; from Binghamton to Buffalo, 203 mi.; from New York to Elmira, which is between Binghamton and Buffalo, 250 mi. How far is it from Elmira to Buffalo?

Multiplication. The process of taking one number as many times as there are units in another is called *multiplication.*

This means multiplication by a whole number. Thus, to multiply \$205 by 3 means that \$205 is taken three times.

The familiar terms "multiplicand," "multiplier," and "product" are illustrated in the annexed example.

\$205	multiplicand
3	multiplier
\$615	product

Abstract and Concrete Numbers. It is common to speak of a number that does not refer to any particular kind of object or measure as an *abstract number.* It is also customary to speak of a number that refers to some particular kind of object or measure as a *concrete number.* Thus, 4 is commonly called an abstract number, and 4 ft. a concrete number.

For a discussion of this subject teachers are referred to page 294.

The multiplier is thought of as abstract, and the product is thought of as like the multiplicand.

Multiple. The product of two abstract whole numbers is called a *multiple* of either. For example, 35 is a multiple of 7 and also a multiple of 5.

Factor. Any one of two or more whole numbers (excluding the number one) which, multiplied together, make another number is called a *factor* of that number. For example, 2, 3, 5, 6, 10, and 15 are the factors of 30.

Power. The result of taking a number any number of times as a factor is called a *power* of the number.

For example, $2 \times 2 = 4$, and 4 is called the *second* power, or the *square*, of 2, and the product 2×2 is often written 2^2. So $2 \times 2 \times 2$, or 2^3, is the *third* power, or the *cube*, of 2.

How to Multiply. You have learned how to multiply one number by another. For example, the following may be studied:

$635.52	3247	3247
215	1305	1350
3177 60	16235	162350
6355 2	9741	9741
127104	3247	3247
$136636.80	4237335	4383450

In the first example above, in which $635.52 is to be multiplied by 215, we multiply as if the problem were $215 \times 63{,}552¢$.

First, multiplying 63,552 by 5 we have 317,760.

Then, multiplying 63,552 by 1 ten, the result is 63,552 tens, which we write with the right-hand figure, 2, in tens' place.

Then, multiplying 63,552 by 2 hundreds, the result is 127,104 hundreds, which we write with the 4 in hundreds' place.

Adding, we have 13,663,680 cents, or $136,636.80.

MULTIPLYING NUMBERS

1. At a certain temperature sound travels 1132 ft. a second. How far will sound travel in 16 sec.? in 90 sec.? in 2 min.?
2. At $175 each, how much will 307 mules cost?
3. At $162.50 each, how much will 204 horses cost?
4. At $875 each, how much will 325 automobiles cost?
5. At $137.50 each, how much will 120 wagons cost?
6. At $406.75 each, how much will 62 house lots cost?
7. At $5270.75 each, how much will 18 houses cost?

USING WHAT YOU HAVE LEARNED

1. There are 196 lb. of flour in a barrel of flour. How many pounds of flour are there in 175 barrels?

2. If the average consumption of meat in a year is 194 lb. for each person, how many pounds will be consumed in a year by the inhabitants of a village of 1673 people?

3. Mr. Kane saves \$6.75 out of his wages each week. How much does he save in 36 wk.? in 52 wk.? in 108 wk.?

4. Mr. Roberts bought 14 tons of coal for \$11.25 a ton, and put it in his cellar, where he had 3 tons left over from last year for which he had paid \$10.75 a ton. Find the cost of all the coal in the cellar.

5. Mr. Sinclair owns a house and a garage which he rents to Mr. Robinson, receiving \$67.50 a month for the house and \$7.50 a month for the garage. How much rent does he receive from Mr. Robinson in a year?

While a pupil should be left to his own ingenuity in solving such a problem, if he does not take a reasonably short method, he should be led to see that his method might be improved.

Find the total cost in each of the following cases:

6. 175 automobiles at \$675.50 each.

7. 136 cows at \$97.50 each.

8. 146 tires at \$72.50 each, and 128 at \$68.75 each.

9. 112 stoves at \$78.24 each, and 69 at \$87.36 each.

10. 136 horses at \$157.50 each, and 96 at \$178.50 each.

11. 128 chairs at \$5.35 each, and 64 more, also at \$5.35 each.

12. 1208 camp chairs at \$1.08 each, 96 tents at \$29.48 each, and 650 camp tables at \$1.08 each.

DRILL WORK IN MULTIPLICATION

Perform the following multiplications, timing each column:

1. 978 × 506.	**12.** 42 × 4437.	**23.** 508 × 4779.
2. 352 × 324.	**13.** 93 × 7080.	**24.** 767 × 7873.
3. 682 × 726.	**14.** 69 × 7236.	**25.** 328 × 5362.
4. 823 × 983.	**15.** 83 × 8192.	**26.** 503 × 8742.
5. 435 × 481.	**16.** 54 × 7762.	**27.** 621 × 8263.
6. 632 × 408.	**17.** 36 × 8087.	**28.** 987 × $12.87.
7. 563 × 481.	**18.** 53 × 8129.	**29.** 602 × $42.75.
8. 408 × 662.	**19.** 66 × 6666.	**30.** 672 × $68.08.
9. 507 × 836.	**20.** 91 × 3473.	**31.** 360 × $127.60.
10. 802 × 906.	**21.** 17 × 4646.	**32.** 666 × $208.09.
11. 532 × 848.	**22.** 33 × 8308.	**33.** 565 × $1274.60.

The following work affords 420 additional exercises, to be used if desired. The teacher should select only as many as may be necessary.

34. Multiply each number by each number below it, and also multiply each number by each number to the right of it:

	I	II	III	IV	V	VI
a	243	307	442	581	226	822
b	376	476	208	622	642	344
c	405	921	179	389	379	296
d	221	330	643	427	496	473
e	392	407	827	462	577	892
f	470	920	906	330	640	487
g	689	821	432	448	807	745
h	573	139	507	229	488	307
i	349	427	572	946	399	700
j	763	396	378	278	420	970

Multiplying by Powers of 10. *To multiply a whole number by 10, annex a zero; to multiply by 100, annex two zeros; to multiply by 1000, annex three zeros.*

To multiply a number representing United States money by 10, move the decimal point one place to the right; to multiply by 100, move the decimal point two places to the right; to multiply by 1000, move the decimal point three places to the right.

Hence to multiply 27 by 40, multiply by 4 and annex 0.

To multiply $25.16 by 300, multiply by 3 and move the decimal point two places to the right.

The reasons involved in the above are probably familiar to the pupils. If not, they should be discussed informally. A formal explanation by the pupils is not necessary.

Aliquot Part. A number that will exactly divide another is called an *aliquot part* of that other number.

For example, 2 and 5 are aliquot parts of 10, and 2, 4, 5, 10, 20, 25, and 50 are aliquot parts of 100.

Therefore 2¢, 4¢, 5¢, 10¢, 20¢, 25¢, and 50¢ are aliquot parts of $1. To these may be added $33\frac{1}{3}$¢, $16\frac{2}{3}$¢, and certain other aliquot parts which will be considered later.

Since $5=\frac{10}{2}$, $50=\frac{100}{2}$, and $25=\frac{100}{4}$, we have the following:

To multiply by 5, multiply by 10 and divide by 2.
To multiply by 50, multiply by 100 and divide by 2.
To multiply by 25, multiply by 100 and divide by 4.

For example, consider the following:

$$5 \times 648 = \tfrac{1}{2} \text{ of } 6480 = 3240.$$
$$50 \times 752 = \tfrac{1}{2} \text{ of } 75{,}200 = 37{,}600.$$
$$25 \times 984 = \tfrac{1}{4} \text{ of } 98{,}400 = 24{,}600.$$

Similar rules are easily found for 500, 250, 75, and 750.

USING WHAT YOU HAVE LEARNED

Numbers 1 to 15, oral

1. At \$6 each, how much will 30 chairs cost?

2. At \$12 each, how much will 40 tables cost?

3. At \$15 each, how much will 10 writing desks cost?

4. If a railroad train goes 40 mi. an hour, how far will it go in 20 hr. at the same rate? in 25 hr. at the same rate?

5. If it costs \$15 a year to educate you, how much does it cost to educate 200 pupils at the same rate?

Multiply the following:

6. 50×7.	8. 60×6.	10. 80×11.	12. 90×20.
7. 30×12.	9. 40×11.	11. 90×10.	13. 90×100.

14. At \$84 an acre, how much will 7 acres of land cost?

Teach the pupils to multiply from the left, thinking of 7×84 as $560 + 28$, or 588.

15. State the value of 7×35; 8×42; 7×33; 6×45.

16. At \$1575 each, how much will 50 automobiles cost?

17. At \$215.50 each, how much will 3000 machines cost?

18. If 2 M (2 thousand) feet of white pine cost \$164.50, how much will 8 M cost?

Pupils should appreciate the importance of recognizing the *relations of numbers*. In this example they should see that 8 M is 4×2 M, and should simply multiply \$164.50 by 4.

19. In Ex. 18 how much will 24 M cost?

Teachers should accustom the pupils to estimate an answer in advance and to write it down. Then if they find a great difference between their result and their estimate, they have either shown poor judgment or made an error in their work. Many absurd answers can be avoided in this way.

USING WHAT YOU HAVE LEARNED

1. If each of the 88,756 persons in a certain county saves on the average $25 a year in food, how much is the total annual saving?

2. If there are 2860 soldiers in a certain regiment and the government pays 45¢ a day for food for each soldier, what is the cost of the food for the regiment during the month of June?

3. When wheat advanced in price from $1.25 per bushel to $2.65 per bushel, how much additional money did a farmer receive for his wheat on 90 acres of land yielding 23 bu. per acre?

4. The average amount of meat consumed annually by each person in this country is 194 lb. If 91 lb. per person is sufficient, what is the cost of the difference at 25¢ per pound?

5. Mr. Mann sold 40 barrels of apples to Mr. Smith for $3.50 a barrel at the orchard. Mr. Smith shipped them to a dealer in the city and figured the cost of shipping and his profit at $1.17 a barrel. How much did the dealer pay Mr. Smith? How much more did the dealer pay for all the apples than Mr. Smith paid Mr. Mann?

6. A man paid $57.50 a month for a flat in the city and furnished his own heat. To secure this heat he used in a year 9 tons of coal at $12.50 a ton. The next year he moved into a steam-heated apartment for which he paid $88.50 a month. Find the increase in his expenses for a year.

7. A man bought for a boarding house 80 bu. of potatoes. During the winter he lost 15¢ worth of potatoes out of every bushel by their rotting. How much money did he lose through the rotting of the potatoes?

DRILL WORK IN MULTIPLICATION

Multiply, in turn, by 2, 20, and 200:

1. 245.	3. 287.	5. 509.	7. 827.
2. $2.45.	4. $2.87.	6. $50.90.	8. $82.70.

Multiply, in turn, by 3, 30, and 300:

9. 682.	11. 725.	13. 933.	15. 640.
10. $6.82.	12. $72.50.	14. $93.30.	16. $6.40.

Multiply, in turn, by 40, 400, 50, and 500:

17. 536.	19. 721.	21. 663.	23. 842.
18. $6.35.	20. $5.93.	22. $4.34.	24. $3.81.

25. Multiply $734.65 by 9; by 90; by 900; by 970.
26. Multiply $875.40 by 8; by 80; by 800; by 875.
27. Multiply $624.45 by 7; by 35; by 305; by 350.
28. Multiply $946.60 by 6; by 60; by 3; by 300; by 600.
29. Multiply by 50: 237, 825 ft., $28.50, $880.40.
30. Multiply by 25: 134, 902 ft., $39.60, $784.40.
31. Multiply by 500: 16, 64, 726, $32.80, $624.40.

Multiply, in turn, 50¢, 25¢, 20¢, and 10¢ by:

32. 40. 33. 200. 34. 160. 35. 240. 36. 360. 37. 4800.

Perform the following multiplications:

38. 210 × $346.80.	43. 320 × $6218.
39. 800 × 2932 ft.	44. 700 × 3864 yd.
40. 9600 × $5400.	45. 4300 × $9250.
41. 7000 × $215.42.	46. 9000 × $421.77.
42. 6520 × $412.50.	47. 2750 × $231.08.

Nature of Division. In multiplication two factors are given to find the product. In division the product and one of the two factors are given to find the other factor.

That is, in the case of $2 \times \$5 = \$10$ we have given the factors 2 and \$5, from which to find the product \$10.

In division we have given the product \$10 and the factor 2, to find the other factor \$5; or we have given the product \$10 and the factor \$5, to find the other factor 2.

Upon the question of abstract and concrete numbers here and on page 31 the teacher should consult pages 22 and 294.

Division. If the product of two factors and one of the factors are given, the process of finding the other factor is called *division.*

Symbols. There are two common symbols used in division. Besides the symbol $\div$ we use the fraction form. Thus, we may write $15 \div 5 = 3$, or $\frac{15}{5} = 3$. That is, regarded as operations to be performed, $15 \div 5$ and $\frac{15}{5}$ have the same meaning.

Dividend. The given product, that is, the number to be divided, is called the *dividend.*

Divisor. The given factor, that is, the number by which we divide, is called the *divisor.*

Quotient. The required factor, that is, the number found by division, is called the *quotient.*

Remainder. Often the divisor does not exactly divide the dividend. Then that part of the dividend which is left is called the *remainder.*

Thus, $14 \div 3 = 4$ with remainder 2. The quotient is usually written $4\frac{2}{3}$, meaning 4 with a remainder 2 which still is to be divided into 3 equal parts.

Such terms, long familiar to you, are given only for a review.

Two Kinds of Division. Since $2 \times \$4 = \$8$, it is evident that $\$8 \div \$4 = 2$ and $\$8 \div 2 = \$4$. That is,

If the dividend and the divisor are both concrete, they must be like numbers, and the quotient is abstract.

If the dividend is concrete and the divisor abstract, the quotient is concrete and like the dividend.

Check. To check the work in division, we multiply the quotient by the divisor; this product added to the remainder, if there is one, should be equal to the dividend.

For example, $45 \div 5 = 9$; then $5 \times 9 = 45$. Again, $48 \div 5 = 9$ with remainder 3; then $5 \times 9 = 45$, and $45 + 3 = 48$.

Short Division. When the divisor is so small that the work may be done mentally, the process of *short division* is used.

1. Divide 42,364 by 7.

Here 7 is not contained in 4, but 7 is contained in 42 six times. Then 7 is not contained in 3, so we write a 0 below the 3. Then 7 is contained in 36 five times, with remainder 1, and in 14 twice. Therefore the quotient is 6052.

$$7\overline{)42364}$$
$$6052$$

Some teachers write the quotient above the dividend in short division, but it is not the business custom and it is inconvenient in elaborate computations requiring successive divisions.

2. Divide \$27.53 by 4.

Dividing as before, we have $\$27 \div 4 = \$6$ with \$3 remaining; then $\$3.50 \div 4 = \$0.80$ with \$0.30 remaining; and $\$0.33 \div 4 = \$0.08$ with \$0.01 remaining. Therefore the quotient may be written $\$6.88\frac{1}{4}$. In practical problems it is the custom to neglect any fraction less than $\frac{1}{2}¢$ in the quotient and to call a full cent any fraction $\frac{1}{2}¢$ or greater.

$$4\overline{)\$27.53}$$
$$\$6.88\tfrac{1}{4}$$

ORAL DRILL WORK IN SHORT DIVISION

Divide the following, stating the results as rapidly as you can:

1.	2)20	2)24	2)42	2)48	2)84
2.	3)21	3)36	3)63	3)96	3)69
3.	4)24	4)36	4)48	4)84	4)56
4.	5)35	5)45	5)55	5)60	5)80
5.	6)36	6)42	6)54	6)72	6)96
6.	7)42	7)49	7)63	7)84	7)91
7.	8)56	8)64	8)88	8)96	8)104
8.	9)63	9)81	9)27	9)45	9)144
9.	7)98	4)96	5)85	3)51	4)680
10.	4)52	3)60	2)94	4)68	6)222
11.	4)72	6)78	3)57	5)95	8)144
12.	6)82	7)60	8)91	9)75	11)121

13. At 12¢ each, how many grapefruit can you buy for 72¢?

14. At 12¢ each, how many grapefruit can you buy for $1, and how much money will you have left?

15. At 8¢ each, how many oranges can you buy for 75¢, and how much money will you have left?

16. How many yards are there in 144 ft.?

17. How many feet are there in 250 in., and how many inches are left over?

Dividing by Powers of 10. To divide a number by 10 is to find how many tens there are in the number.

Thus, in 1230 there are 123 tens and 0 units, and in 2373 there are 237 tens and 3 units; that is, $2373 \div 10 = 237\frac{3}{10}$.

To divide a whole number by 10, cut off the right-hand figure of the dividend. The result is the quotient, and the right-hand figure expresses the remainder.

To divide a whole number by any power of 10, cut off as many figures from the right of the dividend as there are zeros at the right of the divisor. The result is the quotient, and the right-hand figures express the remainder.

The remainder may be written over the divisor as part of the quotient, the fraction being expressed in lowest terms.

Thus, $12{,}400 \div 100 = 124$, and $736{,}117 \div 1000 = 736\frac{117}{1000}$.

Tell the pupils that they will soon study decimals and will then find a much easier way of expressing such a fraction as $\frac{117}{1000}$.

Dividing by Multiples of Powers of 10. We divide by the power of 10, and then by the number indicating the multiple.

To divide 42,800 by 200 we cut off two zeros from the divisor and two zeros from the dividend and then divide by 2.

To divide 27,400 by 300, we cut off the zeros as above and then divide by 3. The integral quotient is 91 with remainder 1 in hundreds' place, and $\frac{100}{300} = \frac{1}{3}$.

To divide 27,407 by 300, we cut off two zeros from the divisor and two figures from the right of the dividend and then divide by 3. The integral quotient is 91 and there is a remainder 1 in hundreds' place plus 7, or 107.

$2\not{0}\not{0})\,428\not{0}\not{0}$
214

$3\not{0}\not{0})\,274\not{0}\not{0}$
$91\frac{1}{3}$

$3\not{0}\not{0})\,274\not{0}\not{7}$
$91\frac{107}{300}$

USING WHAT YOU HAVE LEARNED

1. How many hours are there in 1680 minutes?

2. How many minutes are there in 1740 seconds?

3. At $60 each, how many sets of harness must a manufacturer sell to receive $1860?

4. A dealer sells some city lots for $53,100 at $900 a lot. How many lots does he sell?

5. A stock farmer decides to spend $6880 in adding new stock to his farm. At $80 apiece how many cows can he buy for that amount?

6. An orange grower has 8460 oranges which he wishes to ship in boxes of 100 oranges each. How many boxes can he ship, and how many oranges will he have left over?

7. If a coal pocket contains 786,400 lb. of coal, how many tons (2000 lb.) does it contain, and how much over?

8. A dealer has $3850 with which to buy some village lots at $600 each. What is the greatest number he can buy, and how much money will he have left?

9. An automobile dealer has $85,000 to invest in automobiles at $2000 each. What is the greatest number he can buy for this sum, and how much money will he have left?

Find the quotients, and the remainders if any:

10. $37{,}800 \div 200$. 12. $92{,}400 \div 400$. 14. $229{,}000 \div 4000$.

11. $54{,}300 \div 300$. 13. $438{,}000 \div 3000$. 15. $745{,}500 \div 5000$.

16. In laying a floor in a corridor 80 ft. long a carpenter uses boards 16 ft. long. There are 18 boards in the width of the corridor. Allowing for the loss of one board in matching, how many boards does it take?

Long Division. We shall now consider the more general case of dividing by a number of two figures.

Thomas sells papers with another boy. At their news stand they sold 49,820 papers in 52 wk. To the nearest whole number, what was the average number that they sold per week?

If they sold 49,820 papers in 52 wk., in 1 wk. they sold on the average 49,820 papers $\div$ 52.

The divisor is not contained in 4 or in 49, and so there are no ten-thousands and no thousands in the quotient.

Since $9 \times 52 = 468$, the divisor is contained nine times in 498, with remainder 30. Since this is hundreds, the quotient figure 9 is written over hundreds' place. Bringing down the next figure of the dividend, 2, we have 302 tens to be divided.

Since $5 \times 52 = 260$, the divisor is contained five times in 302, with remainder 42. Since this is tens, the quotient figure 5 is written over tens' place. Bringing down the next figure of the dividend, 0, we have 420 still to be divided.

```
   958
52)49820
   468
   ---
   302
   260
   ---
   420
   416
   ---
     4
```

Since $8 \times 52 = 416$, the units' figure of the quotient is 8 and there is a remainder of 4. Therefore the quotient is 958 and the remainder is 4.

Therefore $49{,}820 \div 52 = 958$ to the nearest whole number, and so the average number of papers sold per week is 958.

Check. $52 \times 958 = 49{,}816$ and $49{,}816 + 4 = 49{,}820$.

In the example above, the remainder is 4, which we may write as $\frac{4}{52}$, or $\frac{1}{13}$ in lowest terms. If the remainder had been 26 or greater, the fraction would be $\frac{1}{2}$ or more in lowest terms, and we should say that the average number of papers sold is 959.

DRILL WORK IN DIVISION

Perform the following divisions:

1. $882 \div 21$.	6. $1924 \div 52$.	11. $1419 \div 33$.
2. $744 \div 31$.	7. $2976 \div 62$.	12. $2838 \div 43$.
3. $1558 \div 41$.	8. $4032 \div 72$.	13. $4851 \div 63$.
4. $2142 \div 51$.	9. $6724 \div 82$.	14. $2701 \div 73$.
5. $4686 \div 71$.	10. $6900 \div 92$.	15. $9207 \div 93$.

Teachers will find it advantageous to begin this review drill with cases like those above, where each divisor ends in 1, 2, or 3 and each quotient is integral. It is desirable to put a time limit upon all such drill work, keeping the records of the pupils. Do not make the rapid drill too long.

The next point of difficulty involves divisors ending in 9 or 8, as in Exs. 16–28; the next, any divisors, as in Exs. 29–41; and the next, cases in which there is possibly a remainder, as in Exs. 42–54.

Perform the following divisions:

16. $957 \div 29$.	29. $2376 \div 36$.	42. $4375 \div 76$.
17. $1638 \div 39$.	30. $1406 \div 37$.	43. $8291 \div 78$.
18. $2597 \div 49$.	31. $2070 \div 45$.	44. $6007 \div 19$.
19. $4248 \div 59$.	32. $4140 \div 46$.	45. $9009 \div 33$.
20. $2553 \div 69$.	33. $3472 \div 56$.	46. $5402 \div 69$.
21. $5372 \div 79$.	34. $8280 \div 23$.	47. $7353 \div 68$.
22. $8722 \div 89$.	35. $6231 \div 67$.	48. $7040 \div 64$.
23. $1596 \div 38$.	36. $2730 \div 78$.	49. $8296 \div 75$.
24. $4466 \div 58$.	37. $3472 \div 28$.	50. $5379 \div 86$.
25. $5372 \div 68$.	38. $3476 \div 44$.	51. $4381 \div 95$.
26. $7744 \div 88$.	39. $4984 \div 56$.	52. $3107 \div 79$.
27. $8722 \div 98$.	40. $7656 \div 88$.	53. $6234 \div 87$.
28. $6396 \div 78$.	41. $7990 \div 85$.	54. $8763 \div 91$.

USING WHAT YOU HAVE LEARNED

1. If $1668 is divided equally among a dozen persons, how much is the share of each?

2. If $6180 is divided equally among fifteen persons, how much is the share of each?

3. If the dividend is 840, the quotient 64, and the remainder 8, what is the divisor?

4. If the quotient is 367, the divisor 12, and the remainder 7, what is the dividend?

5. If the divisor is 36, the quotient 14, and the remainder 23, what is the dividend?

6. At the rate of 200 mi. a day, how long would it take a freight steamer to go from San Francisco to Guam, 5200 mi.?

7. The product of two numbers is 98,441, and one of them is 49. What is the other number?

8. A canning factory ships 60,000 cans of fruit in boxes containing 4 doz. cans each. How many boxes are required?

9. If the circumference of a wagon wheel is 16 ft., and there are 5280 ft. in a mile, how many times will the wheel turn while the wagon is going 14 mi.?

10. In a recent year the price of coal was $8 a ton in April, and it advanced 25¢ on the first of each month thereafter. How much did a man save on the price if he bought 25 tons in April instead of waiting until September?

11. There being 27 cu. ft. in a cubic yard, how many loads (cubic yards) of earth must a contractor remove in order to excavate 135,675 cu. ft.?

12. If 137 cotton mills in a recent year used 382,026,915 lb. of cotton, what was the average per mill to the nearest pound?

Long Division of United States Money. In dividing United States money, we divide as with other numbers, placing the decimal point in the quotient above that in the dividend.

1. A manufacturer sold 372 harnesses and received $21,375.12 in payment. What was the price of each harness?

Each harness cost $21,375.12 ÷ 372.

The operation is as follows:

2137 ÷ 372 = 5, with remainder 277;

2775 ÷ 372 = 7, with remainder 171;

1711 ÷ 372 = 4, with remainder 223;

2232 ÷ 372 = 6.

$$
\begin{array}{r}
\$57.46 \\
372\overline{)\$21375.12} \\
1860 \\
\hline
2775 \\
2604 \\
\hline
1711 \\
1488 \\
\hline
2232 \\
2232 \\
\hline
\end{array}
$$

The decimal point in the quotient is placed above that in the dividend.

Therefore $21,375.12 ÷ 372 = $57.46, the price of each harness.

Check. 372 × $57.46 = $21,375.12.

2. If a mill sells 25 yd. of cloth for $136, what price per yard did the mill receive for the cloth?

The price per yard is found by dividing $136 by 25.

We write $136 as $136.00 and divide as explained above.

We find that $136.00 ÷ 25 = $5.44, the price per yard.

Check. 25 × $5.44 = $136.

DIVIDING UNITED STATES MONEY

Divide the following:

1. $974.58 ÷ 111.
2. $772.12 ÷ 199.
3. $3867.50 ÷ 25.
4. $296 ÷ 40.
5. $128 ÷ 25.
6. $385 ÷ 125.
7. $87,328.80 ÷ 27.
8. $97,747.20 ÷ 64.
9. $39,994.56 ÷ 81.

DRILL WORK IN DIVISION

Perform the following divisions:

1. 55,685 ÷ 301.
2. 53,004 ÷ 631.
3. 48,594 ÷ 623.
4. 364,397 ÷ 211.
5. 976,950 ÷ 501.
6. 972,471 ÷ 111.
7. 161,784 ÷ 321.
8. 324,360 ÷ 306.
9. 140,850 ÷ 225.
10. 183,750 ÷ 175.
11. 148,555 ÷ 407.
12. 234,901 ÷ 503.
13. 208,575 ÷ 309.
14. 870,672 ÷ 204.
15. $3678.48 ÷ 786.
16. $1024.65 ÷ 495.
17. $4729.32 ÷ 604.
18. $3492.06 ÷ 407.
19. $1174.40 ÷ 320.
20. $4196.40 ÷ 538.
21. $2877.12 ÷ 216.
22. $6153.28 ÷ 328.
23. $9715.42 ÷ 286.
24. $9975.86 ÷ 478.
25. $9657.42 ÷ 426.
26. 81,231 ÷ 44.
27. 17,268 ÷ 37.
28. 92,357 ÷ 28.
29. 36,452 ÷ 49.
30. 88,757 ÷ 56.
31. 41,795 ÷ 56.
32. 79,145 ÷ 49.
33. 36,523 ÷ 48.
34. 67,558 ÷ 59.
35. 87,073 ÷ 66.
36. 90,601 ÷ 72.
37. 401,121 ÷ 332.
38. 210,379 ÷ 224.
39. 852,191 ÷ 315.
40. 720,830 ÷ 853.
41. 360,389 ÷ 427.
42. 692,823 ÷ 236.
43. 811,423 ÷ 2878.
44. 487,712 ÷ 4327.
45. 913,204 ÷ 3469.
46. 311,075 ÷ 2481.
47. 716,673 ÷ 6008.
48. 561,685 ÷ 9852.
49. 8,599,571 ÷ 5838.
50. 3,462,428 ÷ 3259.

USING WHAT YOU HAVE LEARNED

1. A wagon dealer pays $1680 for 12 wagons. At this rate, how much do 3 wagons cost? How much do 9 wagons cost?

2. At the rate of $2040 for 16 acres of land, how much will a man have to pay for 8 acres? for 32 acres? for 100 acres?

3. If a stock dealer buys some cattle for $22,100, paying $68 a head, how many head of cattle does he buy?

4. If a jobber sells during the year $164,822 worth of shoes at $122 a case, how many cases does he sell?

5. If a dealer sells $246,897 worth of linseed oil during a year at $63 a barrel, how many barrels does he sell?

6. If a factory sells 7368 ready-made suits of a certain grade for $184,200, what is the price per suit?

7. If a state pays $4,651,596 for constructing 942 mi. of model road, what is the average cost per mile?

8. If it costs $10,473,204 to construct 574 mi. of railroad, what is the average cost per mile?

9. If a company employs 16,018 men on the same schedule of wages, and the pay roll for these men is $28,031,500 a year, find the annual wages of each man.

10. If a company employs 3840 men on the same schedule of wages, and the pay roll for these men is $6,336,000 a year, find the monthly wages of each man.

11. A shop has 1328 men on its pay roll, all receiving the same wages. For 6 da. this pay roll amounts to $43,824. How much does each man receive on the average a day?

12. In a certain savings bank 116,180 persons have deposited $39,827,991. Find to the nearest cent the average deposit of each person.

FINDING PRICES, WAGES, AND COST

1. Under a certain contract a boss carpenter paid 27 men $2193.75 for 13 da. work, all the men receiving the same wages. Find the daily wages of each.

2. A dealer sold 3 sets of furniture each week for 4 wk. and received $825. The sets were all sold at the same price per set. Find the price at which each set was sold.

3. A merchant bought 1 doz. suits of clothes and found they were in such demand that he bought 2 doz. more of the same kind, the prices being the same. He paid $1157.40 for all the suits. How much did he pay per suit?

4. A manufacturer has 240 men on his pay roll, the wages of all being the same. He pays them $6900 in 5 da. Find the daily wage per man.

5. In a certain school of 43 pupils the teacher's salary is $1100 a year. The cost of fuel is $164.80; of books and other school supplies, $148.75; of repairs and improvements, $86.29. Find the total cost per pupil for the year.

6. In a certain city a man's gas bill for a certain month is $4.25. He uses 5000 cu. ft. of gas in that month. Find how much he has to pay per 1000 cu. ft.

7. The clerks in a certain store bought 17 Government Savings Stamps in a certain month, paying $71.91 for the lot. What was the price of each stamp?

8. The men in a certain railroad shop paid $3299.40 for 39 Treasury Savings Certificates. How much did they pay for each certificate?

9. A wholesale dealer paid $40,150 for 7300 pairs of shoes. If he sold the shoes for $6.25 a pair, how much more did he receive for them than he paid?

USING WHAT YOU HAVE LEARNED

1. If 27 acres of land are worth $2227.50, what is the average value per acre? How much are 4 acres worth?

2. If a dealer pays $413.42 for 14 suits of ready-made clothes, how much does he pay per suit? At the same rate, how much would he pay for 7 suits? for 28 suits? for 21 suits?

3. A man pays $1430 for the rent of an apartment in the city for 11 mo. At this rate, how much does he pay for 5 mo.? for 7 mo.? for 3 mo.? for 8 mo.?

4. A man bought 7 M ft. of lumber at $48.50 per M. He sold half the lumber at $8.75 per C, and the other half at $47.50 per M. How much did he gain?

The teacher should explain that 7 M means 7000, and $8.75 per C means $8.75 per hundred feet. Lumber is usually sold by the M; that is, by the 1000 feet. The method of measuring lumber is considered later in the course.

5. At an athletic meet 860 spectators bought tickets at 45¢ each, and 1240 at 25¢ each. What is the average amount paid per spectator?

In all such cases give the result to the nearest cent.

6. An owner made a net profit of $865 on one house, $540 on a second, $690 on a third, and lost $275 on a fourth. What was his average profit per house?

Net profit means the profit after all expenses have been paid.

7. At 60 lb. to the bushel, how many bushels in 28,750 lb. of wheat, and how many pounds over?

8. An experiment station reports that the corn raised on an acre of good land should contain 147 lb. of fat. How many acres would be needed to produce 5586 lb. of fat?

The chief ingredients in corn are protein, carbohydrates, and fat.

II. COMMON FRACTIONS

MEANING OF FRACTIONS

All work oral

1. Which is the greater, $\frac{1}{2}$ of an apple or $\frac{1}{3}$ of the same apple?

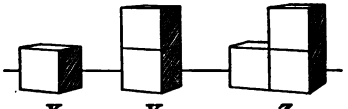
2. Which is the greater, 2 or 3? $\frac{1}{2}$ or $\frac{1}{3}$? 3 or 4? $\frac{1}{3}$ or $\frac{1}{4}$?

3. Which number is the greatest, 2, 3, or 4? Which fraction is the greatest, $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$? Which fraction is the least?

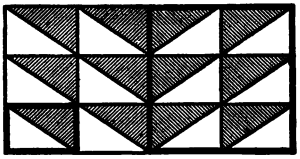
4. In the figures here shown, what fractional part of figure *A* is shaded? What part of figure *B* is shaded? What part of figure *C* is shaded?

5. From figure *A* we see that $\frac{1}{2}$ is equal to how many sixths? From figure *B* we see that $\frac{1}{3}$ is equal to how many sixths?

6. In the figures here shown, *X* is what part as large as *Y*? *X* is what part as large as *Z*? *Z* is how many times *X*? *Y* is what part as large as *Z*? *Z* is how many times *Y*?



7. Each of these small rectangles is what part of the large rectangle? Each of these shaded triangles is what part of the large rectangle? What part of the rectangle is $\frac{1}{2}$ of $\frac{1}{12}$ of it? What part is $\frac{1}{12}$ of $\frac{1}{2}$ of it? How much is $\frac{1}{2}$ of $\frac{1}{12}$ of anything? How much is $\frac{1}{12}$ of $\frac{1}{2}$ of anything?



This page is typical of the work with objects that should introduce this review of fractions.

The definitions on page 44 should not be learned verbatim, but the teacher should be certain that the pupils can use the terms correctly.

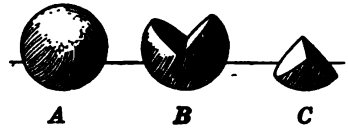
Fraction. A number which shows how many equal parts of anything are taken is called a *fraction.*

Explain that $\frac{3}{4}$ shows that 3 of the 4 equal parts are taken.

Denominator. The number in a fraction which shows into how many equal parts anything has been divided is called the *denominator* of the fraction.

Numerator. The number in a fraction which shows how many equal parts of anything have been taken is called the *numerator* of the fraction.

In this figure, *B* contains three of the four equal parts of a sphere; that is, *B* is $\frac{3}{4}$ of a sphere, 4 being the denominator and 3 the numerator.



Terms. The numerator and the denominator together are called the *terms* of the fraction.

Common Fraction. A fraction like $\frac{3}{4}$, in which both terms are expressed, is called a *common fraction.*

The teacher should explain that we write $\frac{25}{100}$ of a dollar thus, \$0.25; and that in the case of \$0.25 both terms of the fraction are not expressed.

Unit Fraction. A fraction like $\frac{1}{3}$, in which the numerator is 1, is called a *unit fraction.*

Proper and Improper Fractions. A fraction like $\frac{3}{5}$, with the numerator less than the denominator, is called a *proper fraction.*

A fraction like $\frac{4}{4}$ or $\frac{5}{4}$, in which the numerator equals or exceeds the denominator, is called an *improper fraction.*

Integer. A number like 7, that is, a number expressed without fractions, is called an *integer.*

Mixed Number. A number like $2\frac{3}{4}$, composed of an integer and a fraction taken together, is called a *mixed number.*

READING AND WRITING FRACTIONS

Numbers 1 to 12, oral

1. If Irene lives $\frac{1}{2}$ mi. from school, and Margaret lives $\frac{1}{4}$ mi. from school, and Jennie lives $\frac{3}{4}$ mi. from school, which lives nearest the school? Which next? Which the farthest away?

2. In Ex. 1 tell the number of parts into which the mile has been divided in each case and the number of these parts taken to make the fraction.

Read the following aloud:

3. $\frac{3}{8}$. 5. $\frac{2}{5}$. 7. $\frac{3}{16}$. 9. $12\frac{1}{2}$. 11. $8\frac{5}{32}$.

4. $\frac{5}{8}$. 6. $\frac{7}{8}$. 8. $\frac{7}{16}$. 10. $33\frac{1}{3}$. 12. $9\frac{7}{64}$.

The fractions most commonly used in commercial transactions have as denominators 2, 3, 4, 8, 12, and 16; those most commonly used in measurements in machine work and similar industries have, in addition to these, the denominators 24, 48, 32, 64, and 96.

Write the following in figures:

13. One half. 15. Two thirds. 17. Five sixths.

14. One fourth. 16. Three fifths. 18. Five eighths.

Write the following in words:

19. $\frac{5}{8}$. 21. $\frac{5}{16}$. 23. $\frac{7}{12}$. 25. $2\frac{3}{8}$. 27. $9\frac{15}{32}$.

20. $\frac{7}{8}$. 22. $\frac{9}{16}$. 24. $\frac{5}{24}$. 26. $3\frac{7}{8}$. 28. $7\frac{21}{64}$.

Write the following in words and also write the number of dollars and cents that would be taken in making out bills:

29. \$25.12$\frac{1}{2}$. 30. \$6.66$\frac{2}{3}$. 31. \$33.37$\frac{1}{2}$. 32. \$75.16$\frac{7}{8}$.

In a result follow the business custom of neglecting any fraction of a cent less than $\frac{1}{2}$, and calling a fraction of a cent $\frac{1}{2}$ or greater a full cent. Some business concerns call any fraction of a cent a full cent, but this custom is not to be followed in this book.

CHANGING THE NAME OF A FRACTION

All work oral

1. Look at circle *A* and state how many sixths of the circle you see in $\frac{1}{2}$ of the circle.

2. How many sixths of a pie are there in $\frac{1}{2}$ of a pie?

Explain that we have simply changed the name of $\frac{1}{2}$ (half) to sixths.

3. Look at the circles marked *B* and state how many eighths of the circle you see in $\frac{1}{2}$ of the circle; in $\frac{3}{4}$ of the circle.

4. How many eighths of an inch are there in $\frac{1}{2}$ in.?

5. Look at the circles marked *C* and state how many tenths of the circle you see in $\frac{1}{2}$ of the circle; in $\frac{4}{5}$ of the circle.

6. How many tenths of an inch are there in $\frac{1}{2}$ in.? in $\frac{1}{5}$ in.? in $\frac{2}{5}$ in.? in $\frac{4}{5}$ in.?

7. From the figure below, $\frac{1}{2}$ is equal to how many eighths? how many fourths?

8. From the figure, $\frac{6}{8}$ is equal to how many fourths?

9. If you call the above rectangle *one*, how many halves do you see in 1? How many fourths? How many eighths?

10. How many fourths do you see in $\frac{1}{2}$? in 1?

11. How many eighths do you see in $\frac{1}{4}$? in $\frac{1}{2}$? in 1?

12. How many fifths are there in 1? in 2? in 3?

13. How many sixths are there in 1? in 2? in 5? in 10?

Reducing a Fraction. From the examples which we have just studied we see that

Both terms of a fraction may be multiplied by the same number without changing the value of the fraction.

Both terms of a fraction may be divided by the same number without changing the value of the fraction.

When we change the terms of a fraction without changing the value of the fraction, we *reduce* the fraction.

For example, when we say that $\frac{1}{2} = \frac{2}{4}$, we reduce the $\frac{1}{2}$ to fourths. When we say that $\frac{6}{9} = \frac{2}{3}$, we reduce the $\frac{6}{9}$ to thirds.

Lowest Terms. When both terms cannot be divided exactly by the same number, the fraction is said to be *in lowest terms.*

A fraction is *reduced to lowest terms* when both terms cannot be divided exactly by the same number.

For example, $\frac{4}{8}\frac{4}{8} = \frac{2}{4}\frac{2}{4} = \frac{1}{2}\frac{1}{2}$, and $\frac{4}{8}\frac{4}{8}$ is reduced to lowest terms when reduced to $\frac{1}{2}\frac{1}{2}$.

Prime Number. A number which has no factor except itself and 1 is called a *prime number.*

Explain that 3 is a factor of 12 as defined on page 22, and that 3 is a prime number.

State to the class that in a case like $\frac{1}{1}\frac{1}{2}$ the terms are sometimes said to be *prime to each other.*

Prime Factor. When a factor is a prime number, it is called a *prime factor.*

Explain that 2 and 3 are prime factors of 12 and that 4 is also a factor of 12 but not a prime factor.

Common Factor. A factor of two or more numbers is called a *common factor* or a *common divisor* of the numbers.

Show that 3 is a common factor of 12, 15, and 45; but that 5, although a common factor of 15 and 45, is not a common factor of 12, 15, and 45.

Canceling Factors. In making a toy airplane Frank found that he must use $\frac{8}{12}$ of a piece of canvas. This seemed to him an awkward fraction to use. How can he easily reduce it to a simpler fraction?

He remembers that he can divide both terms by the same number without changing the value of the fraction, and he sees that he will save himself labor if he divides by the largest factor that is found in both terms; that is, by the largest factor *common* to both terms. This factor is 4; therefore

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}, \text{ lowest terms.}$$

In this example the factor 4 is said to be *canceled* from both terms when the work is written as here shown.

$$\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}} = \frac{2}{3}$$

Reducing to Lowest Terms. *To reduce a fraction to lowest terms, cancel all factors common to both terms.*

In practice we can tell what factors to try, for they are usually 2, 3, or 5, these factors being repeated in many numbers.

REDUCING TO LOWEST TERMS

Reduce to lowest terms:

1. $\frac{4}{8}$. 2. $\frac{4}{6}$. 3. $\frac{8}{9}$. 4. $\frac{6}{10}$. 5. $\frac{4}{12}$. 6. $\frac{6}{15}$.

Simplify the following fractions occasionally met in quotients:

7. $\frac{72}{104}$. 8. $\frac{60}{105}$. 9. $\frac{72}{144}$. 10. $\frac{111}{123}$. 11. $\frac{65}{70}$. 12. $\frac{385}{700}$.

Such fractions as those given in Exs. 7 to 12 are not used in business, but since we meet them in division it is necessary to know how to reduce them to lowest terms.

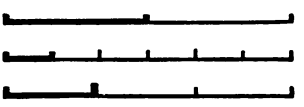
Teachers should make it clear that this cancellation is a case of *division*; that we cannot *subtract* the same number from both terms of the fraction.

REDUCING FRACTIONS

Numbers 1 to 12, oral

1. Calling the line *one*, how many sixths do you see in $\frac{1}{2}$?

2. 1 = how many sixths?



3. How many sixths do you see in $\frac{1}{3}$? in $\frac{2}{3}$? in $\frac{3}{3}$? in 1?

Reduce these fractions as indicated:

4. $\frac{1}{2}$ to eighths. **7.** $\frac{1}{2}$ to fourths. **10.** $\frac{4}{5}$ to tenths.

5. $\frac{1}{2}$ to tenths. **8.** $\frac{1}{4}$ to eighths. **11.** $\frac{3}{4}$ to twelfths.

6. $\frac{1}{2}$ to twelfths. **9.** $\frac{2}{6}$ to fourths. **12.** $\frac{6}{16}$ to eighths.

Reduce these fractions to halves:

13. $\frac{3}{6}$. **14.** $\frac{2}{4}$. **15.** $\frac{4}{4}$. **16.** $\frac{8}{4}$. **17.** $\frac{12}{4}$. **18.** $\frac{9}{6}$.

Reduce these fractions to fourths:

19. $\frac{1}{2}$. **20.** $\frac{6}{2}$. **21.** $\frac{18}{8}$. **22.** $\frac{24}{8}$. **23.** $\frac{28}{8}$. **24.** $\frac{32}{8}$.

Reduce these fractions to eighths:

25. $\frac{3}{4}$. **26.** $\frac{6}{6}$. **27.** $\frac{5}{2}$. **28.** $\frac{14}{4}$. **29.** $\frac{4}{16}$. **30.** $\frac{8}{16}$.

Reduce these fractions to twelfths:

31. $\frac{1}{3}$. **32.** $\frac{3}{4}$. **33.** $\frac{2}{3}$. **34.** $\frac{7}{6}$. **35.** $\frac{9}{6}$. **36.** $\frac{4}{24}$.

Reduce these fractions to lowest terms:

37. $\frac{6}{8}$. **38.** $\frac{8}{12}$. **39.** $\frac{3}{9}$. **40.** $\frac{5}{10}$. **41.** $\frac{6}{12}$. **42.** $\frac{5}{15}$.

43. Express $\frac{2}{3}$ in. and $\frac{1}{2}$ in. as sixths of an inch, and in this way find which is the greater.

44. Express $\frac{2}{3}$ in. and $\frac{3}{4}$ in. as twelfths of an inch, and in this way find which is the greater.

USING WHAT YOU HAVE LEARNED

1. Margaret needs 24 in. of cloth for some work she is doing. Her teacher tells her that this is the same as $\frac{24}{36}$ yd. This is an awkward fraction, and she sees that it would be easier to understand it if the terms were smaller. Reduce the fraction to lowest terms.

2. In measuring an iron rod in the shop, Jack found that the diameter was $\frac{14}{16}$ in. Express this in eighths of an inch. Is the diameter less than $\frac{1}{2}$ in.? Is it less than $\frac{3}{4}$ in.?

3. A piece of plate glass is $\frac{8}{32}$ in. thick. Express this fraction in lowest terms. Express the thickness in eighths of an inch; in sixteenths of an inch.

4. A man ordered some iron strips $\frac{1}{8}$ in. thick and some that were $\frac{5}{32}$ in. thick. Reduce $\frac{1}{8}$ to thirty-seconds and show which strips were the thicker.

5. Which wire has the greater diameter, that which is $\frac{16}{64}$ in. across or that which is $\frac{3}{8}$ in. across? How do you know?

6. A workman has several sizes of wire. They are marked $\frac{8}{16}$ in., $\frac{14}{32}$ in., $\frac{3}{8}$ in., $\frac{8}{4}$ in., and $\frac{5}{8}$ in., these being their diameters. By reducing these fractions to thirty-seconds, arrange them in order of size, beginning with the smallest.

Reduce these fractions to sixty-fourths:

7. $\frac{7}{16}$. 8. $\frac{9}{16}$. 9. $\frac{9}{32}$. 10. $\frac{15}{32}$. 11. $\frac{16}{32}$. 12. $\frac{128}{128}$.

Reduce these fractions to thirty-sixths:

13. $\frac{2}{3}$. 14. $\frac{3}{4}$. 15. $\frac{1}{9}$. 16. $\frac{1}{6}$. 17. $\frac{5}{9}$. 18. $\frac{12}{24}$.

Reduce these fractions to twenty-fourths:

19. $\frac{7}{12}$. 20. $\frac{1}{3}$. 21. $\frac{2}{48}$. 22. $\frac{6}{48}$. 23. $\frac{5}{12}$. 24. $\frac{11}{12}$.

REDUCING TO AN IMPROPER FRACTION

Numbers 1 to 3, oral

1. How many halves of a circle do you see in one circle? in $1\frac{1}{2}$ circles? Express $\frac{3}{2}$ as a mixed number.

2. How many halves of a circle do you see in 2 circles? Express $\frac{4}{2}$ as a whole number.

3. How many thirds do you see in 1? in $1\frac{1}{3}$? in $1\frac{2}{3}$? in 2? Express $\frac{4}{3}$ and $\frac{5}{3}$ as mixed numbers; $\frac{6}{3}$ as a whole number.

4. How many fourths are there in 1? in $1\frac{1}{4}$? in $1\frac{3}{4}$? in 2? Write only the results, thus: $1 = \frac{4}{4}$, and so on.

5. How many fifths are there in 1? in $1\frac{1}{5}$? in $1\frac{2}{5}$? in $1\frac{3}{5}$?

6. How many sixths are there in 1? in $1\frac{1}{6}$? in $1\frac{5}{6}$? in 2? in $2\frac{1}{6}$? in $2\frac{5}{6}$? in 3?

7. How many sevenths are there in $1\frac{1}{7}$? in $2\frac{2}{7}$? in $3\frac{1}{7}$?

8. How many eighths are there in 1? in 2? in $2\frac{1}{8}$? in $2\frac{3}{8}$?

9. How many ninths are there in $1\frac{1}{9}$? in $2\frac{2}{9}$? in 3?

10. How many tenths are there in $3\frac{3}{10}$? in $3\frac{7}{10}$? in 4?

11. How many twelfths are there in 1? in 2? in 3? in 4? in $2\frac{1}{12}$? in $3\frac{5}{12}$? in $4\frac{7}{12}$? in $4\frac{11}{12}$?

12. How many sixteenths are there in 1? in 2? in 3? in 4? in $1\frac{1}{16}$? in $1\frac{3}{16}$? in $2\frac{1}{16}$? in $2\frac{5}{16}$? in $3\frac{7}{16}$? in $4\frac{9}{16}$?

Reducing to an Improper Fraction. The Red Cross workers need for bandages some strips of cloth 1 yd. long and $\frac{1}{8}$ yd. wide. Ruth has a piece of the cloth $4\frac{5}{8}$ yd. long and 1 yd. wide. To use this to best advantage she cuts it into pieces $\frac{1}{8}$ yd. wide. How many such pieces are there in $4\frac{5}{8}$ yd.?

Ruth found the number by reducing $4\frac{5}{8}$ to eighths, and this is how she did it:

Since $1 = \frac{8}{8}$, we see that $4 = \frac{4 \times 8}{8} = \frac{32}{8}$.

Therefore $4\frac{5}{8} = \frac{32}{8} + \frac{5}{8} = \frac{37}{8}$.

Having found that $4\frac{5}{8} = \frac{37}{8}$, Ruth went on and finished the solution. What was the result?

REDUCING TO AN IMPROPER FRACTION

1. How many pieces of cloth $\frac{1}{4}$ yd. long are there in $7\frac{3}{4}$ yd. of cloth? in $8\frac{1}{4}$ yd.? in $9\frac{3}{4}$ yd.? in $5\frac{1}{2}$ yd.?

2. How many packages of pepper, each weighing $\frac{1}{8}$ lb., can be put up from $3\frac{3}{8}$ lb. of pepper? from $4\frac{7}{8}$ lb.?

Reduce as indicated:

3. 3 to eighths.
4. 5 to thirds.
5. $2\frac{1}{2}$ to halves.
6. $3\frac{4}{5}$ to fifths.
7. $3\frac{2}{3}$ to thirds.
8. $4\frac{5}{6}$ to sixths.
9. $3\frac{1}{9}$ to ninths.
10. $2\frac{3}{5}$ to fifths.
11. $3\frac{1}{3}$ to thirds.
12. $4\frac{2}{5}$ to fifths.
13. $5\frac{4}{9}$ to ninths.
14. $6\frac{3}{5}$ to fifths.
15. $7\frac{2}{3}$ to thirds.
16. $8\frac{5}{9}$ to ninths.
17. $3\frac{2}{9}$ to ninths.
18. 32 to tenths.
19. $3\frac{1}{4}$ to fourths.
20. $6\frac{5}{8}$ to eighths.
21. $4\frac{3}{4}$ to fourths.
22. $5\frac{1}{8}$ to eighths.
23. $6\frac{7}{8}$ to eighths.

24. How many packages of candy, each weighing $\frac{1}{4}$ lb., can be put up from $25\frac{1}{2}$ lb. of candy?

Reducing to an Integer or a Mixed Number. James sold the tickets for the athletic meet. When he came to count his money he had among the bills and coins 78 "quarters." How many dollars were these quarters worth?

James found that he must reduce $\frac{78}{4}$ to dollars; that is, to an integer or a mixed number.

Since $\frac{1}{4}$ means that 1 has been divided into 4 equal parts, or $1 \div 4$, we easily see that

$$\frac{78}{4} = 78 \div 4 = 19\frac{2}{4} = 19\frac{1}{2},$$

and so 78 "quarters" are worth $\$19\frac{1}{2}$, or \$19.50.

A fraction is an indicated division.

To reduce an improper fraction to an integer or a mixed number, divide the numerator by the denominator and express the result in lowest terms.

REDUCING TO AN INTEGER OR A MIXED NUMBER

Numbers 1 and 2, oral

1. If you have earned 14 half dollars, how many dollars have you earned?

2. If you have counted 33 "quarters," how many dollars' worth of "quarters" have you counted?

Reduce the following to integers or mixed numbers:

3. $\frac{33}{3}$.	7. $\frac{125}{24}$.	11. $\frac{632}{32}$.	15. $\frac{127}{4}$.	19. $\frac{81}{8}$.
4. $\frac{47}{6}$.	8. $\frac{298}{16}$.	12. $\frac{190}{12}$.	16. $\frac{87}{12}$.	20. $\frac{92}{12}$.
5. $\frac{42}{6}$.	9. $\frac{236}{32}$.	13. $\frac{425}{32}$.	17. $\frac{93}{32}$.	21. $\frac{76}{32}$.
6. $\frac{78}{5}$.	10. $\frac{349}{32}$.	14. $\frac{276}{16}$.	18. $\frac{198}{16}$.	22. $\frac{725}{32}$.

23. Express 29 "quarters" as dollars.

24. Express as dollars 24 half dollars; 25 half dollars.

Fractional Parts. If our school baseball team plays 28 games and wins 16 of them, what fraction of the games does it win?

Here we wish to find what fraction 16 is of 28.

Just as 1 is $\frac{1}{2}$ of 2, and 2 is $\frac{2}{4}$ of 4, so 16 is $\frac{16}{28}$ of 28.

So we see that our school team has won $\frac{16}{28}$ of the games. But we can reduce this to the simpler fraction $\frac{4}{7}$, and so we say that the team has won $\frac{4}{7}$ of the games.

To find what fraction the first of two numbers is of the second, divide the first number by the second and express the result in lowest terms.

FINDING FRACTIONAL PARTS

Numbers 1 to 6, oral

1. In a class of 38 pupils 20 of them are girls. What fraction of the class are girls?

2. Two boys plan to walk 4 mi. After walking 2 mi. they have walked what part of the distance?

Find what fraction the first of these numbers is of the second:

3. 5; 15. 4. 8; 24. 5. 16; 32. 6. 12; 18.

7. If you are taking a railway journey of 180 mi., what part of it have you finished when you have gone 60 mi.?

8. In Ex. 7 what part of the journey have you finished when you have gone 30 mi.? 100 mi.? 120 mi.?

Find what fraction the first of these numbers is of the second:

9. 35 ft.; 140 ft. 11. 18 mi.; 72 mi. 13. 17 yd.; 68 yd.

10. \$90; \$270. 12. 27 in.; 54 in. 14. 19 ft.; 133 ft.

15. A man's income is \$2500. If he spends \$1500, what fraction of his income does he spend?

Least Common Denominator. If several fractions have the same denominator, these fractions are said to be *similar* and to have a *common denominator*; and if this denominator is the smallest denominator possible, the fractions are said to have the *least common denominator* (L.C.D.).

For example, $\frac{4}{8}$ and $\frac{2}{8}$ are similar fractions. They have a common denominator, but the fractions may be reduced to $\frac{2}{4}$ and $\frac{1}{4}$, and in this case 4 is the L.C.D.

Reducing to the L.C.D. Three notebooks are placed in a pile. The first is $\frac{1}{4}$ in. thick, the second $\frac{3}{8}$ in., and the third $\frac{5}{12}$ in. How thick are the three together?

To answer such a question we must know how to add $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{5}{12}$, and to do this we must reduce them to fractions having the least common denominator.

Since the denominator must contain 4, 8, and 12 it must be a multiple of each of these numbers and contain all their factors. That is, the L.C.D. must be a *common multiple*, and the *least common multiple*, of 4, 8, and 12.

We see that 4, 8, and 12 all contain 4, that 8 contains 2 also, and that 12 contains 3 also. Therefore the L.C.D. must contain the factors 4, 2, and 3, and is $4 \times 2 \times 3$, or 24.

$$\frac{1}{4} = \frac{6 \times 1}{6 \times 4} = \frac{6}{24}$$
$$\frac{3}{8} = \frac{3 \times 3}{3 \times 8} = \frac{9}{24}$$
$$\frac{5}{12} = \frac{2 \times 5}{2 \times 12} = \frac{10}{24}$$

Since $24 \div 4 = 6$, we multiply by 6 both terms of $\frac{1}{4}$ to reduce $\frac{1}{4}$ to 24ths, and we have $\frac{1}{4} = \frac{6}{24}$.

Since $24 \div 8 = 3$, we multiply by 3 both terms of $\frac{3}{8}$ to reduce $\frac{3}{8}$ to 24ths, and we have $\frac{3}{8} = \frac{9}{24}$.

Since $24 \div 12 = 2$, we multiply by 2 both terms of $\frac{5}{12}$ to reduce $\frac{5}{12}$ to 24ths, and we have $\frac{5}{12} = \frac{10}{24}$.

COMMON DENOMINATORS

Numbers 1 to 9, oral

1. In making some panels for a door a workman has to glue together two pieces of wood, one $\frac{1}{2}$ in. thick and the other $\frac{1}{4}$ in. thick. In order to find the thickness of the panel he must add $\frac{1}{2}$ in. and $\frac{1}{4}$ in. What common denominator shall he use? Is this the L.C.D. of the fractions? What other common denominator might he take?

Reduce to fractions having any common denominator:

2. $\frac{3}{4}, \frac{1}{6}$. 3. $\frac{5}{6}, \frac{7}{8}$. 4. $\frac{3}{7}, \frac{5}{14}$. 5. $\frac{3}{8}, \frac{7}{10}$.

Reduce to fractions having the least common denominator:

6. $\frac{1}{2}, \frac{3}{4}$. 7. $\frac{8}{10}, \frac{3}{5}$. 8. $\frac{7}{8}, \frac{1}{16}$. 9. $\frac{3}{5}, \frac{3}{10}$.

Reduce to fractions having the least common denominator:

10. $\frac{2}{3}, \frac{5}{6}$.
11. $\frac{3}{4}, \frac{1}{6}$.
12. $\frac{3}{4}, \frac{5}{6}$.
13. $\frac{1}{3}, \frac{5}{6}$.
14. $\frac{5}{8}, \frac{3}{4}$.
15. $\frac{7}{8}, \frac{1}{3}$.
16. $\frac{5}{6}, \frac{2}{3}$.
17. $\frac{7}{8}, \frac{2}{3}$.
18. $\frac{1}{5}, \frac{2}{3}$.
19. $\frac{3}{5}, \frac{7}{8}$.
20. $\frac{7}{12}, \frac{11}{24}$.
21. $\frac{3}{16}, \frac{5}{24}$.
22. $\frac{7}{8}, \frac{5}{12}$.
23. $\frac{5}{8}, \frac{11}{24}$.
24. $\frac{7}{8}, \frac{13}{24}$.
25. $\frac{7}{8}, \frac{5}{32}$.
26. $\frac{5}{8}, \frac{11}{32}$.
27. $\frac{7}{16}, \frac{5}{32}$.
28. $\frac{15}{16}, \frac{31}{32}$.
29. $\frac{11}{16}, \frac{27}{32}$.
30. $\frac{3}{8}, \frac{7}{24}$.
31. $\frac{7}{8}, \frac{17}{24}$.
32. $\frac{5}{12}, \frac{3}{32}$.
33. $\frac{7}{12}, \frac{5}{32}$.
34. $\frac{11}{12}, \frac{15}{32}$.
35. $\frac{1}{24}, \frac{1}{32}$.
36. $\frac{5}{24}, \frac{3}{32}$.
37. $\frac{7}{24}, \frac{9}{32}$.
38. $\frac{7}{24}, \frac{9}{48}$.
39. $\frac{13}{24}, \frac{11}{48}$.
40. $\frac{5}{8}, \frac{7}{64}$.
41. $\frac{5}{16}, \frac{11}{64}$.
42. $\frac{11}{32}, \frac{15}{64}$.
43. $\frac{1}{12}, \frac{3}{64}$.
44. $\frac{5}{12}, \frac{7}{64}$.
45. $\frac{1}{24}, \frac{1}{64}$.
46. $\frac{7}{24}, \frac{25}{64}$.
47. $\frac{3}{8}, \frac{5}{16}, \frac{7}{24}$.
48. $\frac{7}{8}, \frac{5}{12}, \frac{11}{24}$.
49. $\frac{3}{8}, \frac{11}{12}, \frac{17}{32}$.

Reduce the fractional parts of the following groups of numbers to fractions having the least common denominator:

50. $3\frac{5}{12}, 5\frac{7}{16}$.
51. $2\frac{1}{4}, 3\frac{5}{16}, 8\frac{7}{24}$.
52. $1\frac{3}{8}, 2\frac{13}{24}, 3\frac{11}{32}$.

Adding Fractions. As already learned, if we add 3 marbles and 5 pencils we have a sum that is merely 8 things of two different kinds. So if we add $\frac{3}{4}$ and $\frac{5}{6}$ we shall have merely 8 parts of a unit of two different sizes. Therefore, to obtain a definite sum in adding fractions that do not have the same denominator, we must first reduce them to similar fractions.

For example, add $\frac{3}{5}$, $\frac{9}{10}$, and $\frac{7}{12}$.

The least common denominator (L.C.D.) of the fractions, as learned on page 55, is $5 \times 2 \times 2 \times 3$, or 60. Dividing 60 by 5, we have 12, and so we see that to reduce $\frac{3}{5}$ to 60ths we multiply both terms of $\frac{3}{5}$ by 12.

Similarly, dividing 60 by 10, we have 6, and we multiply both terms of $\frac{9}{10}$ by 6. We reduce $\frac{7}{12}$ to 60ths by multiplying both terms by 5.

$$\frac{3}{5} = \frac{12 \times 3}{12 \times 5} = \frac{36}{60}$$

$$\frac{9}{10} = \frac{6 \times 9}{6 \times 10} = \frac{54}{60}$$

$$\frac{7}{12} = \frac{5 \times 7}{5 \times 12} = \frac{35}{60}$$

$$\frac{125}{60}$$

$$= 2\frac{5}{60}$$

$$= 2\frac{1}{12}$$

Adding the numerators, we have $\frac{125}{60}$, which we reduce to a mixed number $2\frac{5}{60}$. We then reduce the fraction $\frac{5}{60}$ to lowest terms. The result is $2\frac{1}{12}$, and the work should appear as above.

To add fractions, reduce them to fractions having the least common denominator and add the numerators for the numerator of the sum, writing this over the least common denominator. Then reduce to lowest terms.

Many objective illustrations of fractions are given in the preceding work, but from now on the objective work acquires more force if done by the pupil. Addition and subtraction of fractions should be taught objectively, exactly as addition and subtraction of integers were taught in the primary grades. A reasonable use of paper cutting and of diagrams of circles, rectangles, and lines should precede the written work in fractions.

ADDING FRACTIONS

Numbers 1 to 6, oral

1. Express $\frac{1}{4}$ as eighths. To the result add $\frac{5}{8}$.

2. If you fasten a strip of molding $\frac{1}{4}$ in. thick to a strip of wood $\frac{5}{8}$ in. thick, how thick are the two together?

3. Express $\frac{4}{5}$ as tenths. To the result add $\frac{3}{10}$.

4. If you lay an arithmetic $\frac{4}{5}$ in. thick on a notebook $\frac{3}{10}$ in. thick, how thick are the two together?

5. Express $\frac{2}{3}$ as sixths. To the result add $\frac{1}{6}$.

6. If you sew a piece of cloth $\frac{2}{3}$ yd. wide to a strip $\frac{1}{6}$ yd. wide, how wide will the new piece be?

7. Some plaster $\frac{3}{8}$ in. thick is coated with a finer plaster $\frac{3}{16}$ in. thick. How thick is the plaster then?

8. A plate of brass $\frac{1}{32}$ in. thick is laid on a plate of iron $\frac{3}{16}$ in. thick. What is the total thickness?

9. An iron rod of diameter $1\frac{5}{16}$ in. is covered with brass plating $\frac{1}{32}$ in. thick. What is the diameter of the rod then?

10. A piece of cardboard $\frac{1}{64}$ in. thick is laid on a book $\frac{5}{8}$ in. thick. How thick are the two together?

Add the following fractions:

11. $\frac{3}{4}$, $\frac{7}{24}$.
12. $\frac{3}{4}$, $1\frac{3}{24}$.
13. $\frac{5}{16}$, $\frac{7}{24}$.
14. $\frac{3}{16}$, $1\frac{1}{24}$.
15. $\frac{3}{4}$, $\frac{1}{48}$.
16. $\frac{3}{4}$, $\frac{7}{48}$.
17. $\frac{3}{8}$, $1\frac{3}{48}$.
18. $\frac{7}{8}$, $2\frac{3}{48}$.

19. Add $\frac{1}{2}$ in., $\frac{1}{3}$ in., $\frac{2}{3}$ in., $\frac{1}{4}$ in., $\frac{1}{6}$ in., and $\frac{1}{12}$ in.

Add the following fractions as indicated:

20. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.
21. $\frac{1}{2} + \frac{3}{4} + \frac{1}{8}$.
22. $\frac{1}{3} + \frac{1}{4} + \frac{1}{12}$.
23. $\frac{2}{3} + \frac{3}{4} + \frac{1}{12}$.
24. $\frac{1}{3} + \frac{1}{8} + \frac{1}{12}$.
25. $\frac{2}{3} + \frac{5}{8} + \frac{1}{12}$.

Adding Mixed Numbers. If Alice needs $5\frac{2}{3}$ yd. of cloth for one piece of work and $3\frac{2}{3}$ yd. for another, how many yards does she need in all?

We must add $5\frac{2}{3}$ yd. and $3\frac{2}{3}$ yd. to find the total number of yards.

First adding the thirds, we have

$$\frac{2}{3}+\frac{2}{3}=\frac{4}{3}=1\frac{1}{3}.$$

$$\begin{array}{r} 5\frac{2}{3} \\ 3\frac{2}{3} \\ \hline 9\frac{1}{3} \end{array}$$

We write $\frac{1}{3}$ in fractions' column and add 1 to units' column.

We then have $1+3+5=9$, and we write 9 under units.

The sum is $9\frac{1}{3}$, and so Alice needs $9\frac{1}{3}$ yd. of cloth.

ADDING MIXED NUMBERS

1. One of Clara's books weighs $1\frac{3}{4}$ lb. and another weighs $\frac{3}{4}$ lb. How much do the two weigh together?

2. If a girl needs $1\frac{3}{4}$ yd. of gingham to make one kind of apron, $1\frac{1}{4}$ yd. for another kind, and $1\frac{3}{4}$ yd. for a third kind, how much gingham does she need to make the three aprons?

3. If your desk is $21\frac{3}{4}$ in. long and $14\frac{3}{4}$ in. wide, how many inches is it around the four sides of the desk?

4. Milton is putting some picture molding around a room $14\frac{1}{4}$ ft. long and 12 ft. wide. How much molding does he need?

Add the following:

5. $4\frac{1}{2}$	7. $3\frac{1}{3}$	9. $32\frac{2}{3}$	11. $46\frac{2}{3}$	13. $28\frac{1}{3}$	15. $37\frac{2}{3}$
3	2	$12\frac{1}{3}$	$15\frac{2}{3}$	18	$15\frac{2}{3}$
$3\frac{1}{2}$	$6\frac{1}{3}$	$13\frac{2}{3}$	$14\frac{1}{3}$	$7\frac{2}{3}$	$6\frac{2}{3}$
6. $3\frac{1}{2}$	8. $4\frac{1}{2}$	10. $12\frac{1}{2}$	12. $29\frac{1}{3}$	14. $32\frac{2}{3}$	16. $64\frac{1}{3}$
$3\frac{1}{2}$	$2\frac{1}{2}$	$14\frac{1}{2}$	$16\frac{1}{3}$	$23\frac{2}{3}$	$27\frac{2}{3}$
2	$3\frac{1}{2}$	$36\frac{1}{2}$	$4\frac{1}{3}$	$5\frac{1}{3}$	$18\frac{2}{3}$

Adding Mixed Numbers Continued. Each of three girls, Irene, Kate, and Emily, is making a dress. Irene uses $2\frac{1}{2}$ yd. of cloth, Kate uses $4\frac{2}{3}$ yd., and Emily $3\frac{3}{4}$ yd. How much cloth do they use in all?

$$2\tfrac{1}{2} = 2\tfrac{6}{12}$$
$$4\tfrac{2}{3} = 4\tfrac{8}{12}$$
$$3\tfrac{3}{4} = 3\tfrac{9}{12}$$
$$9\tfrac{23}{12} = 10\tfrac{11}{12}$$

The L.C.D. is $2 \times 2 \times 3$, or 12.

Reducing each fraction to a fraction with denominator 12, by the method shown on pages 55 and 57, we have

$$2\tfrac{6}{12} + 4\tfrac{8}{12} + 3\tfrac{9}{12} = 9\tfrac{23}{12} = 10\tfrac{11}{12},$$

and so the girls use $10\frac{11}{12}$ yd. of cloth in all.

To add mixed numbers, add separately the whole numbers and the fractions and find the sum of the results.

ADDING MIXED NUMBERS

1. A teacher's desk is $4\frac{2}{3}$ ft. long and $3\frac{5}{12}$ ft. wide. What is the perimeter, that is, the distance around it?

2. The three sides of a triangle are $4\frac{3}{8}$ in., $5\frac{1}{2}$ in., and $5\frac{3}{16}$ in. How many inches are there in the perimeter?

3. If a boy weighs $58\frac{3}{8}$ lb., his books $2\frac{3}{4}$ lb., and his dog $23\frac{5}{8}$ lb., how much do they all weigh together?

4. The three sides of a triangular lot of land are $36\frac{3}{8}$ yd., $23\frac{3}{4}$ yd., and $34\frac{1}{8}$ yd. How long is the perimeter of the lot?

Add the following:

5. $1\frac{1}{2} + \frac{3}{4} + \frac{7}{8}$.
6. $3\frac{1}{3} + \frac{5}{6} + \frac{1}{2}$.
7. $2\frac{7}{8} + \frac{2}{3} + \frac{3}{4}$.
8. $3\frac{2}{3} + \frac{3}{5} + \frac{7}{15}$.
9. $2\frac{1}{2} + 3\frac{3}{4} + 2\frac{7}{8}$.
10. $3\frac{2}{3} + 1\frac{5}{6} + \frac{1}{2}$.
11. $2\frac{3}{8} + 3\frac{3}{4} + \frac{5}{16}$.
12. $3\frac{2}{3} + 2\frac{5}{6} + \frac{1}{2}$.
13. $2\frac{3}{4} + 3\frac{3}{8} + 5\frac{7}{16}$.
14. $3\frac{3}{4} + 2\frac{7}{8} + 5\frac{9}{16}$.
15. $2\frac{5}{8} + 3\frac{1}{2} + 2\frac{15}{16}$.
16. $3\frac{2}{3} + 2\frac{4}{5} + 2\frac{7}{15}$.

Subtracting Fractions. Emily has $\frac{1}{2}$ yd. of lace, and she uses $\frac{3}{10}$ yd. in trimming a doll's waist. How much lace has she left after trimming the waist?

We must subtract $\frac{3}{10}$ yd. from $\frac{1}{2}$ yd.

The two denominators are 2 and 10, and since 2 is a factor of 10, the L.C.D. of the fractions is 10. Then we have

$$\frac{1}{2} = \frac{5}{10},$$

and $$\frac{5}{10} - \frac{3}{10} = \frac{2}{10} = \frac{1}{5}.$$

$$\begin{array}{r} \frac{1}{2} = \frac{5}{10} \\ \frac{3}{10} = \frac{3}{10} \\ \hline \frac{2}{10} = \frac{1}{5} \end{array}$$

Hence the difference is $\frac{1}{5}$, and so Emily has $\frac{1}{5}$ yd. of lace left.

To subtract fractions, reduce them to fractions having the least common denominator, and subtract the numerators for the numerator of the difference, writing this over the least common denominator. Then reduce to lowest terms.

SUBTRACTING FRACTIONS

1. If Margaret has $\frac{5}{8}$ yd. of ribbon and uses $\frac{1}{2}$ yd. in trimming some sleeves, how much ribbon has she left?

2. If you live $\frac{3}{4}$ mi. from school, how far have you still to go when you have walked $\frac{3}{8}$ mi. towards home?

Subtract the following:

3. $\frac{1}{2} - \frac{1}{12}$.
4. $\frac{5}{8} - \frac{1}{16}$.
5. $\frac{7}{8} - \frac{1}{4}$.
6. $\frac{2}{3} - \frac{5}{12}$.
7. $\frac{3}{4} - \frac{3}{16}$.
8. $\frac{7}{8} - \frac{1}{2}$.
9. $\frac{5}{8} - \frac{1}{4}$.
10. $\frac{1}{2} - \frac{3}{8}$.
11. $\frac{7}{8} - \frac{1}{24}$.
12. $\frac{3}{4} - \frac{5}{12}$.
13. $\frac{5}{12} - \frac{1}{4}$.
14. $\frac{1}{4} - \frac{1}{12}$.
15. $\frac{7}{16} - \frac{1}{8}$.
16. $\frac{1}{3} - \frac{1}{4}$.
17. $\frac{1}{4} - \frac{1}{16}$.
18. $\frac{3}{4} - \frac{5}{16}$.
19. $\frac{5}{6} - \frac{2}{3}$.
20. $\frac{1}{5} - \frac{1}{6}$.
21. $\frac{3}{5} - \frac{1}{2}$.
22. $\frac{5}{12} - \frac{3}{8}$.
23. $\frac{9}{10} - \frac{1}{5}$.
24. $\frac{7}{8} - \frac{5}{12}$.
25. $\frac{7}{8} - \frac{9}{16}$.
26. $\frac{5}{6} - \frac{3}{4}$.
27. $\frac{7}{8} - \frac{3}{4}$.
28. $\frac{7}{8} - \frac{2}{3}$.
29. $\frac{2}{3} - \frac{1}{2}$.
30. $\frac{5}{8} - \frac{1}{2}$.
31. $\frac{5}{8} - \frac{3}{32}$.
32. $\frac{9}{10} - \frac{3}{5}$.
33. $\frac{1}{4} - \frac{3}{16}$.
34. $\frac{1}{2} - \frac{5}{16}$.

Subtracting a Mixed Number. John saws a strip $4\frac{3}{8}$ in. wide from a board 14 in. wide. How wide is the board that is left?

We must subtract $4\frac{3}{8}$ in. from 14 in.

We see that we cannot take $\frac{3}{8}$ from nothing, although we can take $4\frac{3}{8}$ from 14.

We therefore think of 14 as $13\frac{8}{8}$, which we can do because $\frac{8}{8}=1$. We then have

$$\frac{8}{8}-\frac{3}{8}=\frac{5}{8},$$

and

$$13-4=9.$$

$$\begin{array}{r} 14 \\ 4\frac{3}{8} \\ \hline 9\frac{5}{8} \end{array}$$

Therefore the difference between 14 and $4\frac{3}{8}$ is $9\frac{5}{8}$, and so $9\frac{5}{8}$ in. is the width of the board that John has left.

SUBTRACTING A MIXED NUMBER

1. In making a bird house Jack saws a piece $4\frac{7}{8}$ in. long from a board 16 in. long. How long is the piece that remains?

2. Mary has a piece of ribbon 34 in. long. She uses $18\frac{1}{2}$ in. in trimming a bag. How much ribbon has she left?

3. On an automobile trip of 54 mi., if you make a call $9\frac{3}{4}$ mi. down the road, how much farther have you still to go?

Subtract the following:

4. $7-3\frac{1}{3}$.
5. $4-2\frac{3}{8}$.
6. $9-6\frac{2}{5}$.
7. $8-4\frac{5}{8}$.
8. $14-3\frac{2}{5}$.
9. $34-18\frac{5}{8}$.
10. $26-13\frac{3}{5}$.
11. $38-9\frac{7}{16}$.
12. $36-4\frac{9}{16}$.
13. $30-8\frac{7}{16}$.
14. 8 in. $-2\frac{3}{4}$ in.
15. 9 in. $-4\frac{5}{32}$ in.

16. From the sum of $2\frac{1}{2}$ and $7\frac{1}{2}$ subtract the sum of $3\frac{1}{4}$ and $2\frac{1}{2}$.

17. From the sum of $14\frac{3}{8}$, $1\frac{7}{8}$, and $1\frac{3}{4}$ subtract $3\frac{2}{3}$.

18. From the difference between $10\frac{1}{4}$ and $5\frac{1}{4}$ subtract the sum of $2\frac{1}{4}$ and $1\frac{1}{8}$.

Subtracting from a Mixed Number. From a piece of wire $23\frac{1}{3}$ in. long Ralph uses $8\frac{5}{6}$ in. for a toy airplane. How much wire has he left?

From $23\frac{1}{3}$ in. we must subtract $8\frac{5}{6}$ in.

The two denominators are 3 and 6. Since 6 contains 3 the L.C.D. is 6.

Since $23\frac{1}{3} = 23\frac{2}{6}$, we see that

$$23\frac{1}{3} - 8\frac{5}{6} = 23\frac{2}{6} - 8\frac{5}{6}.$$

We may now think of $23\frac{2}{6}$ as $22\frac{8}{6}$, and then $22\frac{8}{6} - 8\frac{5}{6} = 14\frac{3}{6} = 14\frac{1}{2}$.

$$\begin{array}{r} 23\frac{1}{3} = 23\frac{2}{6} \\ 8\frac{5}{6} \\ \hline 14\frac{1}{2} \end{array}$$

This process should be developed at the blackboard.

SUBTRACTING FROM A MIXED NUMBER

1. If Martha has $32\frac{1}{4}$ in. of cloth and uses $9\frac{3}{4}$ in., how much cloth has she left?

2. If one book is $8\frac{1}{4}$ in. long and another is $5\frac{3}{16}$ in. long, what is the difference between their lengths?

3. If we have a piece of board $16\frac{1}{8}$ in. long and we need a piece only $13\frac{3}{4}$ in. long, how much shall we cut off?

Subtract the following:

4. $32\frac{5}{8} - 8$.
5. $6\frac{3}{8} - \frac{3}{4}$.
6. $9\frac{1}{2} - 4\frac{3}{8}$.
7. $32\frac{7}{8} - 9$.
8. $8\frac{3}{8} - \frac{3}{4}$.
9. $12\frac{1}{4} - 3\frac{5}{8}$.
10. $26\frac{3}{4} - 19$.
11. $8\frac{5}{12} - \frac{7}{8}$.
12. $32\frac{2}{5} - 18\frac{3}{8}$.
13. $23\frac{5}{6} - 18$.
14. $5\frac{5}{12} - \frac{5}{6}$.
15. $24\frac{1}{3} - 8\frac{3}{4}$.
16. $23\frac{7}{12} - 8\frac{1}{2}$.
17. $9\frac{2}{3} - 2\frac{4}{5}$.
18. $23\frac{1}{5} - 16\frac{5}{8}$.

19. How much must you add to $32\frac{7}{8}$ in. to make a yard?

20. How much must you take from 1 yd. to leave $18\frac{5}{8}$ in.?

DRILL WORK IN FRACTIONS

1. In adding the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{12}$ we need to find the L.C.D. of the fractions, that is, the least common multiple of 2, 3, 4, and 12. Find this least common multiple.

Find the least common multiples of the following:

2. 3, 7.	**5.** 8, 12.	**8.** 2, 3, 6.	**11.** 2, 4, 6, 8.
3. 3, 9.	**6.** 8, 14.	**9.** 3, 4, 6.	**12.** 3, 4, 6, 8.
4. 6, 9.	**7.** 8, 16.	**10.** 3, 5, 6.	**13.** 4, 5, 7, 10.

Add the following:

14.	**15.**	**16.**	**17.**	**18.**	**19.**
$3\frac{1}{4}$	$4\frac{2}{3}$	$5\frac{1}{2}$	$3\frac{2}{5}$	$6\frac{2}{9}$	$24\frac{3}{8}$
$2\frac{2}{3}$	$3\frac{3}{4}$	$2\frac{1}{3}$	$4\frac{1}{3}$	$3\frac{1}{3}$	$15\frac{5}{16}$
$5\frac{1}{2}$	$5\frac{1}{3}$	$4\frac{2}{3}$	$3\frac{1}{6}$	$7\frac{5}{6}$	$33\frac{3}{4}$
$3\frac{1}{12}$	$6\frac{7}{12}$	$7\frac{5}{6}$	$5\frac{4}{5}$	$5\frac{1}{2}$	$17\frac{1}{2}$

Such exercises should be speed tests. A pupil should copy **Exs. 14–19**, add, and check the results in less than 5 min.

20. From the sum of $14\frac{7}{8}$ and $23\frac{3}{4}$ take the sum of $9\frac{1}{2}$ and $7\frac{1}{16}$.

Add the following:

21.	**22.**	**23.**	**24.**	**25.**	**26.**
$3\frac{1}{2}$	$5\frac{2}{3}$	$3\frac{1}{8}$	$5\frac{1}{5}$	$2\frac{3}{4}$	$41\frac{1}{5}$
$4\frac{2}{3}$	$4\frac{1}{6}$	$4\frac{3}{4}$	$2\frac{2}{3}$	$5\frac{7}{8}$	$28\frac{3}{4}$
$2\frac{3}{4}$	$7\frac{3}{4}$	$9\frac{1}{4}$	$3\frac{5}{6}$	$3\frac{1}{2}$	$16\frac{5}{8}$
$5\frac{3}{4}$	$5\frac{7}{8}$	$6\frac{7}{8}$	$4\frac{3}{5}$	$4\frac{3}{8}$	$9\frac{1}{2}$
$7\frac{1}{12}$	$9\frac{5}{6}$	$4\frac{1}{2}$	$2\frac{1}{2}$	$9\frac{1}{4}$	$14\frac{9}{10}$
$9\frac{5}{6}$	$8\frac{7}{12}$	$3\frac{5}{8}$	$6\frac{7}{10}$	$61\frac{5}{16}$	$3\frac{7}{10}$

A pupil should copy **Exs. 21–26**, add, and check the results in **5 min.**

27. Add $24\frac{1}{2}$ in., $13\frac{3}{4}$ in., $16\frac{1}{4}$ in., $21\frac{7}{8}$ in., and $15\frac{3}{8}$ in.

28. From $17\frac{1}{2}+16\frac{3}{4}+5\frac{3}{4}$ take $15\frac{7}{8}-9\frac{3}{4}$.

DRILL WORK IN FRACTIONS

1. Find the least common multiple of 3, 5, 7, and 9.

2. Find the L.C.D. of the fractions $\frac{3}{4}$, $\frac{3}{5}$, and $\frac{1}{7}$.

3. How much must you add to $\frac{4}{5}$ to make $\frac{7}{8}$?

4. How much must you subtract from $\frac{3}{4}$ yd. to leave $\frac{1}{8}$ yd.?

Subtract in each of the following cases:

5. $8\frac{3}{4}$	7. $7\frac{1}{2}$	9. $5\frac{7}{8}$	11. $9\frac{2}{3}$	13. $19\frac{3}{16}$
$2\frac{1}{2}$	$3\frac{3}{8}$	$2\frac{1}{4}$	$5\frac{3}{4}$	$14\frac{1}{3}$
6. $9\frac{5}{8}$	8. $8\frac{5}{8}$	10. $9\frac{2}{3}$	12. $7\frac{1}{4}$	14. $32\frac{11}{16}$
$3\frac{1}{2}$	$3\frac{1}{4}$	$4\frac{1}{6}$	$3\frac{2}{3}$	$18\frac{2}{5}$

Exs. 5–14 should be copied, solved, and checked in 3 min.

15. A man has a board $15\frac{7}{8}$ in. long. He wishes to use a piece of it $8\frac{2}{3}$ in. long for the end of a box. How long is the piece which he will have left?

16. A woman has a piece of velvet ribbon 6 yd. long. She cuts off $3\frac{7}{8}$ yd. How long is the piece that is left?

Subtract the following:

17. $3 - \frac{1}{4}$.	25. $17\frac{7}{24} - \frac{2}{3}$.	33. $15\frac{3}{4} - 5\frac{7}{8}$.
18. $16 - \frac{7}{8}$.	26. $\frac{9}{10} - \frac{1}{3}$.	34. $19\frac{3}{8} - 7\frac{3}{4}$.
19. $14 - \frac{3}{16}$.	27. $3\frac{7}{8} - \frac{1}{16}$.	35. $23\frac{1}{3} - 2\frac{1}{5}$.
20. $35 - \frac{7}{12}$.	28. $5\frac{7}{8} - 1\frac{5}{16}$.	36. $36\frac{2}{3} - 7\frac{4}{5}$.
21. $3\frac{1}{2} - 1\frac{1}{4}$.	29. $1\frac{7}{8} - \frac{15}{32}$.	37. $13\frac{7}{12} - 5\frac{1}{3}$.
22. $7\frac{1}{2} - 3\frac{5}{8}$.	30. $5\frac{1}{8} - \frac{7}{32}$.	38. $26\frac{7}{12} - 8\frac{2}{3}$.
23. $9\frac{1}{2} - 4\frac{3}{16}$.	31. $6\frac{5}{6} - \frac{2}{3}$.	39. $35\frac{15}{32} - 7\frac{5}{16}$.
24. $37\frac{1}{4} - 5\frac{11}{16}$.	32. $9\frac{1}{6} - \frac{2}{3}$.	40. $49\frac{5}{48} - 16\frac{3}{16}$.

III. LITTLE EXAMINATIONS

i. 1. MDCXXI = (?).
2. 857,982 − 598,891.
3. $702\frac{1}{8} - 338\frac{3}{4}$.
4. $901\frac{3}{4} - 348\frac{7}{8}$.
5. 85 × $57,675.
6. 57 × $63,250.
7. 89 × $41.77.
8. $28.71 ÷ 9.

ii. 1. MDXCI = (?).
2. 787,859 + 358,583.
3. 700,203 − 386,774.
4. 6076 × 48,305.
5. 75 × 628,385.
6. $1780.02 ÷ 99.
7. $42,346.20 ÷ 7.
8. 35,500 ÷ 500.

iii. 1. MCCXCVI = (?).
2. 229,628 + 69,837.
3. 402,120 − 36,879.
4. 7008 × 82,635.
5. 65 × 426,908.
6. 600 × 74,374.
7. $180.18 ÷ 91.
8. 46,500 ÷ 250.

iv. 1. CLXXXIV = (?).
2. 893,434 + 98,877.
3. 911,034 − 38,048.
4. 489 × $662.75.
5. 70 × 386,469.
6. 85 × 643,667.
7. $446.22 ÷ 67.
8. 52,320 ÷ 125.

v. 1. Reduce $8\frac{2}{3}$ to thirds.
2. $8\frac{1}{2} + 3\frac{3}{8} + 2\frac{1}{4}$.
3. $4\frac{7}{8} + 8\frac{3}{4} + 2\frac{1}{2}$.
4. $702\frac{1}{8} - 238\frac{1}{4}$.
5. $\frac{5}{8} + \frac{3}{8} + 2\frac{1}{4}$.
6. $3\frac{3}{4} + \frac{1}{2} - \frac{1}{8}$.
7. $8\frac{3}{4} + 3\frac{1}{2}$.
8. $33\frac{2}{5} - 18\frac{3}{8}$.

vi. 1. Reduce $7\frac{4}{5}$ to fifths.
2. Reduce $\frac{25}{5}$ to units.
3. $7\frac{3}{4} + 6\frac{3}{4} + 4\frac{7}{8}$.
4. $3\frac{2}{3} + 8\frac{3}{4} - 7\frac{1}{2}$.
5. $701\frac{7}{8} - 348\frac{3}{4}$.
6. $64.96 ÷ 8.
7. 3376 ÷ 44.
8. $8\frac{3}{8}$ ft. + $6\frac{7}{16}$ ft.
9. $9\frac{3}{8}$ in. − $2\frac{1}{2}$ in.
10. 79 × $64.21.

IV. REVIEW AND DRILL

Nature and Purpose of Review and Drill. At the end of each chapter in this book there occurs a section like this one. The work is called Review and Drill, and it is optional with the teacher whether or not it shall be used or how much of it shall be taken. None of these sections is merely a review of the chapter immediately preceding it, for review work is already provided in the chapter itself; on the contrary, the review is general and covers many of the important processes in arithmetic that have been studied. This renders possible a continual return to fundamental and first principles, a constant brushing up in the essentials of arithmetic. This constant brushing up, drilling, and reviewing of the essentials is necessary for the attainment of the best results. The work is easily made interesting; it contributes to efficiency in computation; and it will, if carried out in the right spirit, help to remove the charge that most pupils are not quick and accurate in their calculations.

When the length of the school year permits, the study of these Review and Drill sections is recommended strongly; but the work being supplementary and optional, it doubtless will often be desirable for the teachers to select from these exercises only what is specially adapted to their immediate needs and to omit the rest. A certain flexibility is thus offered that will make the textbook adaptable to varying local requirements.

Each section consists of three parts: first, the Minimum Essentials, chiefly a drill in operations, emphasizing the *mechanical* side of arithmetic; second, an exercise in Problems without Numbers and Problems for Completion, emphasizing the *analytic* side of arithmetic; and, finally, live Industrial Problems that emphasize the *practical* side of arithmetic.

It will at once occur to every teacher that in these Review and Drill sections, with their threefold, inclusive emphasis, there is ready at hand a series of *efficiency tests* which will make it easy at frequent intervals to check up the work of the school as well as the work of the individual pupils.

MINIMUM ESSENTIALS

1. Write in words the numbers 3,628,309 and MDXCIII.
2. Write in figures the number nineteen million, six.
3. Add $3296 + 49{,}248 + 674 + 3462 + 58{,}779 + 6364$.
4. Add $\$38.75 + \$93.46 + \$7.75 + \$329.50 + \$4.36$.
5. From 701,020 subtract 86,748.
6. Find the difference between \$942.65 and \$98.78.
7. Multiply 53,095 by 3278.
8. Multiply \$478.70 by 964.
9. Divide the sum of 87,960 and 48,290 by 250.
10. Divide 774,932 by 302, and 945,260 by 604.
11. Reduce to lowest terms: $\frac{6}{8}$, $\frac{12}{16}$, $\frac{16}{32}$, $\frac{18}{32}$, $\frac{24}{32}$, $\frac{32}{32}$.
12. How many fourths are there in 7? in $7\frac{1}{4}$? in $9\frac{3}{4}$?
13. Reduce $25\frac{3}{4}$ to fourths; to eighths.
14. Reduce to integers or mixed numbers: $\frac{34}{2}$, $\frac{47}{4}$, $\frac{61}{8}$.
15. Find the value of $7\frac{1}{2} + \frac{1}{8} + 3\frac{3}{4} + 9\frac{3}{4} + 3\frac{5}{8}$.
16. From $9\frac{1}{4}$ in. take the sum of $3\frac{1}{2}$ in. and $4\frac{7}{8}$ in.
17. Find the value of $6\frac{3}{16} + 2\frac{5}{8} + 3\frac{1}{6} + 2\frac{7}{12} - 8\frac{11}{16}$.
18. How many strips of molding 8 in. long can be cut from a strip 5 ft. long, and how much will be left over?

Multiply or divide as indicated:

19. 625×425.	24. 569×839.	29. $65{,}664 \div 456$.
20. 772×996.	25. 3287×5008.	30. $17{,}336 \div 102$.
21. 2268×3872.	26. $1728 \div 12$.	31. $77{,}112 \div 189$.
22. 3081×4235.	27. $5184 \div 24$.	32. $328{,}320 \div 360$.
23. 2007×5280.	28. $4128 \div 32$.	33. $509{,}928 \div 528$.

PROBLEMS WITHOUT NUMBERS

All work oral

1. How do you subtract a number of three figures from a greater number when the units' figure to be subtracted is greater in value than the units' figure of the minuend, and the tens' figure to be subtracted is greater than the tens' figure of the minuend? Give your reasons.

2. How do you multiply by a number of three figures, the last two figures being zero? Give your reasons.

3. How do you divide by a number of three figures, the last two figures being zero? Give your reasons.

4. How do you find the square of a number? How do you find the cube of a number?

5. How do you cancel factors from the terms of a fraction? What are the advantages of cancellation? On what principle does cancellation depend?

6. If you know the cost of some land and the cost per acre, how do you find the number of acres?

7. If you know the weight of a cubic foot of water and the number of cubic feet of water in a cistern, how do you find the weight of the water in the cistern?

8. If you have the difference of two numbers and one of the two numbers given, how do you find the other number?

9. If you have the quotient of two numbers given and know the dividend, how do you find the divisor?

Teachers should make frequent use of Problems without Numbers and should encourage the making of such problems by one group in a class for another group to solve. Expressive and clear English should find close correlation with precise knowledge and succinct statement in arithmetic.

PROBLEMS FOR COMPLETION

1. A man bought three Treasury Savings Certificates and paid $253.80 for them. Complete the problem and solve it.

It usually happens that a problem may be completed in various ways. The pupil should be encouraged to make any reasonable problem out of what is given, so long as he solves that problem correctly.

2. Thrift Stamps cost 25¢ each. With 16 Thrift Stamps and 23¢ it is possible to buy one Government Savings Stamp in a certain month. A man has $5 to invest. Complete the problem and solve it.

Complete each of the following problems and solve it:

3. A man has $1950. He wishes to buy Treasury Savings Certificates at $846 each.

4. A man takes his family on an automobile trip. The total distance to be covered in 4 da. is 548 mi. The first day they go 142 mi.; the second day, 131 mi.; the third day, $136\frac{3}{4}$ mi.

5. The expenses of a certain school last year amounted to $1583.15. This included four items: the teacher's salary, $146.90 for books and supplies, $139.50 for fuel and lights, and $96.75 for miscellaneous expenses.

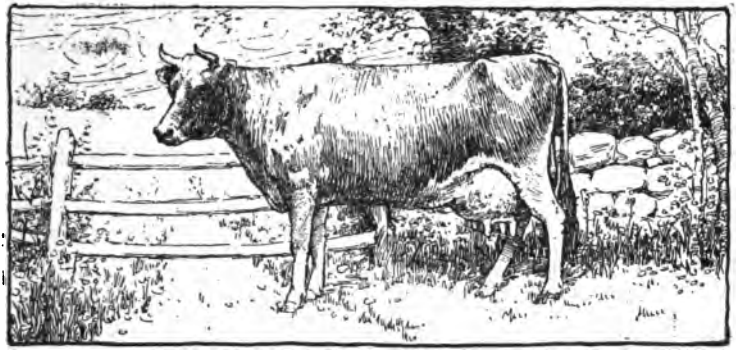
6. A certain school is in session from a quarter before nine to noon, and from a quarter past one to a quarter before four.

7. Kate is knitting a sweater. She needs $2\frac{3}{4}$ hanks of yarn. For a smaller sweater she needs $2\frac{1}{2}$ hanks.

8. A certain baseball team has won 16 games and lost 13, and another team has won 14 games and lost 11.

Teachers should encourage the frequent use of Problems for Completion, particularly such as relate to local conditions. The pupils should be asked to make such problems for use in class contests.

Industrial Problems



BLOOD TELLS IN THE DAIRY

1. Ralph's father explained the meaning of grading cows, keeping their records for milk, and disposing of those with poor records. The following is a record of twelve herds of the same size:

I. With Systematic Grading			II. Without Systematic Grading		
Herd	Annual cost of food per cow	Annual profit per cow	Herd	Annual cost of food per cow	Annual profit per cow
a	$34.28	$34.02	*g*	$31.65	$13.34
b	33.37	36.19	*h*	40.58	8.13
c	47.11	25.83	*i*	38.30	22.22
d	36.72	38.27	*j*	37.40	22.33
e	31.19	48.86	*k*	33.78	38.93
f	31.57	42.81	*l*	32.88	14.91

Ralph calculated to the nearest cent the average annual cost of the food and the profit per cow for group I. He then did the same for group II. Find the results in each of the two cases.

WASTE AND SAVING

1. Mrs. Stevens finds that her cook is wasting one slice of bread every day. If a loaf of bread is cut into 12 slices, in how many days will she waste 3 loaves of bread?

2. Mrs. Campbell finds that if she buys potatoes during the winter, she pays an average of $3 a bushel for them. By buying 10 bu. in the fall she can get them for $1.50 a bushel. Find how much she saves by buying this amount in the fall, even if she loses 1 bu. by their rotting during the winter.

3. Mrs. Brown decided to save money by buying in larger quantities. She made a list of a few items, showing what she had been paying and what the grocer would charge for the same goods in the quantities stated. If she can use the larger amounts, find how much she can save on the following order:

Crackers,	23¢ per lb.;	box of 7 lb.,	$1.40
Canned corn,	18¢ per can;	case of 24 cans,	$3.60
Bacon,	45¢ per lb.;	piece of 8 lb.,	$3.20
Soap,	10¢ per bar;	box of 100 bars,	$8.75

4. By the use of a fireless cooker Mrs. Brown reduced her gas bills for January, February, and March from a total of $8.95 for these months in the previous year to $7.25. At this rate how much will she save if she uses the cooker all the year?

5. In July a coal dealer telephoned Mr. Field that he could buy a winter's supply of coal for $11.60 a ton. Mr. Field waited until October, however, and then paid $12.85 a ton for the same grade of coal. He required 16 tons. How much could he have saved by buying his coal in July?

The pupils are not yet prepared to consider the question of interest on the money, but should simply solve in accordance with what is given.

CHAPTER II

I. COMMON FRACTIONS

USING WHAT YOU HAVE LEARNED

1. In a relay race the first boy of one team ran his lap in $14\frac{4}{5}$ sec., the second boy in $13\frac{3}{5}$ sec., the third in $14\frac{3}{5}$ sec., and the fourth in $13\frac{4}{5}$ sec. What was the total time of the race?

2. Miriam has a piece of goods $2\frac{3}{4}$ yd. long to dress her two dolls. She uses $\frac{7}{8}$ yd. for one doll and $\frac{1}{2}$ yd. for the other. With the rest she makes a jacket for the larger doll. How much does she use for the jacket?

3. Making no allowance for doors and windows, how many feet of picture molding will be needed to go around a classroom $22\frac{3}{4}$ ft. long and $18\frac{1}{2}$ ft. wide?

4. Malcolm makes a box to put under a shelf in a space $21\frac{1}{4}$ in. high. The wood he has is $\frac{5}{8}$ in. thick. If he needs a space of $3\frac{1}{8}$ in. between the cover and the shelf, what must be the inside height of the box?

5. A man spent \$60 in visiting a certain city. Of this amount \$22 was for railroad fare, \$8 for sleeping-car tickets, and \$30 for hotel bills and other expenses. What part of the money did he spend for each?

In all such examples the results must be expressed in lowest terms.

This page affords a brief review of the addition and subtraction of fractions as studied in Chapter I. We shall now take up the multiplication and division of fractions.

Cancellation. On page 48 the work in canceling common factors was sufficient to meet the need at that time. It will, however, be found helpful to learn a little more about canceling.

Greatest Common Divisor. The greatest factor common to two or more numbers is called their *greatest common divisor* (G.C.D.). For example, although 3 is a common divisor of 12, 18, and 48, the greatest common divisor is 6.

The greatest common divisor of such numbers as we have met or shall meet in fractions is easily found. For example, consider 12 and 30. We see that

$$12 = 2 \times 3 \times 2, \qquad 30 = 2 \times 3 \times 5,$$

and that 2 and 3 are the only common factors. Therefore 2×3, or 6, is the greatest common divisor.

The treatment of fractions has changed so greatly with the omission of those having large terms that the subject of G.C.D. has become practically obsolete in connection with this kind of work. The subject is included here for those who may care to use it, but it may safely be omitted.

Indeed, the whole question of the divisibility of numbers has no longer the practical value that it once had. The work on pages 74–76 should therefore be taken up as presenting some interesting properties of numbers rather than as being of great practical value.

It should be explained informally to the class that when we speak of a factor of a number, or of a common divisor of two or more numbers, or of the divisibility of numbers, we always refer to *exact* division of integers.

Even Number. A number that is divisible by 2 is called an *even number*. For example, 20, 38, and 1112 are even numbers.

Odd Number. A number that is not even is called an *odd number*. For example, 21, 39, and 127 are odd numbers.

Such definitions as these are given as matters of general information rather than as being essential to the reduction of fractions. They are related to the divisibility of numbers now to be considered.

Divisibility by 2. *A number is divisible by 2 if the units are so divisible.* For example, in the number 54 we see that 4 is divisible by 2, and so we know that 54 is divisible by 2.

Divisibility by 3. *A number is divisible by 3 if the sum of its digits is so divisible.* For example, since $4+1+1$, or 6, is divisible by 3, we see that 411 is divisible by 3.

Divisibility by 5. *A number is divisible by 5 if it ends in 0 or 5.* This tells us at a glance that 180 and 275 are divisible by 5.

These are the most important cases of divisibility the pupil will use. The reasons are better understood later, when the pupil studies algebra.

Divisibility by 4. *A number is divisible by 4 if the number represented by its two right-hand figures is so divisible.* This tells us at a glance that 9784 is divisible by 4.

Divisibility by 6. *A number is divisible by 6 if it is even and if the sum of its digits is divisible by 3.* This tells us at a glance that 1122 is divisible by 6.

Divisibility by 8. *A number is divisible by 8 if the number represented by its three right-hand figures is so divisible.* This tells us at a glance that 175,640 is divisible by 8.

Divisibility by 9. *A number is divisible by 9 if the sum of its digits is so divisible.* For example, in the number 70,812 we see that $7+0+8+1+2$, or 18, is divisible by 9, and so we know that 70,812 is divisible by 9.

Divisibility by 11. *A number is divisible by 11 if the difference between the sums of the digits in its even and odd places is 0 or is divisible by 11.* This tells us at a glance that 430,507 is divisible by 11, for $7+5+3=15$, $0+0+4=4$, and $15-4=11$.

The pupils should see that they now have a new power over numbers. It seems remarkable that we can tell almost at a glance that a number like 111,210,121,440 is divisible by 2, 3, 4, 5, 6, 8, 9, 10, and 11.

USING WHAT YOU HAVE LEARNED

Numbers 1 to 27, oral

1. In an example in division the fraction $\frac{405}{495}$ appeared in the quotient. Name at once three different factors that are common to both terms. Which factor is it best to cancel first if we wish to have as small numbers to work with as possible?

2. If you have the fraction $\frac{845}{480}$, what factor will you cancel first? What factor will you cancel next?

Name such of the following numbers as are divisible by 3:

3. 3819. 4. 7720. 5. 4554. 6. 7281. 7. 3845.

Name such of the following numbers as are divisible by 4:

8. 6964. 9. 7830. 10. 5708. 11. 7316. 12. 3975.

Name such of the following numbers as are divisible by 6:

13. 9396. 14. 3969. 15. 7480. 16. 3462. 17. 3003.

Name such of the following numbers as are divisible by 9:

18. 7047. 19. 4077. 20. 6832. 21. 5445. 22. 8008.

Name such of the following numbers as are divisible by 11:

23. 1001. 24. 2002. 25. 7007. 26. 8118. 27. 1212.

Reduce the following fractions to lowest terms:

28. $\frac{77}{1001}$. 29. $\frac{75}{375}$. 30. $\frac{33}{2002}$. 31. $\frac{405}{576}$. 32. $\frac{77}{561}$.

Teachers should make it clear that in practice such fractions appear only in quotients. Business men have no need for using the old method of G.C.D. for reducing fractions. They simply cancel such factors as are apparent, or else they reduce the fractions to decimals.

Multiplying by a Whole Number. If you have 4 "quarters," how many dollars have you?

The question shows the practical need of multiplying a fraction.

How many dollars have you if you have 8 "quarters"? 5 "quarters"? 7 "quarters"?

How much is 2 times 3 in.? 2 times $\frac{3}{7}$? 5 times $\frac{2}{11}$? How did you multiply the fractions by the integers?

How much is 4 times $\frac{1}{4}$? 4 times $\frac{3}{4}$? 2 times $\frac{3}{4}$? 5 times $\frac{7}{40}$? 7 times $\frac{4}{49}$? How did you multiply these fractions?

To multiply a fraction by a whole number, multiply the numerator by the multiplier.

Before actually multiplying, indicate the multiplication and cancel common factors in both terms if possible.

Teachers will observe that this includes the special case of dividing the denominator by the multiplier.

$$\frac{\overset{3}{\cancel{8}} \times 7}{\underset{4}{\cancel{8}}} = \frac{21}{4} = 5\tfrac{1}{4}$$

For example, to multiply $\frac{7}{8}$ by 6. Since we have 7 eighths, if we have 6 times as many eighths we shall have 6×7 eighths. Canceling as shown above, the result is $5\frac{1}{4}$.

MULTIPLYING BY A WHOLE NUMBER

Numbers 1 to 5, oral

1. Mary needs for some work 8 pieces of cloth, each $\frac{5}{8}$ yd. long. How many yards does she need in all?

Multiply the following, using cancellation when possible:

2. $6 \times \frac{2}{3}$. **4.** $5 \times \frac{4}{5}$. **6.** $32 \times \frac{5}{72}$. **8.** $96 \times \frac{9}{32}$.

3. $8 \times \frac{7}{8}$. **5.** $5 \times \frac{7}{10}$. **7.** $33 \times \frac{7}{36}$. **9.** $32 \times \frac{15}{64}$.

10. Find the product of 16, $\frac{3}{8}$, 9, and $\frac{2}{3}$.

Fractional Part of a Whole Number. If there are 240 pupils in a school, and $\frac{3}{4}$ of them are in the primary grades, how many are in the primary grades?

Because $\frac{1}{4}$ of 240 is 60,
and $\frac{3}{4}$ of 240 is 3 times as many,
hence $\frac{3}{4}$ of 240 is 3×60, or 180.

This work is simply a review of that done in the primary grades.

$$\frac{3}{4} \text{ of } 240 = \frac{3 \times \overset{60}{\cancel{240}}}{\cancel{4}} = 180$$

To find a fractional part of a whole number, divide the number by the denominator of the fraction and multiply the result by the numerator. Cancel whenever possible.

It is important to remember that cancellation should be used before the other operations are attempted.

FINDING FRACTIONAL PARTS

Numbers 1 to 4, oral

Find the indicated parts of the following numbers:

1. $\frac{1}{8}$ of 32. **2.** $\frac{3}{8}$ of 32. **3.** $\frac{1}{6}$ of 36. **4.** $\frac{5}{6}$ of 36.

5. A man has to go 126 mi. After going $\frac{5}{9}$ of the distance, how many miles has he gone? How many miles has he still to go?

6. A man owns 128 head of cattle. If he sells $\frac{5}{8}$ of them, how many does he sell? How many has he left?

7. A man owns 224 acres of land. If he sells $\frac{7}{8}$ of it, how many acres does he sell? How many acres has he left?

Find the indicated parts of the following numbers:

8. $\frac{3}{8}$ of 1464. **11.** $\frac{5}{8}$ of 9768. **14.** $\frac{5}{8}$ of 3376.

9. $\frac{7}{8}$ of 3272. **12.** $\frac{1}{6}$ of 7740. **15.** $\frac{5}{6}$ of 1758.

10. $\frac{5}{6}$ of 7764. **13.** $\frac{5}{6}$ of 7776. **16.** $\frac{3}{8}$ of 4976.

FRACTIONAL PART OF A FRACTION

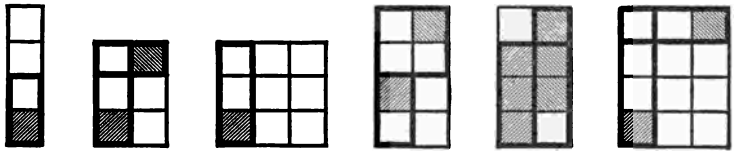
All work oral

1. How much is $\frac{1}{2}$ of \$4? $\frac{1}{2}$ of 4 blocks? $\frac{1}{2}$ of $\frac{4}{5}$?

2. In taking $\frac{1}{2}$ of $\frac{4}{5}$, how do you proceed?

A *B* *C*

3. In the figures shown above, *A* represents a whole sphere. Which figure represents $\frac{3}{4}$ of the sphere? Which figure represents $\frac{1}{3}$ of $\frac{3}{4}$ of the sphere? How much is $\frac{1}{3}$ of $\frac{3}{4}$ of the sphere?



A *B* *C* *D* *E* *F*

4. What part of the rectangle *A* is $\frac{1}{2}$ of $\frac{1}{2}$ of *A*?

5. Show that $\frac{1}{2}$ of $\frac{1}{3}$ of *B* is the same as $\frac{1}{3}$ of $\frac{1}{2}$ of *B*.

6. What part of *B* is $\frac{1}{3}$ of $\frac{1}{2}$ of *B*?

7. What part of *C* is $\frac{1}{3}$ of $\frac{1}{3}$ of *C*?

8. What part of *D* is $\frac{1}{2}$ of $\frac{1}{4}$ of *D*, or $\frac{1}{4}$ of $\frac{1}{2}$ of *D*?

9. Point to $\frac{1}{2}$ of $\frac{3}{4}$ of *E*; to $\frac{3}{4}$ of $\frac{1}{2}$ of *E*. What part of *E* is $\frac{3}{4}$ of $\frac{1}{2}$ of *E*?

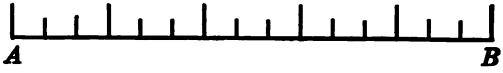
10. Point to $\frac{1}{3}$ of *F*; to $\frac{1}{4}$ of $\frac{1}{3}$ of *F*. Also point to $\frac{1}{4}$ of *F* and to $\frac{1}{3}$ of $\frac{1}{4}$ of *F*. What part of *F* is $\frac{1}{4}$ of $\frac{1}{3}$ of *F*? $\frac{1}{3}$ of $\frac{1}{4}$ of *F*?

11. What part of a foot is $\frac{1}{2}$ of $\frac{1}{3}$ ft.? $\frac{1}{3}$ of $\frac{1}{2}$ ft.? $\frac{1}{3}$ of $\frac{1}{4}$ ft.? $\frac{1}{4}$ of $\frac{1}{3}$ ft.? $\frac{1}{6}$ of $\frac{1}{2}$ ft.? $\frac{1}{2}$ of $\frac{1}{6}$ ft.?

Multiplying by a Fraction. Taking a fractional part of a number is called *multiplying by a fraction.*

We write "$\frac{1}{2}$ of 4" or "$\frac{1}{2} \times 4$," the $\times$ being read "of."

Multiplying a Fraction by a Fraction. For example, let us suppose that we have to multiply $\frac{4}{5}$ by $\frac{2}{3}$.



In this figure let the line AB be divided into five equal parts, and let each of these five equal parts be divided into three equal parts. Then each part is evidently $\frac{1}{15}$ of AB.

That is, $\frac{1}{3}$ of $\frac{1}{5}$ of the line $= \frac{1}{15}$ of the line.

Then $\frac{1}{3}$ of $\frac{4}{5}$ must be 4 times as much, or $\frac{4}{15}$,

and $\frac{2}{3}$ of $\frac{4}{5}$ must be twice $\frac{4}{15}$, or $\frac{8}{15}$.

To multiply a fraction by a fraction, multiply the numerators together for the required numerator and multiply the denominators together for the required denominator. Cancel whenever possible.

For example, find $\frac{11}{12}$ of $\frac{15}{22}$.

We arrange the work as here shown. Cancel 11 from 11 and 22, and 3 from 15 and 12. The result is found to be $\frac{5}{8}$.

$$\frac{11}{12} \text{ of } \frac{15}{22} = \frac{\cancel{11}^{1} \times \overset{5}{\cancel{15}}}{\underset{4}{\cancel{12}} \times \underset{2}{\cancel{22}}} = \frac{5}{8}$$

Teachers are advised to limit the work to simple fractions, such as are used in practical life. In this illustration we have used fractions with larger terms than those commonly needed, this being done for the purpose of illustrating cancellation in as difficult a case as is likely to arise.

The pupils should consider cases like the following:

Draw on the blackboard a rectangle $2\frac{1}{2}$ ft. long and 1 ft. high; mark it off into fifths; mark off each fifth into halves. Then how much is $\frac{1}{2}$ of $\frac{1}{5}$? How much is $\frac{1}{5}$ of $\frac{1}{2}$?

FRACTION OF A FRACTION

1. If Jack walks 1 mi. in $\frac{1}{4}$ hr., how long will it take him to walk $\frac{1}{2}$ mi.? to walk $\frac{1}{3}$ mi.?

In all such cases it is understood that the same rate is maintained.

2. If an express train runs 1 mi. in $\frac{3}{4}$ min., how long will it take the train to run $\frac{5}{8}$ mi.?

3. If a train runs $\frac{2}{3}$ of a certain distance in 1 hr., what part of the distance can it run in $\frac{3}{4}$ hr.?

4. If it takes a train $\frac{3}{4}$ hr. to reach a certain station, in what fraction of an hour will it go $\frac{5}{6}$ of the distance?

5. If Tom lives $\frac{4}{5}$ mi. from school, and Fred lives $\frac{5}{8}$ as far, how far does Fred live from school?

6. If $\frac{5}{8}$ yd. of lace is needed to trim two neckbands, how much lace will be needed to trim one neckband?

7. In a certain race an automobile runs 1 mi. in 49 sec., that is, in $\frac{49}{60}$ min. What part of a minute does it take to run $\frac{5}{8}$ mi.?

Multiply the following:

8. $\frac{1}{8} \times \frac{4}{5}$.
9. $\frac{4}{5} \times \frac{5}{8}$.
10. $\frac{3}{5} \times \frac{5}{9}$.
11. $\frac{3}{4} \times \frac{8}{15}$.
12. $\frac{5}{9} \times \frac{3}{5}$.
13. $\frac{3}{8} \times \frac{7}{8}$.
14. $\frac{2}{3} \times \frac{9}{10}$.
15. $\frac{2}{3} \times \frac{15}{16}$.
16. $\frac{3}{4} \times \frac{5}{24}$.
17. $\frac{7}{16} \times \frac{5}{7}$.
18. $\frac{7}{16} \times \frac{8}{21}$.
19. $\frac{15}{32} \times \frac{4}{5}$.

20. If Jane lives $\frac{7}{8}$ mi. from school, and Madge lives $\frac{1}{3}$ as far, how far from school does Madge live? If Isabel lives $\frac{1}{2}$ as far as Madge, how far from school does Isabel live?

21. Mrs. Sinclair used $\frac{7}{8}$ yd. of velvet for one hat and $\frac{2}{5}$ as much for another hat. How much velvet did she use for both hats?

22. Find the value of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$; the value of $\frac{3}{8}$ of $\frac{4}{5}$ of $\frac{5}{12}$.

Multiplying a Mixed Number by a Fraction. 1. If you buy $4\frac{1}{2}$ lb. of butter at the store and use half of it, how much butter do you use?

$\frac{1}{2}$ of 4 lb. = 2 lb.

$\frac{1}{2}$ of $\frac{1}{2}$ lb. = $\frac{1}{4}$ lb.

Adding 2 lb. and $\frac{1}{4}$ lb., we have $2\frac{1}{4}$ lb., the amount used.

2. If you have $5\frac{1}{4}$ yd. of ribbon and use $\frac{2}{3}$ of it, how much do you use?

$\frac{2}{3}$ of $5\frac{1}{4} = \frac{2}{3}$ of $\frac{21}{4}$.

Canceling as here shown, the result is found to be $3\frac{1}{2}$.

That is, $3\frac{1}{2}$ yd. is the amount of ribbon you use.

$$\frac{2}{3} \text{ of } 5\tfrac{1}{4} = \frac{\cancel{2}}{\cancel{3}} \times \frac{\overset{7}{\cancel{21}}}{\underset{2}{\cancel{4}}} = \frac{7}{2} = 3\tfrac{1}{2}$$

USING WHAT YOU HAVE LEARNED

1. If you have $8\frac{1}{2}$ lb. of sugar and use $\frac{3}{4}$ of it, how many pounds do you use?

2. If you have $7\frac{1}{2}$ lb. of sugar and use $\frac{3}{8}$ of it, how many pounds do you use?

Find the value of each of the following:

3. $\frac{2}{3}$ of $6\frac{1}{2}$ in.
4. $\frac{5}{8}$ of $3\frac{3}{4}$ ft.
5. $\frac{3}{4}$ of $5\frac{7}{8}$ in.
6. $\frac{3}{8}$ of $4\frac{3}{4}$ yd.
7. $\frac{1}{2}$ of $17\frac{3}{4}$ mi.
8. $\frac{2}{3}$ of $25\frac{1}{2}$ mi.
9. $\frac{1}{8}$ of $14\frac{1}{2}$ in.
10. $\frac{3}{8}$ of $21\frac{1}{2}$ lb.
11. $\frac{1}{4}$ of $23\frac{1}{4}$ in.
12. $\frac{3}{4}$ of $15\frac{1}{2}$ yd.
13. $\frac{3}{8}$ of $20\frac{1}{4}$ yd.
14. $\frac{5}{8}$ of $15\frac{3}{4}$ mi.

15. If you are motoring $24\frac{1}{2}$ mi., how far have you gone when you have motored $\frac{5}{8}$ of the distance?

Find the value of each of the following:

16. $\frac{2}{3}$ of $\frac{5}{8}$ of 15 ft.
17. $\frac{3}{8}$ of $\frac{2}{3}$ of $48\frac{1}{2}$ ft.

Multiplying Mixed Numbers. 1. A grocer sells washing soda in boxes of $1\frac{3}{4}$ lb. each. How many pounds of soda would you buy if you bought 6 boxes?

We have to multiply 1 lb., and also $\frac{3}{4}$ lb., each by 6. The work is as here shown.

The result is found to be $10\frac{1}{2}$ lb.

$$\begin{array}{r} 1\frac{3}{4} \\ 6 \\ \hline 6 \times \frac{3}{4} = 4\frac{1}{2} \\ 6 \times 1 = 6 \\ \hline 6 \times 1\frac{3}{4} = 10\frac{1}{2} \end{array}$$

Teachers should place the work on the blackboard and lead the class to take every step. It is not necessary to label the numbers in work of this kind, but it may be done if desired.

This is the case of multiplying a mixed number by a whole number.

2. If washing soda costs 12¢ a pound, how much will $6\frac{1}{2}$ lb. cost?

We have to multiply 12¢ by $6\frac{1}{2}$. We multiply first by $\frac{1}{2}$ and then by 6. Then we add these products. The work is as here shown.

The result is found to be 78¢.

$$\begin{array}{r} 12 \\ 6\frac{1}{2} \\ \hline \frac{1}{2} \text{ of } 12 = 6 \\ 6 \times 12 = 72 \\ \hline 6\frac{1}{2} \times 12 = 78 \end{array}$$

Teachers should place the work on the blackboard and proceed as suggested under Ex. 1.

This is the case of multiplying a whole number by a mixed number.

3. In knitting a scarf, if Mary knits $3\frac{1}{2}$ in. in 1 hr., how many inches can she knit in $2\frac{1}{2}$ hr.?

To multiply $3\frac{1}{2}$ in. by $2\frac{1}{2}$, we reduce each to an improper fraction. Then we multiply as here shown.

$$\begin{array}{r} 3\frac{1}{2} = \frac{7}{2} \\ 2\frac{1}{2} = \frac{5}{2} \\ \hline 2\frac{1}{2} \times 3\frac{1}{2} = \frac{35}{4} \\ = 8\frac{3}{4} \end{array}$$

Teachers should place the work on the blackboard and proceed as suggested under Ex. 1.

This is the case of multiplying a mixed number by a mixed number.

The simpler forms recommended for pupils, after the above work is understood, are given on page 84.

How to Multiply Mixed Numbers. We shall now state how to solve each of the three cases studied on page 83.

To multiply a mixed number by an integer, multiply the fractional part and the whole part separately and add the products.

For example, take the case of $6 \times 3\frac{2}{5}$.

Here we have

$$6 \times \tfrac{2}{5} = 2\tfrac{2}{5}$$
$$6 \times 3 = 18$$
$$6 \times 3\tfrac{2}{5} = 20\tfrac{2}{5}$$

The pupil's work should appear as shown at the right.

$$\begin{array}{r} 3\frac{2}{5} \\ 6 \\ \hline 2\frac{2}{5} \\ 18 \\ \hline 20\frac{2}{5} \end{array}$$

To multiply an integer by a mixed number, multiply by the fractional part and the whole part separately and add the products.

For example, take the case of $6\frac{3}{5} \times 12$.

Here we have

$$\tfrac{3}{5} \times 12 = 7\tfrac{1}{5}$$
$$6 \times 12 = 72$$
$$6\tfrac{3}{5} \times 12 = 79\tfrac{1}{5}$$

The pupil's work should appear as shown at the right.

$$\begin{array}{r} 12 \\ 6\frac{3}{5} \\ \hline 7\frac{1}{5} \\ 72 \\ \hline 79\frac{1}{5} \end{array}$$

To multiply a mixed number by a mixed number, reduce each to an improper fraction and multiply the resulting improper fractions.

For example, take the case of $5\frac{2}{5} \times 3\frac{3}{4}$.

Here we have

$$5\tfrac{2}{5} = \tfrac{27}{5}$$
$$3\tfrac{3}{4} = \tfrac{15}{4}$$
$$\tfrac{27}{\cancel{5}} \times \tfrac{\cancel{15}^{3}}{4} = \tfrac{81}{4} = 20\tfrac{1}{4}$$

The pupil's work should appear as shown at the right.

$$\begin{array}{r} 5\frac{2}{5} = \frac{27}{5} \\ 3\frac{3}{4} = \frac{15}{4} \\ 3 \\ \hline \frac{27}{\cancel{5}} \times \frac{\cancel{15}^{3}}{4} = \frac{81}{4} \\ = 20\frac{1}{4} \end{array}$$

DRILL WORK IN MULTIPLICATION

Multiply the following:

1. $6 \times 3\frac{1}{3}$.
2. $8 \times 3\frac{3}{4}$.
3. $8 \times 2\frac{5}{8}$.
4. $9 \times 3\frac{7}{8}$.
5. $9 \times 4\frac{5}{6}$.
6. $1\frac{1}{2} \times 6$.
7. $2\frac{1}{2} \times 7$.
8. $4\frac{1}{4} \times 12$.
9. $6\frac{3}{4} \times 21$.
10. $9\frac{5}{8} \times 35$.
11. $2\frac{1}{2} \times 2\frac{1}{2}$.
12. $3\frac{1}{3} \times 4\frac{1}{2}$.
13. $6\frac{3}{8} \times 8\frac{3}{4}$.
14. $5\frac{3}{5} \times 4\frac{2}{3}$.
15. $6\frac{5}{8} \times 9\frac{7}{8}$.
16. A rod is $16\frac{1}{2}$ ft. How many feet in 7 rd.?
17. A rod is $5\frac{1}{2}$ yd. How many yards in $2\frac{1}{2}$ rd.?

Multiply the following:

18. $20 \times 1\frac{1}{2}$.
19. $30 \times 2\frac{1}{3}$.
20. $36 \times 4\frac{2}{3}$.
21. $24 \times 1\frac{3}{4}$.
22. $40 \times 3\frac{5}{6}$.
23. $3\frac{1}{2} \times 7$.
24. $5\frac{1}{3} \times 8$.
25. $4\frac{2}{3} \times 50$.
26. $7\frac{1}{6} \times 90$.
27. $8\frac{5}{8} \times 24$.
28. $1\frac{1}{2} \times 2\frac{7}{8}$.
29. $3\frac{1}{2} \times 4\frac{5}{16}$.
30. $1\frac{3}{4} \times 2\frac{7}{16}$.
31. $2\frac{3}{8} \times 4\frac{1}{16}$.
32. $4\frac{7}{8} \times 5\frac{3}{16}$.
33. At $27\frac{3}{4}$ mi. an hour, how far will an automobile go in 3 hr.?
34. At $36\frac{1}{2}$ mi. an hour, how far will a train go in $1\frac{3}{4}$ hr.?

Multiply the following:

35. $24 \times 3\frac{1}{2}$.
36. $36 \times 3\frac{2}{3}$.
37. $48 \times 6\frac{3}{4}$.
38. $40 \times 5\frac{7}{8}$.
39. $96 \times 2\frac{1}{16}$.
40. $3\frac{1}{16} \times 32$.
41. $4\frac{3}{16} \times 64$.
42. $1\frac{5}{16} \times 20$.
43. $2\frac{5}{12} \times 48$.
44. $3\frac{7}{12} \times 15$.
45. $4\frac{3}{8} \times 4\frac{3}{8}$.
46. $2\frac{7}{8} \times 6\frac{7}{8}$.
47. $1\frac{1}{4} \times 3\frac{1}{12}$.
48. $3\frac{3}{4} \times 4\frac{7}{12}$.
49. $2\frac{5}{8} \times 1\frac{11}{12}$.

In each of the groups of 15 abstract problems in the above drill work, test the pupils as to the number of correct results they have found in 4 min., keeping the records for use in future reviews.

USING WHAT YOU HAVE LEARNED

1. If Frank picks on an average $3\frac{3}{4}$ qt. of blueberries in an hour, how many quarts does he pick in 2 hr.?

2. If Alice picks 3 qt. of raspberries per hour, how many quarts does she pick in $2\frac{1}{4}$ hr.?

3. If the length of a paddle of a water wheel in a mill is $\frac{3}{7}$ of the diameter and the diameter is 63 in., what is the length of the paddle?

4. A certain recipe calls for 7 oz. of sugar. To make $3\frac{1}{2}$ times the recipe, how many ounces of sugar must you use?

5. A certain recipe calls for 2 cups of cream. To make $2\frac{1}{2}$ times the recipe, how many cups of cream must you use?

6. If an automobile goes a mile in 57 sec., or $\frac{57}{60}$ min., in a certain race, what part of a minute will it take to go $\frac{3}{5}$ mi. at the same rate?

7. A man has a city lot that contains $\frac{16}{25}$ of an acre. If he sells $\frac{5}{8}$ of it, what part of an acre does he sell? What part of an acre has he left?

8. If it takes $2\frac{5}{8}$ yd. of cloth to make a boy's suit, how many yards will it take to make a dozen suits?

9. A tin cup is found to hold $\frac{15}{16}$ pt. When it is $\frac{2}{3}$ full the cup contains what part of a pint?

10. If a glass jar holds $\frac{15}{16}$ qt., what part of a quart will it contain when $\frac{4}{5}$ full? What part of a quart will it contain when $\frac{2}{5}$ full? when $\frac{3}{5}$ full?

Multiply the following:

11. $16 \times 3\frac{19}{32}$.
12. $18 \times 31\frac{5}{9}$.
13. $37\frac{4}{9} \times 72$.
14. $29\frac{7}{8} \times 64$.
15. $4\frac{1}{3} \times 25\frac{7}{8}$.
16. $12\frac{3}{4} \times 15\frac{2}{3}$.

Parts of a Dollar. Mrs. Roberts can buy preserves at 35¢ a jar or she can buy 3 jars for $1. Since she pays $1 for 3 jars, she pays $\frac{1}{3}$ of $1 for 1 jar. Since $\frac{1}{3}$ of $1 is $33\frac{1}{3}$¢, Mrs. Roberts saves a little on each jar when she buys 3 jars for $1.

We see from this example that we often have to use parts of $1, such as $\frac{1}{3}$ of $1, although we could not pay exactly $33\frac{1}{3}$¢ for any one thing, because there is no American coin that is $\frac{1}{3}$ of a cent.

We need to know certain parts of a dollar for just such cases as the one Mrs. Roberts met in the above example. The following table should therefore be learned:

$5¢ = \frac{1}{20}$ of $1	$20¢ = \frac{1}{5}$ of $1	$62\frac{1}{2}¢ = \frac{5}{8}$ of $1
$6\frac{1}{4}¢ = \frac{1}{16}$ of $1	$25¢ = \frac{1}{4}$ of $1	$66\frac{2}{3}¢ = \frac{2}{3}$ of $1
$8\frac{1}{3}¢ = \frac{1}{12}$ of $1	$33\frac{1}{3}¢ = \frac{1}{3}$ of $1	$75¢ = \frac{3}{4}$ of $1
$12\frac{1}{2}¢ = \frac{1}{8}$ of $1	$37\frac{1}{2}¢ = \frac{3}{8}$ of $1	$83\frac{1}{3}¢ = \frac{5}{6}$ of $1
$16\frac{2}{3}¢ = \frac{1}{6}$ of $1	$50¢ = \frac{1}{2}$ of $1	$87\frac{1}{2}¢ = \frac{7}{8}$ of $1

Teachers will observe that the fifteen different parts of $1 given in the above table are the more important ones that are used in ordinary computation in making purchases. The pupil knows that $1¢ = \frac{1}{100}$ of $1, that $3¢ = \frac{3}{100}$ of $1, and so on, and he does not need drill upon such facts. He will not make much use of the fact that $60¢ = \frac{3}{5}$ of $1, but he will often use the fact that $12\frac{1}{2}¢$ is $\frac{1}{8}$ of $1. The number of facts memorized in arithmetic is not of so much consequence as the importance of these facts in the problems of daily life.

For example, Fred can buy paper pads in quantities of 4 or more at $12\frac{1}{2}¢$ each. How much will 16 such pads cost?

In solving this example it is much easier to find the value of $16 \times \$\frac{1}{8}$, which is $2, than to find the value of $16 \times 12\frac{1}{2}¢$. If you do not think so, multiply in each case and see which is the easier.

Illustrative Problems. 1. At the rate of 3 for \$1, or $33\frac{1}{3}¢$ each, how much will 20 collars cost?

$20 \times \$\frac{1}{3} = \$\frac{20}{3} = \$6\frac{2}{3} = \$6.66\frac{2}{3}$, or \$6.67, the cost of 20 collars.

We write \$6.67 as the result to the nearest cent.

2. At the rate of 6 for \$1, or $16\frac{2}{3}¢$ each, how much will 15 bottles of ink cost?

$15 \times \$\frac{1}{6} = \$2\frac{1}{2} = \$2.50$, the cost of 15 bottles of ink.

3. At $\$1.12\frac{1}{2}$ each, how much will 7 doormats cost?

$\$1.12\frac{1}{2} = \$1 + \$\frac{1}{8}$, the cost of 1 mat.

Multiplying by 7, we have $\$7 + \$\frac{7}{8}$, or $\$7\frac{7}{8}$.

$\$7\frac{7}{8} = \$7.87\frac{1}{2}$, or \$7.88, the cost of 7 mats.

Remember that $\frac{1}{2}¢$ is counted as a whole cent in an answer of this kind.

PARTS OF A DOLLAR

1. At the rate of 3 for \$1, or $33\frac{1}{3}¢$ each, how much must a school pay for 26 spelling books? for 228 spelling books?

2. At the rate of 6 for \$1, how much must a school pay for 160 boxes of crayons? for 112 boxes of crayons?

3. At $33\frac{1}{3}¢$ each, find the cost of 75 handkerchiefs.

4. At $\$1.33\frac{1}{3}$ a book, find the cost of 124 books.

5. At $\$1.16\frac{2}{3}$ a yard, find the cost of 252 yd. of lace.

6. At $\$1.12\frac{1}{2}$ a yard, find the cost of 128 yd. of velvet.

Multiply $\$0.33\frac{1}{3}$ and $\$0.66\frac{2}{3}$ by each of the following:

7. 42. 8. 66. 9. 78. 10. 96. 11. 540.

Multiply $\$0.16\frac{2}{3}$ and $\$0.12\frac{1}{2}$ by each of the following:

12. 48. 13. 96. 14. 144. 15. 192. 16. 240.

DRILL ON PARTS OF A DOLLAR

Numbers 1 to 15, oral

1. At the rate of 3 for \$2, how much will 1 necktie cost?

Express $\frac{1}{3}$ of \$2, or \$$\frac{2}{3}$, in cents as learned in the table on page 87.

2. At the rate of 8 for \$5, how much will 1 comb cost?
3. At the rate of 8 for \$7, how much will 1 book cost?

Multiply the following:

4. $6 \times 33\frac{1}{3}¢$.
5. $8 \times 6\frac{1}{4}¢$.
6. $12 \times 8\frac{1}{3}¢$.
7. $16 \times 12\frac{1}{2}¢$.
8. $18 \times 16\frac{2}{3}¢$.
9. $45 \times 20¢$.
10. $9 \times 33\frac{1}{3}¢$.
11. $8 \times 37\frac{1}{2}¢$.
12. $48 \times 50¢$.
13. $8 \times 62\frac{1}{2}¢$.
14. $9 \times 66\frac{2}{3}¢$.
15. $6 \times 83\frac{1}{3}¢$.
16. At $87\frac{1}{2}¢$ each, how much will 16 plates cost?
17. At $83\frac{1}{3}¢$ each, how much will 36 pitchers cost?

Multiply the following:

18. $120 \times 5¢$.
19. $32 \times 6\frac{1}{4}¢$.
20. $48 \times 8\frac{1}{3}¢$.
21. $48 \times 12\frac{1}{2}¢$.
22. $48 \times 16\frac{2}{3}¢$.
23. $75 \times 20¢$.
24. $96 \times 25¢$.
25. $64 \times 37\frac{1}{2}¢$.
26. $75 \times 50¢$.
27. $72 \times 62\frac{1}{2}¢$.
28. $93 \times 66\frac{2}{3}¢$.
29. $64 \times 75¢$.
30. $78 \times 83\frac{1}{3}¢$.
31. $72 \times 87\frac{1}{2}¢$.
32. $60 \times 8\frac{1}{3}¢$.
33. $72 \times 12\frac{1}{2}¢$.
34. $99 \times 33\frac{1}{3}¢$.
35. $88 \times 37\frac{1}{2}¢$.
36. $176 \times 37\frac{1}{2}¢$.
37. $126 \times 83\frac{1}{3}¢$.
38. $75 \times 62\frac{1}{2}¢$.
39. $125 \times 66\frac{2}{3}¢$.
40. $28 \times 83\frac{1}{3}¢$.
41. $132 \times 87\frac{1}{2}¢$.

Find the number of cents in:

42. $\frac{1}{16}$ of \$1.
43. $\frac{5}{12}$ of \$1.
44. $\frac{5}{16}$ of \$1.
45. $\frac{7}{16}$ of \$1.
46. $\frac{1}{32}$ of \$1.
47. $\frac{3}{32}$ of \$1.

Cash Check. In large stores the clerks are required to fill out cash checks in which they must frequently use the multiplication of fractions. This is a sample cash check:

Sold by No. *68*		Amount rec'd, $*15.00*		
$\frac{1}{2}$ yd.	Silk	1.75		88
$4\frac{8}{8}$ "	Summer silk	.96	4	20
$11\frac{1}{2}$ "	Trimming	.75	8	63
	Total		13	71

In this book, fractions of a cent less than $\frac{1}{2}$¢ in an answer should be discarded, $\frac{1}{2}$¢ or more being counted as 1¢.

CASH CHECKS

Make out cash checks for the following sales, and write after each one the amount of change due:

1. $26\frac{1}{2}$ yd. silk @ $1.75, 8 yd. cotton @ $37\frac{1}{2}$¢, $14\frac{1}{2}$ yd. madras @ 55¢. Amount received, $60.

2. $5\frac{1}{2}$ yd. cheviot @ $3.40, $6\frac{1}{2}$ yd. velveteen @ $2.60, $5\frac{3}{8}$ yd. silk @ $1.75. Amount received, $50.

3. $8\frac{1}{4}$ yd. India linen @ 65¢, $4\frac{5}{8}$ yd. dimity @ 45¢, $12\frac{1}{8}$ yd. linen suiting @ $1.20. Amount received, $25.

4. $9\frac{1}{2}$ yd. India linen @ 64¢, $4\frac{1}{8}$ yd. cheviot @ $3.75, $12\frac{1}{2}$ yd. cotton @ 40¢. Amount received, $30.

5. $3\frac{1}{2}$ yd. linen @ 72¢, $5\frac{1}{2}$ yd. ribbon @ 42¢, $2\frac{7}{8}$ yd. velvet @ $3.20. Amount received, $20.

6. $7\frac{1}{4}$ yd. muslin @ 48¢, $2\frac{1}{8}$ yd. velvet @ $4.40, $3\frac{3}{8}$ yd. silk @ $3.20, $8\frac{1}{4}$ yd. lining @ 24¢, $3\frac{3}{4}$ yd. suiting @ $1.60, $5\frac{1}{2}$ yd. ribbon @ 35¢. Amount received, $50.

DIVIDING FRACTIONS

All work oral

1. How much is 15 in. ÷ 3? $\frac{15}{16} \div 3$? $\frac{15}{32} \div 3$?

State the answers to the following:

2. 20 in. ÷ 4, and $\frac{20}{21} \div 4$.
3. 14 in. ÷ 7, and $\frac{14}{15} \div 7$.
4. 25 qt. ÷ 5, and $\frac{25}{32} \div 5$.
5. 18 yd. ÷ 6, and $\frac{18}{25} \div 6$.

6. Dividing the numerator of a fraction by a whole number does what to the value of the fraction?

7. If you divide $\frac{1}{2}$ of this rectangle into two equal parts, each is what part of the rectangle? How much is $\frac{1}{2} \div 2$?

8. If you divide $\frac{3}{4}$ of the rectangle into two equal parts, how many eighths of the rectangle do you then have? How much is $\frac{3}{4} \div 2$? What have you done to the denominator?

9. Multiplying the denominator of a fraction by a whole number does what to the value of the fraction? Illustrate.

Divide the following by dividing the numerators:

10. $\frac{4}{5} \div 2$.
11. $\frac{6}{7} \div 3$.
12. $\frac{15}{16} \div 3$.
13. $\frac{12}{25} \div 6$.
14. $\frac{15}{16} \div 5$.
15. $\frac{16}{25} \div 8$.
16. $\frac{25}{42} \div 5$.
17. $\frac{27}{32} \div 9$.

Divide the following by multiplying the denominators:

18. $\frac{1}{3} \div 5$.
19. $\frac{4}{5} \div 9$.
20. $\frac{2}{3} \div 7$.
21. $\frac{7}{8} \div 3$.
22. $\frac{3}{5} \div 4$.
23. $\frac{3}{8} \div 2$.
24. $\frac{3}{8} \div 7$.
25. $\frac{5}{8} \div 9$.

Divide the following:

26. $\frac{4}{15} \div 2$.
27. $\frac{15}{16} \div 10$.
28. $\frac{3}{5} \div 2$.
29. $\frac{25}{32} \div 5$.
30. $\frac{7}{8} \div 4$.
31. $\frac{5}{12} \div 4$.
32. $\frac{5}{8} \div 2$.
33. $\frac{15}{8} \div 5$.

Dividing a Fraction or a Mixed Number by a Whole Number. From the work given on page 91 we have the following:

Dividing the numerator of a fraction by a whole number divides the fraction by that number.

Multiplying the denominator of a fraction by a whole number divides the fraction by that number.

We also learned on page 47 that

Both terms of a fraction may be multiplied by the same number without changing the value of the fraction.

Both terms of a fraction may be divided by the same number without changing the value of the fraction.

Illustrative Problems. 1. Laura has a piece of cloth $2\frac{1}{3}$ yd. long, and she uses $\frac{1}{4}$ of it in making a dress for her sister's doll. What part of a yard does she use?

We have to find $\frac{1}{4}$ of $2\frac{1}{3}$ yd.

Think of $2\frac{1}{3}$ as $\frac{7}{3}$. We then have an expression in the form of a fraction the numerator of which is $\frac{7}{3}$ and the denominator of which is 4, as shown at the right.

$$\frac{2\frac{1}{3}}{4} = \frac{\frac{7}{3}}{4} = \frac{7}{12}$$

Multiply this fractional numerator ($\frac{7}{3}$) and the denominator (4) by 3, and the result is $\frac{7}{12}$.

The pupil should do all this mentally, simply thinking of $3 \times 2\frac{1}{3}$ as 7, and of 3×4 as 12. Indeed, a little practice will lead him to say "6, 7, 12ths."

2. Divide $4\frac{2}{3}$ by 7.

Multiply by 3 the numerator ($4\frac{2}{3}$) and the denominator (7) of the fractional form here shown.

The numerator is then $3 \times 4 + 2$, or 14.

The denominator is 3×7, or 21.

The result is $\frac{14}{21}$, or $\frac{2}{3}$ in the lowest terms.

$$\frac{4\frac{2}{3}}{7} = \frac{14}{21} = \frac{2}{3}$$

As before, a pupil will soon come to say simply "12, 14, 21sts."

Further Illustrative Problems. 1. Divide $\frac{4}{7}$ by 2.

Simply divide the numerator by 2, the denominator remaining unchanged, as here shown.

$$\frac{4 \div 2}{7} = \frac{2}{7}$$

In dividing a fraction by a whole number, always divide the numerator, if it is exactly divisible, instead of multiplying the denominator. It makes the work shorter.

2. Divide $\frac{5}{8}$ by 3.

Simply multiply the denominator by 3, the numerator remaining unchanged, as here shown.

$$\frac{5}{3 \times 8} = \frac{5}{24}$$

3. Divide $\frac{15}{16}$ by 10.

Indicate the multiplying of the denominator by 10 and then cancel as here shown.

$$\frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}} \times 16} = \frac{3}{32}$$

This is shorter than to multiply the denominator and then cancel. By shortening the work we save time and avoid errors.

4. Divide $45\frac{1}{3}$ by 3.

It is often easier to divide the whole part first. That is, $45 \div 3 = 15$, and $\frac{1}{3} \div 3 = \frac{1}{9}$.

$$\frac{45\frac{1}{3}}{3} = 15\frac{1}{9}$$

If we wished, we could have said that

$45\frac{1}{3} = \frac{136}{3}$, and $\frac{136}{3} \div 3 = \frac{136}{9} = 15\frac{1}{9}$,

but the work would have been longer.

5. Divide $244\frac{1}{2}$ by 3.

First reduce the mixed number to the improper fraction $\frac{489}{2}$, and then divide this fraction by 3 as shown. We divide the numerator since it is exactly divisible by 3, as shown in Ex. 1, and reduce the result to a mixed number.

$$\frac{244\frac{1}{2}}{3} = \frac{489}{2} \div 3$$
$$= \frac{163}{2}$$
$$= 81\frac{1}{2}$$

DIVIDING BY A WHOLE NUMBER

1. Mary has $\frac{1}{2}$ lb. of sugar. She divides this into 3 equal parts, as her cookbook directs. What part of a pound has she in each part?

2. Ruth is making some children's dresses. She is given $13\frac{1}{8}$ yd. of cloth with which to make 5 dresses. How much cloth should she allow for each dress?

3. A sewing club uses $6\frac{3}{4}$ yd. of flannel in making 4 long wrappers for babies. How much flannel should be allowed for each wrapper?

4. A girl finds that she can knit 4 in. on a stocking in $1\frac{1}{4}$ hr. What part of an hour does it take per inch? How much time should she allow for knitting 8 in.?

5. Martha finds that she needs 1 lb. 9 oz., or $1\frac{9}{16}$ lb. of wool for knitting 5 socks. What part of a pound does she need for 1 sock? for a pair of socks?

6. If an automobile goes $67\frac{1}{2}$ mi. in 3 hr., what is the average rate per hour? What would be the average rate per hour if it goes $112\frac{1}{2}$ mi. in 5 hr.?

Divide the following:

7. $\frac{1}{2} \div 3$.	15. $\frac{1}{8} \div 3$.	23. $\frac{1}{4} \div 3$.	31. $24\frac{1}{2} \div 3$.
8. $\frac{1}{2} \div 4$.	16. $\frac{2}{3} \div 2$.	24. $\frac{1}{4} \div 4$.	32. $25\frac{1}{2} \div 4$.
9. $\frac{2}{5} \div 5$.	17. $\frac{4}{5} \div 2$.	25. $\frac{1}{8} \div 3$.	33. $39\frac{3}{5} \div 3$.
10. $\frac{5}{8} \div 5$.	18. $\frac{5}{8} \div 3$.	26. $\frac{5}{8} \div 4$.	34. $28\frac{1}{8} \div 9$.
11. $\frac{6}{7} \div 3$.	19. $\frac{6}{7} \div 4$.	27. $\frac{6}{7} \div 6$.	35. $64\frac{4}{5} \div 4$.
12. $\frac{6}{5} \div 2$.	20. $\frac{3}{8} \div 4$.	28. $\frac{7}{8} \div 7$.	36. $78\frac{3}{4} \div 5$.
13. $\frac{5}{8} \div 7$.	21. $\frac{3}{8} \div 6$.	29. $\frac{5}{8} \div 8$.	37. $85\frac{5}{16} \div 5$.
14. $\frac{6}{7} \div 2$.	22. $\frac{3}{5} \div 4$.	30. $\frac{3}{4} \div 9$.	38. $634\frac{1}{2} \div 5$.

Dividing by a Unit Fraction. How many thirds of an inch in 1 in.? Then $1 \div \frac{1}{3}$ is how many? What can you say about the values of $1 \div \frac{1}{3}$ and 3×1?

In the same way, what can you say about the values of $3 \div \frac{1}{2}$ and 2×3?

What can you say about the values of $\frac{3}{4} \div \frac{1}{8}$ and $3 \times \frac{3}{4}$?

To divide a number by a unit fraction, multiply the number by the denominator of the fraction.

Thus, $1 \div \frac{1}{8} = 8 \times 1 = 8$,

$3 \div \frac{1}{8} = 8 \times 3 = 24$,

and $\frac{3}{4} \div \frac{1}{8} = 8 \times \frac{3}{4} = 6$.

USING WHAT YOU HAVE LEARNED

1. If a sheet of veneer is $\frac{1}{16}$ in. thick, how many sheets pressed together will have a thickness of $\frac{3}{16}$ in.?

2. If a sheet of veneer is $\frac{1}{16}$ in. thick, how many sheets pressed together will have a thickness of $2\frac{1}{4}$ in.?

3. If a sheet of blotting paper is $\frac{1}{32}$ in. thick, how many sheets are there in a pile that is $12\frac{7}{8}$ in. thick?

4. If a wagon wheel makes $\frac{1}{12}$ of a revolution in going 1 ft., how many feet will it go in making 144 revolutions?

Divide the following:

5. $32 \div \frac{1}{8}$.
6. $24 \div \frac{1}{16}$.
7. $15 \div \frac{1}{12}$.
8. $24 \div \frac{1}{12}$.
9. $36 \div \frac{1}{16}$.
10. $\frac{5}{6} \div \frac{1}{12}$.
11. $3\frac{1}{2} \div \frac{1}{2}$.
12. $4\frac{3}{4} \div \frac{1}{4}$.
13. $5\frac{7}{8} \div \frac{1}{8}$.
14. $3\frac{4}{5} \div \frac{1}{5}$.
15. $2\frac{2}{5} \div \frac{1}{5}$.
16. $3\frac{5}{8} \div \frac{1}{8}$.
17. $4\frac{3}{4} \div \frac{1}{12}$.
18. $7\frac{3}{8} \div \frac{1}{16}$.
19. $5\frac{3}{16} \div \frac{1}{16}$.
20. $2\frac{3}{4} \div \frac{1}{24}$.
21. $3\frac{2}{3} \div \frac{1}{24}$.
22. $5\frac{7}{8} \div \frac{1}{32}$.
23. $2\frac{5}{8} \div \frac{1}{64}$.
24. $3\frac{7}{8} \div \frac{1}{64}$.

DIVIDING BY A FRACTION

All work oral

1. Suppose that you have a room 18 ft. wide and wish to find how many strips of carpet $\frac{3}{4}$ yd. wide will be needed to run lengthwise in the room. First you must find how many yards there are in 18 ft. How many yards are there?

2. If you have to divide 6 yd. by $\frac{3}{4}$ yd., you may think of the 6 yd. as fourths of a yard. Express 6 as fourths.

3. If now you have to divide $\frac{24}{4}$ by $\frac{3}{4}$, you may think of the results in the cases of $24¢ \div 3¢$ and 24 in. $\div$ 3 in. What are the results?

4. Then how much is $\frac{24}{4} \div \frac{3}{4}$? How many strips of carpet will you need to carpet the room?

5. Divide $\frac{6}{5}$ by $\frac{2}{5}$. Multiply $\frac{6}{5}$ by $\frac{5}{2}$.

6. Show that $\frac{4}{5} \div \frac{2}{5}$ gives the same result as $\frac{5}{2} \times \frac{4}{5}$.

7. Study the following:

$$15 \div \frac{3}{5} = \frac{5}{\cancel{3}} \times \overset{5}{\cancel{15}} = 25 \qquad \frac{3}{8} \div \frac{5}{16} = \frac{\overset{2}{\cancel{16}}}{5} \times \frac{3}{\cancel{8}} = \frac{6}{5} = 1\tfrac{1}{5}$$

It should now be clear to the pupil that division by a fraction is a case of multiplication; that is, we divide by multiplying by the fraction inverted. Since the multiplier is always placed first, this practice should be continued, as in the examples above, in order to avoid learning a new order of the factors for this special case.

8. How many paper pads, each $\frac{5}{16}$ in. thick, will it take to make a package $\frac{10}{16}$ in., or $\frac{5}{8}$ in., thick?

Divide the following:

9. $2 \div \frac{2}{3}$.
10. $4 \div \frac{2}{3}$.
11. $3 \div \frac{3}{4}$.
12. $9 \div \frac{3}{4}$.
13. $4 \div \frac{4}{5}$.
14. $\frac{5}{8} \div \frac{5}{4}$.
15. $\frac{7}{8} \div \frac{7}{8}$.
16. $5 \div \frac{1}{2}$.

Dividing by a Fraction or a Mixed Number. If the terms of a fraction are interchanged, the resulting fraction is called the *reciprocal* of the fraction. The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.

On page 96 we found that we often need to divide by a fraction. We also found how to do this. We know that when we divide by $\frac{1}{3}$ we may multiply by 3, and when we multiply a divisor by 2 we divide the quotient by 2.

That is, $6 \div \frac{1}{3} = 3 \times 6$,

and $6 \div \frac{2}{3} =$ half as much.

That is, $6 \div \frac{2}{3} = \frac{3}{2} \times 6 = 9$.

$$\frac{3}{\frac{3 \times \cancel{6}}{\cancel{2}}} = 9$$

Therefore, *to divide any number by a fraction, multiply the number by the reciprocal of that fraction.*

Illustrative Problems. 1. Divide $\frac{3}{8}$ by $\frac{5}{6}$.

Since $\frac{3}{8} \div \frac{5}{6} = \frac{6}{5} \times \frac{3}{8}$,

we may cancel as here shown, and the result is found to be $\frac{9}{20}$.

$$\frac{3}{8} \div \frac{5}{6} = \frac{\cancel{6}}{5} \times \frac{3}{\cancel{8}_{4}} = \frac{9}{20}$$

This plan is usually less liable to error than the plan of considering the fractional form $\frac{\frac{3}{8}}{\frac{5}{6}}$ and multiplying both terms by 6 × 8, or even by 24.

2. Divide $4\frac{2}{3}$ by $2\frac{4}{5}$.

Since $4\frac{2}{3} = 1\frac{4}{3}$

and $2\frac{4}{5} = 1\frac{4}{5}$,

we have $1\frac{4}{3} \div 1\frac{4}{5} = \frac{5}{14} \times 1\frac{4}{3}$.

$$4\tfrac{2}{3} \div 2\tfrac{4}{5} = \frac{14}{3} \div \frac{14}{5} = \frac{5}{\cancel{14}} \times \frac{\cancel{14}}{3} = \frac{5}{3} = 1\tfrac{2}{3}$$

Canceling as here shown we have the improper fraction $\frac{5}{3}$. We reduce this improper fraction to a mixed number and the result is found to be $1\frac{2}{3}$, as shown above.

Further Illustrative Problems. 1. Divide 3 by $2\frac{2}{3}$.

Instead of reducing $2\frac{2}{3}$ to an improper fraction, the more mature pupils may proceed as on page 92, thus:

Think of the fractional form $\frac{3}{2\frac{2}{3}}$ and multiply both terms by 3.

We then have $\frac{9}{3 \times 2 + 2} = \frac{9}{6 + 2} = \frac{9}{8} = 1\frac{1}{8}$.

2. Divide $7\frac{3}{5}$ by $4\frac{1}{3}$.

Multiply by 3×5 both terms of the fractional form $\frac{7\frac{3}{5}}{4\frac{1}{3}}$ and we have

$$\frac{15 \times 7 + 9}{15 \times 4 + 5} = \frac{105 + 9}{60 + 5} = \frac{114}{65} = 1\frac{49}{65}.$$

DRILL WORK IN DIVISION

In Exs. 1–16 first divide as indicated and then divide the second fraction by the first:

1. $\frac{7}{8} \div \frac{2}{3}$.	**5.** $\frac{5}{8} \div \frac{3}{5}$.	**9.** $\frac{9}{10} \div \frac{4}{5}$.	**13.** $\frac{5}{64} \div \frac{7}{32}$.
2. $\frac{5}{6} \div \frac{3}{4}$.	**6.** $\frac{3}{8} \div \frac{3}{4}$.	**10.** $\frac{5}{12} \div \frac{7}{8}$.	**14.** $\frac{3}{8} \div \frac{5}{16}$.
3. $\frac{3}{8} \div \frac{5}{6}$.	**7.** $\frac{3}{4} \div \frac{5}{8}$.	**11.** $\frac{3}{8} \div \frac{5}{12}$.	**15.** $\frac{3}{40} \div \frac{6}{25}$.
4. $\frac{5}{8} \div \frac{1}{5}$.	**8.** $\frac{5}{6} \div \frac{5}{8}$.	**12.** $\frac{7}{16} \div \frac{3}{32}$.	**16.** $\frac{7}{48} \div \frac{7}{12}$.

17. How many strips of ribbon, each $2\frac{3}{4}$ yd. long, can be cut from a piece of ribbon that is $13\frac{3}{4}$ yd. long?

Divide the following:

18. $3 \div \frac{7}{8}$.	**22.** $2 \div 3\frac{1}{4}$.	**26.** $3\frac{1}{16} \div 2\frac{1}{8}$.	**30.** $9\frac{7}{12} \div 5\frac{3}{4}$.
19. $5 \div \frac{2}{3}$.	**23.** $3 \div 2\frac{1}{2}$.	**27.** $2\frac{1}{8} \div 3\frac{1}{16}$.	**31.** $5\frac{1}{4} \div 18\frac{3}{8}$.
20. $8 \div \frac{3}{16}$.	**24.** $5 \div 2\frac{2}{3}$.	**28.** $4\frac{3}{16} \div 3\frac{3}{8}$.	**32.** $3\frac{9}{20} \div 2\frac{3}{10}$.
21. $15 \div \frac{5}{32}$.	**25.** $2 \div 5\frac{1}{3}$.	**29.** $3\frac{3}{8} \div 4\frac{3}{16}$.	**33.** $4\frac{5}{9} \div 13\frac{2}{3}$.

USING WHAT YOU HAVE LEARNED

1. How many half-yard pieces can be made by cutting 9 yd. of ribbon? by cutting 15 yd. of ribbon?

2. If your recipe for making cake calls for $\frac{1}{2}$ lb. of sugar, $7\frac{1}{2}$ lb. of sugar is enough for how many times as much cake?

3. How many mats, each $\frac{3}{4}$ yd. long, can be made from a strip of carpet 12 yd. long?

4. How many towels, each requiring $\frac{5}{8}$ yd. of toweling, can be made from a strip $11\frac{7}{8}$ yd. long?

5. How many strips of cloth, each $5\frac{3}{8}$ yd. long, can we get from a strip $48\frac{3}{8}$ yd. long?

6. How many city lots, each with $31\frac{3}{4}$ ft. frontage, can a man get from a piece of land having $158\frac{3}{4}$ ft. frontage?

7. How many yards of cloth at $66\frac{2}{3}¢$ a yard can be bought for \$8? for \$17? for \$36?

8. How many books, each $\frac{7}{8}$ in. thick, will it take to make a pile $23\frac{5}{8}$ in. high?

9. How many strips of wood, each $\frac{9}{16}$ in. thick, will it take to make a pile $19\frac{1}{16}$ in. high?

10. How many sheets of bookbinding board, each $\frac{3}{32}$ in. thick, will it take to make a pile $9\frac{3}{8}$ in. high?

11. If the product of two numbers is 58, and one of the numbers is $3\frac{5}{8}$, what is the other number?

Divide the following:

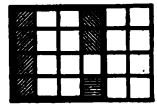
12. $21 \div 4\frac{2}{3}$.
13. $48 \div 3\frac{5}{6}$.
14. $36 \div 1\frac{1}{5}$.
15. $24 \div 4\frac{4}{5}$.
16. $16\frac{4}{5} \div 4\frac{1}{5}$.
17. $15\frac{1}{3} \div 3\frac{5}{6}$.
18. $17\frac{1}{2} \div 2\frac{1}{2}$.
19. $42\frac{2}{5} \div 10\frac{2}{3}$.
20. $20\frac{1}{8} \div 15\frac{1}{3}$.
21. $30\frac{1}{4} \div 15\frac{1}{8}$.
22. $27\frac{3}{4} \div 9\frac{1}{4}$.
23. $45\frac{3}{5} \div 15\frac{1}{5}$.

REVIEW OF IMPORTANT PRINCIPLES

All work oral

1. Point to $\frac{1}{2}$ and also to $\frac{1}{4}$ of square *A*. Point to $\frac{8}{16}$ and also to $\frac{4}{16}$ of square *A*. We see that $\frac{1}{2}$ is how many times $\frac{1}{4}$?

A



B

C

2. In rectangle *B* show that $\frac{2}{8} = \frac{4}{16} = \frac{8}{32} = \frac{16}{24}$.

3. In circle *C* show that $\frac{4}{8} = \frac{1}{2}$.

4. Point to $\frac{3}{16}$ of this line; also to $\frac{6}{16}$ of the line. We see that $\frac{6}{16}$ is how many times $\frac{3}{16}$? How is the value of the fraction changed when we multiply the numerator by 2? when we divide the numerator by 2?

These figures can be used more advantageously if drawn on the board.

5. From Exs. 1 to 4 we again see the truth of certain important principles. Read the following statements aloud and give an example illustrating each:

Multiplying the numerator of a fraction by any number multiplies the fraction by that number.

Dividing the numerator of a fraction by any number divides the fraction by that number.

Multiplying the denominator of a fraction by any number divides the fraction by that number.

Dividing the denominator of a fraction by any number multiplies the fraction by that number.

Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

Fractional Parts. On page 54 we studied about fractional parts. We shall now review that work and study more about such parts.

1. In a running race the distance is 150 yd. After Will has run 60 yd., what part of the distance has he run?

He has run $\frac{60}{150}$ of the distance.

Reducing $\frac{60}{150}$ to lowest terms by dividing both terms by 30, we find that $\frac{60}{150} = \frac{2}{5}$.

So Will has run $\frac{2}{5}$ of the distance.

2. Suppose that the distance in Ex. 1 is $\frac{3}{4}$ mi. and that Will has run $\frac{1}{16}$ mi., what part of the distance has he run?

The fraction in Ex. 1 now takes the fractional form $\frac{\frac{1}{16}}{\frac{3}{4}}$, and we have

$$\frac{1}{16} \div \frac{3}{4} = \frac{\cancel{4}}{3} \times \frac{1}{\cancel{16}_{4}} = \frac{1}{12}.$$

So Will has run $\frac{1}{12}$ of the distance.

We see, as on page 54, that the method is as follows:

To find what fraction the first of two numbers is of the second, divide the first number by the second and express the result in lowest terms.

3. In a spelling test Fred had only $\frac{3}{4}$ of his words correctly spelled. If he had 60 correctly spelled, how many words were there in the test?

$\frac{3}{4}$ of some number is 60,

so $\frac{1}{4}$ of the number is $\frac{1}{3}$ of 60,

and $\frac{4}{4}$ of the number, that is, the number itself, is $\frac{4}{3}$ of 60.

Thus $\frac{4}{\cancel{3}}$ of $\cancel{60}^{20} = 80$, the number of words in the test.

FRACTIONAL PARTS

1. Mabel has $\frac{3}{4}$ yd. of lace and she needs $\frac{3}{8}$ yd. for trimming. What part of the $\frac{3}{4}$ yd. does she need?

2. Maude has $2\frac{1}{2}$ yd. of velvet and needs $\frac{5}{8}$ yd. of it for trimming a hat. What part of the velvet does she need?

Find what part the first of these numbers is of the second:

3. $\frac{2}{3}$, $\frac{3}{4}$.
4. $\frac{3}{4}$, $\frac{7}{8}$.
5. $\frac{2}{5}$, $\frac{3}{4}$.
6. $\frac{1}{16}$, $\frac{3}{8}$.
7. $\frac{3}{16}$, $\frac{5}{8}$.
8. $\frac{5}{16}$, $\frac{7}{8}$.
9. $1\frac{1}{2}$, $4\frac{1}{2}$.
10. $2\frac{2}{3}$, 4.
11. $3\frac{5}{8}$, $4\frac{5}{6}$.
12. $16\frac{2}{3}$, $66\frac{2}{3}$.
13. $14\frac{1}{6}$, $42\frac{1}{2}$.
14. $16\frac{2}{3}$, $22\frac{1}{2}$.

15. John is 14 yr. old and his brother is 20 yr. old. John's age is what part of his brother's age?

16. Frank is 12 yr. old and his brother is 18 yr. old. Six years ago Frank's age was what part of his brother's? Six years from now it will be what part of his brother's age?

17. In an examination Arthur answered $\frac{9}{10}$ of the questions correctly. If he answered 18 questions correctly, how many questions were there in the examination?

Find the number of which:

18. 25 is $\frac{5}{7}$.
19. 32 is $\frac{8}{9}$.
20. 42 is $\frac{6}{7}$.
21. $\frac{1}{2}$ is $\frac{1}{2}$.
22. $\frac{1}{2}$ is $\frac{1}{4}$.
23. $\frac{1}{8}$ is $\frac{3}{4}$.
24. $2\frac{1}{3}$ is $\frac{1}{2}$.
25. $3\frac{3}{4}$ is $\frac{3}{8}$.
26. $5\frac{2}{5}$ is $\frac{9}{10}$.
27. 6 is $\frac{3}{8}$.
28. 9 is $\frac{6}{7}$.
29. $3\frac{3}{5}$ is $\frac{3}{16}$.

30. Emily starts to walk to Fanny's house. After she has walked $\frac{3}{4}$ mi. she has walked $\frac{3}{8}$ of the distance. Find the distance.

31. George starts in his automobile to visit a friend who lives 60 mi. away. After he has gone $37\frac{1}{2}$ mi., what part of the distance has he gone?

Cancellation. Since a division may be written as a fraction, *equal factors may be canceled from both dividend and divisor without changing the quotient.*

1. Divide $2 \times 7 \times 15$ by $21 \times 5 \times 6$.

Writing this as a fraction, 2 is canceled from 2 and 6; 7 is canceled from 7 and 21; 3 is canceled from 15 and 3 (which is left from the 21); 5 is canceled from 5 (which is left from the 15) and 5.

$$\frac{\overset{5}{\cancel{2}} \times \cancel{7} \times \overset{1}{\cancel{15}}}{\underset{3}{\cancel{21}} \times \cancel{5} \times \underset{3}{\cancel{6}}} = \frac{1}{3}$$

The quotient is $\frac{1}{3}$, for 1 is a factor of the numerator.

2. Find the product of $\frac{1}{5}$, $3\frac{3}{8}$, $18\frac{3}{4}$, and $30\frac{2}{3}$.

Reducing to improper fractions, we have

$$3\tfrac{3}{8} = \tfrac{27}{8},$$

$$18\tfrac{3}{4} = \tfrac{75}{4},$$

and $30\frac{2}{3} = \frac{92}{3}$. Canceling, as shown, the product is $388\frac{1}{8}$.

$$\frac{1 \times \overset{9}{\cancel{27}} \times \overset{15}{\cancel{75}} \times \overset{23}{\cancel{92}}}{\cancel{5} \times 8 \times \cancel{4} \times \cancel{3}} = \frac{3105}{8} = 388\tfrac{1}{8}$$

Never multiply till you have to; always cancel if you can.

CANCELLATION

Find the value of each of the following:

1. $\dfrac{7 \times 21 \times 77 \times 6}{49 \times 33 \times 42}$.

2. $\dfrac{20 \times 25 \times 36 \times 48}{27 \times 40 \times 15 \times 12}$.

3. $\dfrac{32 \times 27 \times 44 \times 39}{55 \times 72 \times 36 \times 8 \times 3}$.

4. $\dfrac{125 \times 108 \times 51 \times 8}{40 \times 45 \times 85 \times 12 \times 3}$.

Multiply, using cancellation:

5. $13\frac{1}{3} \times 32\frac{1}{5} \times 16\frac{7}{8}$.

6. $27\frac{1}{5} \times 24\frac{1}{2} \times 37\frac{1}{2}$.

7. $37\frac{1}{2} \times 22\frac{1}{4} \times 12\frac{4}{5}$.

8. $42\frac{2}{3} \times 66\frac{2}{3} \times 74\frac{1}{4}$.

9. $7\frac{3}{5} \times 2\frac{1}{7} \times 5\frac{1}{4} \times 21\frac{1}{3}$.

10. $6\frac{3}{4} \times 7\frac{3}{4} \times 9\frac{1}{3} \times 12\frac{1}{3}$.

REVIEW OF FRACTIONS

Add the following:

1. $\frac{1}{2} + \frac{1}{3}$.
2. $\frac{1}{5} + \frac{1}{8}$.
3. $7\frac{1}{2} + 3\frac{1}{3}$.
4. $2\frac{1}{5} + 5\frac{1}{8}$.
5. $2\frac{1}{4} + 3\frac{1}{6}$.
6. $5\frac{1}{5} + 6\frac{2}{3}$.
7. $9\frac{3}{4} + 7\frac{1}{5}$.
8. $7\frac{5}{6} + 9\frac{3}{8}$.

Subtract the following:

9. $\frac{4}{5} - \frac{1}{2}$.
10. $\frac{3}{4} - \frac{2}{5}$.
11. $9\frac{4}{5} - 2\frac{1}{2}$.
12. $9\frac{3}{4} - 7\frac{2}{5}$.
13. $8\frac{3}{8} - 7\frac{1}{2}$.
14. $3\frac{3}{8} - 1\frac{7}{8}$.
15. $8\frac{3}{4} - 3\frac{7}{8}$.
16. $8\frac{1}{2} - 6\frac{7}{8}$.

Multiply the following:

17. $5 \times 2\frac{2}{5}$.
18. $8 \times 9\frac{7}{8}$.
19. $1\frac{3}{4} \times 9$.
20. $6\frac{1}{3} \times 7\frac{1}{5}$.
21. $6\frac{3}{8} \times 4\frac{1}{2}$.
22. $5\frac{1}{3} \times 3\frac{1}{2}$.
23. $2\frac{7}{8} \times 5\frac{1}{2}$.
24. $7\frac{1}{5} \times 4\frac{1}{4}$.

Divide the following:

25. $\frac{5}{8} \div 2\frac{1}{5}$.
26. $\frac{2}{3} \div 3\frac{1}{3}$.
27. $3\frac{5}{8} \div \frac{3}{8}$.
28. $7\frac{1}{3} \div \frac{2}{3}$.
29. $5\frac{3}{8} \div 4\frac{2}{3}$.
30. $\frac{4}{5} \div 13\frac{1}{2}$.
31. $7\frac{3}{8} \div 1\frac{3}{4}$.
32. $5\frac{5}{8} \div 1\frac{1}{3}$.

Find the value of each of the following:

33. $\frac{3}{5}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of 1 cu. ft.
34. $\frac{1}{2}$ of 5×86 sq. ft.
35. $\frac{7}{8}$ of 4×164 sq. in.
36. $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{3}{8}$ of 1 cu. in.
37. $\frac{3}{5}$ of 15×485 sq. ft.
38. $\frac{3}{7}$ of 13×147 sq. in.

39. Allowing $3\frac{3}{8}$ yd. to a suit of clothes, how many suits can be made from a roll of cloth containing 108 yd.?

40. To divide a sheet of paper $7\frac{1}{8}$ in. wide into 4 equal columns, what distance must be spaced off for each column?

41. Allowing 196 lb. of flour to a barrel and $\frac{3}{4}$ lb. to a loaf, how many loaves of bread can be baked from 63 bbl. of flour?

42. What is the cost of $6\frac{1}{4}$ lb. of mackerel @ 32¢, $3\frac{1}{2}$ lb. of codfish @ 28¢, and 4 cans of salmon at the rate of 2 for 45¢?

II. COMPOUND NUMBERS

Denominate Number. A concrete number that denotes measure is called a *denominate number.* Thus, $8 and 6 bu. 2 pk. are denominate numbers, but 7 men and 8 chairs are not.

Compound Number. A denominate number in which two or more units are expressed is called a *compound number.* For example, 7 pk. 4 qt. and 2 mi. 8 rd. 5 ft. are compound numbers.

Reduction. We may express 4 ft. 3 in. as 51 in., or as $4\frac{1}{4}$ ft. Such a change from one form of a denominate number to another without changing the value is called *reduction.*

Illustrative Examples. 1. Reduce $8\frac{1}{2}$ ft. to inches.

Since $1 \text{ ft.} = 12 \text{ in.}$,
we see that

$$8\tfrac{1}{2} \text{ ft.} = 8\tfrac{1}{2} \times 12 \text{ in.}$$

We therefore have to multiply 12 in. by $8\frac{1}{2}$, and the result is 102 in.

$$1 \text{ ft.} = 12 \text{ in.}$$
$$8\tfrac{1}{2} \text{ ft.} = 8\tfrac{1}{2} \times 12 \text{ in.}$$
$$= 102 \text{ in.}$$

2. Reduce 102 in. to feet.

Since $1 \text{ in.} = \frac{1}{12} \text{ ft.}$,
we see that

$$102 \text{ in.} = 102 \times \tfrac{1}{12} \text{ ft.}$$

We therefore have to multiply $\frac{1}{12}$ ft. by 102, which gives $\frac{102}{12}$ ft.

$$1 \text{ in.} = \tfrac{1}{12} \text{ ft.}$$
$$102 \text{ in.} = 102 \times \tfrac{1}{12} \text{ ft.}$$
$$= 8\tfrac{1}{2} \text{ ft.}$$

Dividing 102 by 12, we have $8\frac{1}{2}$, and so the result is $8\frac{1}{2}$ ft.

Reduction to a lower denomination was formerly called *reduction descending,* while reduction to a higher denomination was called *reduction ascending.* Terms and definitions like these have little value, the subject of compound numbers having lost its former importance. We speak of 2 ft. 3 in. instead of $2\frac{1}{4}$ ft., but we rarely use other such denominations together. Thus we usually speak of 2 lb. 4 oz. as $2\frac{1}{4}$ lb.

Table of Length. The following is the table of length:

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

The teacher may call attention to such additional facts as these:

The *hand* (4 in.) is used in measuring the height of horses at the shoulder; sailors use the *fathom* (6 ft.) for measuring depths, and the *knot* (nautical mile = $1\frac{3}{10}$ common, or statute, miles = $6080\frac{1}{2}$ ft.) for distances at sea. Such facts need not be learned.

REVIEW OF TABLE OF LENGTH

1. If you live $\frac{1}{4}$ mi. from school, how many feet must you walk in coming to school?

2. In Ex. 1 how many yards must you walk?

Find the number of inches in the following:

3. $8\frac{1}{4}$ ft. 4. $16\frac{1}{2}$ ft. 5. $\frac{1}{2}$ yd. 6. $3\frac{1}{2}$ yd.

Find the number of feet in the following:

7. $8\frac{1}{4}$ yd. 8. 18 yd. 9. $\frac{1}{2}$ rd. 10. $\frac{1}{2}$ mi.

11. If you buy $3\frac{3}{4}$ yd. of velvet, how many inches of velvet do you buy?

12. A field is 40 rd. long. Express the length in feet; in yards; as a fraction of a mile.

13. The distance between two railway stations is known to be $9\frac{3}{4}$ mi. Express the distance in miles and feet.

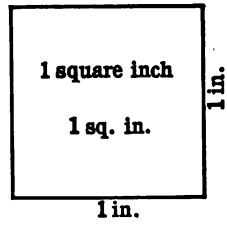
14. If a horse is 16 hands high, and a hand is 4 in., what is the height of the horse in inches? in feet and inches?

Table of Square Measure. The following is the table of square measure:

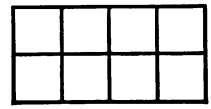
144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Carpenters, architects, and mechanics often write 8″ for 8 in., and 5′ for 5 ft. They also use sq.″ and sq.′ for square inches and square feet. In this book all of these forms are used.

If a square is 1 in. on a side, it is 1 in. square, and its area is 1 sq. in.



If a rectangle is 4 in. long and 2 in. wide, its area is 4 × 2 sq. in., or 8 sq. in.



The pupils may express this in either of these two forms:

4 × 2 sq. in. = 8 sq. in.,

or 2 × 4 sq. in. = 8 sq. in.

In practical work we simply multiply 4 by 2, thinking, "There are *as many* square inches as 2 × 4, or 8 sq. in."

REVIEW OF SQUARE MEASURES

1. Find the number of square inches in the top of a desk the area of which is $3\frac{1}{2}$ sq. ft.

2. The school platform has an area of $3\frac{1}{2}$ sq. yd. What is its area in square feet?

3. A flower bed has an area of 2 sq. rd. What is its area in square yards? in square feet?

4. Find the number of square feet in 7 sq. yd.; in 1 sq. rd.

MEASURING AND DRAWING

1. Measure this page of the arithmetic. Draw a plan of the page to the scale $\frac{1}{4}$.

As the pupil probably learned in primary arithmetic, to draw to the scale of 1 in. to 100 rd., we represent 100 rd. by 1 in. To draw to the scale $\frac{1}{12}$, we represent 12 in. by 1 in. This should be explained at the blackboard.

2. Measure the top of your desk. Draw a plan to the scale of 1 in. to 1 ft.

3. Draw a plan of the top of your desk to the scale $\frac{1}{8}$.

4. Measure the length and width of the schoolroom. Draw a plan and write below it the scale that you have used.

5. Measure the size of the paper upon which you make your drawings. Draw a plan to the scale $\frac{1}{8}$.

6. Measure the size of any other book you are studying. Draw a plan of a page to the scale $\frac{1}{8}$.

7. A drawing has been made of a box cover. The cover is 14 in. long and the drawing is $3\frac{1}{2}$ in. long. What scale was used?

8. A drawing of a floor 24 ft. long and 20 ft. wide is 6 in. long and 5 in. wide. What scale was used?

9. A drawing is made of a rectangle. The drawing has an area of 6 sq. in. If a scale twice as large were used, what would be the area of the drawing? Draw two plans of the rectangle, so as to show that the answer is correct.

10. A drawing is made of a lamp $18\frac{3}{4}$ in. high, and the drawing is $3\frac{1}{8}$ in. high. What scale was used?

The teacher should have the pupils make measurements of desks, doors, blackboards, and other objects in the schoolroom and make scale drawings from these measurements. The pupils should gradually come to know the meaning of the working drawings used by the carpenter, the builder, and the manufacturer.

Table of Cubic Measure. The following is the table of cubic measure:

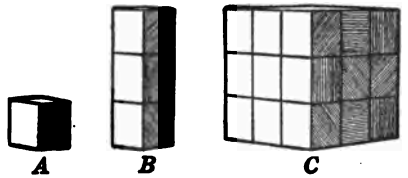
1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

A *perch* of stone or masonry is usually considered as 1 rd. long, 1 ft. high, and $1\frac{1}{2}$ ft. thick, and it contains $24\frac{3}{4}$ cu. ft. It varies, however, in different parts of the country. A cubic yard of earth is considered a *load*.

If a block is 1 in. on an edge, and is in the form of a cube, it is a 1-inch cube, and the volume is 1 cu. in., as in *A*.

If a block is 1 in. square at the base and 3 in. high, the volume is 3 cu. in., as in *B*.

If a block is 3 in. square at the base, the base contains 3 × 3 sq. in., or 9 sq. in. If the block is 3 in. high, the volume is 3 × 9 cu. in., or 27 cu. in., as in *C*.



If a block is 4 in. long, 3 in. wide, and 2 in. high, the volume is

$$4 \times 3 \times 2 \text{ cu. in.} = 24 \text{ cu. in.}$$

REVIEW OF CUBIC MEASURES

1. If your schoolroom is 30 ft. long, 20 ft. wide, and 12 ft. high, how many cubic feet does it contain?

2. The ordinary brick is 2 in. thick, twice as wide as thick, and twice as long as wide. How many cubic inches in a brick?

Find the volumes of boxes of these dimensions:

3. $7\frac{1}{2}$ in., 14 in., 18 in.
4. $4\frac{1}{2}$ in., 8 in., $10\frac{1}{2}$ in.
5. 34 in., 28 in., 16 in.
6. 64 in., 36 in., $18\frac{1}{2}$ in.

Table of Weight. The following is the table of weight:

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)

The teacher should speak to the class about the following matters, but this work is not important enough to be memorized.

The ton of 2000 lb. is sometimes called the *short ton*, there being the *long ton* of 2240 lb., which is used in some transactions in mining products.

Goldsmiths still use an old table of *troy weight*, and apothecaries and physicians use the table of *apothecaries' weight* in prescriptions of medicine, but neither is of enough importance to justify teaching it.

The avoirdupois pound contains 7000 grains, while the troy pound (12 troy ounces) contains 5760 grains.

A *carat* weight, used in weighing diamonds, is (since 1913) 200 milligrams, or 3.08647 grains. Milligrams are weights in the metric system, to be studied later. The word "carat" is also used in speaking of the purity of gold and means $\frac{1}{24}$; 16 carats fine means $\frac{16}{24}$ pure gold.

REVIEW OF MEASURES OF WEIGHT

Numbers 1 to 3, oral

1. If you buy $\frac{1}{2}$ lb. of cheese, how many ounces do you buy?
2. A man buys $\frac{1}{2}$ T. of coal. This is how many pounds?
3. A dealer sells 4000 lb. of coal. This is how many tons?

Find the number of pounds in the following:

4. $2\frac{1}{2}$ T. 5. $7\frac{3}{4}$ T. 6. $15\frac{3}{4}$ T. 7. $17\frac{1}{4}$ T. 8. $26\frac{3}{4}$ T.

Find the number of tons in the following:

9. 2500 lb. 10. 3000 lb. 11. 3750 lb. 12. 4250 lb.

Find the number of ounces in the following:

13. $2\frac{1}{2}$ lb. 14. $2\frac{3}{4}$ lb. 15. $3\frac{1}{4}$ lb. 16. $7\frac{1}{2}$ lb. 17. $9\frac{3}{4}$ lb.

Table of Liquid Measure. The following is the table of liquid measure:

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Teachers should ask the pupils to name the measures in the picture. It should be stated, but not for memorizing, that a gallon contains 231 cu. in., and that barrels vary in size, although in estimating the capacity of tanks and cisterns 31½ gal. are considered a barrel (bbl.), and 2 bbl. a hogshead.

In *apothecaries' liquid measure* 16 fluid ounces make 1 pint.

REVIEW OF LIQUID MEASURES

Find the number of gills in the following:

1. 1½ pt. **2.** 1½ qt. **3.** 3 qt. **4.** ½ gal. **5.** 1½ gal.

Find the number of pints in the following:

6. 8 gi. **7.** 16 gi. **8.** 3½ qt. **9.** ½ gal. **10.** 7½ gal.

Find the number of quarts in the following:

11. 8 gi. **12.** 17 pt. **13.** 16 pt. **14.** ½ gal. **15.** 9½ gal.

Table of Dry Measure. The following is the table of dry measure:

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

REVIEW OF DRY MEASURES

Find the number of pints in the following:

1. $\frac{1}{2}$ qt. **2.** $7\frac{1}{2}$ qt. **3.** 1 pk. **4.** $5\frac{1}{2}$ pk. **5.** 1 bu.

Find the number of quarts in the following:

6. 9 pt. **7.** $\frac{3}{4}$ pk. **8.** $7\frac{1}{2}$ pk. **9.** $2\frac{1}{2}$ bu. **10.** $7\frac{1}{2}$ bu.

Find the number of pecks in the following:

11. 16 pt. **12.** 16 qt. **13.** $3\frac{1}{2}$ bu. **14.** $9\frac{1}{2}$ bu. **15.** $8\frac{3}{4}$ bu.

Find the number of bushels in the following:

16. 32 qt. **17.** 32 pk. **18.** 32 pt. **19.** 64 pk. **20.** 96 pk.

Table of Time. The following is the table of time:

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
About 4 weeks = 1 month (mo.)
12 months = 1 year (yr.)
365 days = 1 common year

April, June, September, and November have 30 da.; the other months have 31 da., except February, which has 28 da. in common years and 29 da. in leap years. The *leap year* has 366 da. A *century* is 100 yr.

For 45 minutes after 7 o'clock we usually write 7.45, 7:45, or 7 45, using A.M. and P.M. respectively to denote forenoon and afternoon.

Years whose numbers are divisible by 4 but not by 100, and those divisible by 400, are leap years. Thus 1928, but not 2100, is a leap year.

Table of Money. The following is the table of money:

10 mills = 1 cent (ct. or ¢)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)

The mill is not coined. The $10 gold piece is sometimes called an eagle.

REVIEW OF MEASURES OF TIME

Find the number of minutes in the following:

1. $2\frac{1}{2}$ hr. 2. 300 sec. 3. 3 hr. 4. $1\frac{1}{2}$ hr. 5. $1\frac{1}{2}$ da.

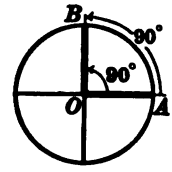
Find the number of hours in the following:

6. $1\frac{1}{2}$ da. 7. 300 min. 8. $2\frac{1}{2}$ da. 9. $2\frac{1}{4}$ da. 10. 1 wk.

Find the number of days in the following:

11. 18 wk. 12. 264 hr. 13. 19 wk. 14. 23 wk. 15. 38 wk.

Tables of Arc and Angle Measures. The circumference of a circle is measured in *degrees*.



There are 360 degrees (written 360°) in a circumference and 90° in a right angle.

The angle at *O*, the center, increases just as fast as the part of the circumference between its arms increases, and so an angle also is measured in degrees. A line that makes a right angle with another line is said to be *perpendicular* to it. A portion of a circumference is called an *arc*.

360° = a circumference

In measuring angles the unit is also the degree. That is,

360° = 4 right angles

In measuring both angles and arcs,

60 seconds (60″) = 1 minute (1′)
60 minutes = 1 degree (1°)

Table of Dozens. This table is often used in counting:

12 units = 1 dozen (doz.)
12 dozen, or 144 units = 1 gross (gro.)

The word "score" is sometimes used for "twenty," as "threescore years."

Table for Counting Sheets of Paper. In counting stationery and folded paper,

24 sheets = 1 quire
20 quires, or 480 sheets = 1 ream

In counting paper in commercial use,

25 sheets = 1 quire
20 quires, or 500 sheets = 1 ream

MISCELLANEOUS MEASURES

1. A machinist wishes to put together two pieces of steel making an angle $\frac{3}{4}$ of a right angle. How many degrees are there in this angle?

2. A sailor speaks of changing his course 3 points, a point being $\frac{1}{8}$ of a right angle. How many degrees does the sailor change his course?

3. The sum of all the angles in a figure of seven sides is 900°. If the angles are all equal, how many degrees are there in each angle?

4. A circular saw is making 1000 R.P.M. (revolutions per minute). Through how many degrees does each tooth of the saw pass per minute? per second?

5. An officer in a battle knows that an enemy battery is concealed in a direction N. 30° E. from where his gun is located, this meaning that the direction is 30° east of a line running directly north from his position. Draw roughly on paper a small map showing where the officer stands and the direction in which the enemy gun is concealed.

6. At \$9.60 a gross, how much will a dozen lamp chimneys cost? How much will $7\frac{1}{2}$ doz. cost?

7. A dealer bought 45 gross of blackboard erasers of which $\frac{1}{10}$ were damaged. How many erasers were in good condition?

8. Which is cheaper, to buy a gross of articles for \$108 or to buy 12 doz. at \$9.50 a dozen? how much cheaper?

9. At 25¢ a quire, how much will 14 quires of paper cost?

See that the pupil remembers the use of aliquot parts.

10. At $16\frac{2}{3}$¢ a quire, how much will 15 quires of paper cost? $7\frac{1}{2}$ quires? 3 doz. quires?

Reducing from Higher to Lower Units. 1. A strip of carpet is $7\frac{1}{4}$ yd. long. Is it long enough to reach the length of a hall that is 22 ft. long?

Here it is convenient to reduce $7\frac{1}{4}$ yd. to feet, or else to reduce 22 ft. to yards, so as to make the comparison.

Because $1 \text{ yd.} = 3 \text{ ft.}$,

we see that $7\frac{1}{4} \text{ yd.} = 7\frac{1}{4} \times 3 \text{ ft.} = 21\frac{3}{4} \text{ ft.}$

$$\begin{array}{r} 7\frac{1}{4} \\ 3 \\ \hline 21\frac{3}{4} \end{array}$$

While we think of multiplying 3 ft. by $7\frac{1}{4}$, we actually multiply $7\frac{1}{4}$ by 3, because this is easier.

We therefore see that the carpet is too short by $\frac{1}{4}$ ft., or 3 in.

We now extend the work on page 105. The above arrangement for computation will be found convenient. The abbreviations for yards and feet are omitted because we interchange multiplier and multiplicand for convenience. We are really multiplying 3 ft. by $7\frac{1}{4}$ and getting feet.

2. Reduce 7 rd. 8 in. to inches.

Because $1 \text{ rd.} = 16\frac{1}{2} \text{ ft.}$,

we see that $7 \text{ rd.} = 7 \times 16\frac{1}{2} \text{ ft.} = 115\frac{1}{2} \text{ ft.}$

Because $1 \text{ ft.} = 12 \text{ in.}$,

we see that $115\frac{1}{2} \text{ ft.} = 115\frac{1}{2} \times 12 \text{ in.} = 1386 \text{ in.}$

Then $1386 \text{ in.} + 8 \text{ in.} = 1394 \text{ in.}$

$$\begin{array}{r} 16\frac{1}{2} \\ 7 \\ \hline 115\frac{1}{2} \\ 12 \\ \hline 1386 \\ 8 \\ \hline 1394 \end{array}$$

In this case we find it easier to multiply $16\frac{1}{2}$ by 7 than to multiply 7 by $16\frac{1}{2}$.

Such an example is rare. We seldom find a practical case more difficult than Ex. 1. It is still more unusual to find a case in which we have to reduce, say, 17 yd. 2 ft. 7 in. to inches.

The great use of decimals in the last few years has made the subject of compound numbers, as formerly taught, nearly obsolete. We shall give in this book only what is commonly needed in business life.

USING WHAT YOU HAVE LEARNED

1. A carpenter saws a length of 9 ft. 9 in. from a board. How many inches does he saw off?

2. Edith is using 3 yd. 9 in. of cloth on some work. How many inches of cloth is she using?

3. A dealer is putting 38 lb. 8 oz. of candy in boxes. How many 4-ounce boxes can he fill with the candy?

4. A man puts 22 gal. of gasoline in a tank. How many quarts of gasoline does he put in?

5. A milk dealer sells 2 gal. 2 qt. of cream. How many pints of cream does he sell?

6. A farmer has 78 bu. of oats in his bin. At 4 qt. each time, how many times can he feed a horse from this amount?

7. A farmer has built 86 rd. 9 ft. of fence. How many feet of fence has he built?

8. How many square rods are there in $\frac{3}{4}$ A.?

9. A strip of carpet is 38 yd. 2 ft. long. How many feet in the length of the strip?

10. In excavating a cellar 540 cu. yd. of earth are removed. How many cubic feet of earth are removed?

11. El Paso County, Texas, has an area of 9331 sq. mi. Find the number of acres in the county.

12. A man buys 9 T. 800 lb. of coal. If he uses 100 lb. a day, how many days will the coal last him?

13. How many more sheets of paper are there in 6 reams of 480 sheets each than in 5 reams of 500 sheets each?

Teachers should frequently call the attention of their pupils to the necessity of estimating an answer in advance of the solution.

Reducing from Lower to Higher Units. 1. If a milk dealer sells 200 pt. of cream in a week, how many gallons does he sell?

Here we have to reduce 200 pt. to gallons.

Because $1 \text{ pt.} = \frac{1}{2} \text{ qt.}$,

we see that $200 \text{ pt.} = 200 \times \frac{1}{2} \text{ qt.} = 100 \text{ qt.}$

Because $1 \text{ qt.} = \frac{1}{4} \text{ gal.}$,

we see that $100 \text{ qt.} = 100 \times \frac{1}{4} \text{ gal.} = 25 \text{ gal.}$

$$\begin{array}{r|l} 2 & 200 \\ \hline 4 & 100 \\ \hline & 25 \end{array}$$

2. Reduce 175 in. to feet and a fraction; to feet and inches.

Because $1 \text{ in.} = \frac{1}{12} \text{ ft.}$,

we see that $175 \text{ in.} = 175 \times \frac{1}{12} \text{ ft.}$

$= 14\frac{7}{12} \text{ ft.}$

$= 14 \text{ ft. } 7 \text{ in.}$

$$\begin{array}{r|l} 12 & 175 \\ \hline & 14\frac{7}{12} \end{array}$$

These reductions are more difficult than those usually found in practice.

USING WHAT YOU HAVE LEARNED

1. If it takes $12\frac{1}{2}$ yd. of cloth for a garment, and you have 86 in. already, how many yards more do you need?

2. If you cut 98 sq. in. from a piece of tin which contains $3\frac{1}{4}$ sq. ft., how many square feet and square inches are left?

3. From a sheet of tin 4 ft. long and $3\frac{1}{2}$ ft. wide a piece is cut containing 600 sq. in. How many square feet are left?

4. From an iron plate 6 ft. long and $3\frac{1}{2}$ ft. wide a piece is cut containing 250 sq. in. How many square feet are left?

Reduce the following:

5. 64 oz. to pounds.
6. 4800 min. to hours.
7. 64 pk. to bushels.
8. 126 in. to feet.
9. 144 in. to yards.
10. 385 ft. to rods and feet.

Adding Compound Numbers. Suppose that your schoolroom is 24 ft. 8 in. wide, and that it is connected with another room 20 ft. 10 in. wide, and that you wish to find the combined width of the two rooms. You might stretch a line through the door and thus measure the combined width, subtracting the thickness of the wall, but it is easier to add the given numbers.

First, 10 in. + 8 in. = 18 in. = 1 ft. 6 in. We write 6 in the column of inches and add 1 to the number of feet.

Then 1 ft. + 20 ft. + 24 ft. = 45 ft.

Therefore the sum is 45 ft. 6 in.

ft.	in.
24	8
20	10
45	6

In practical life we rarely meet cases more difficult than those below. It should be observed that the general principles of adding compound numbers are similar to those of adding ordinary whole numbers. The only difference is that in compound numbers the number of units of a denomination required to make one unit of the next higher varies with the denomination used.

ADDING COMPOUND NUMBERS

Add the following:

	ft.	in.
1.	32	11
	16	8
	14	4
	13	10

	sq. ft.	sq. in.
3.	36	40
	74	75
	31	62
	27	48

	yd.	in.
5.	34	12
	10	24
	26	17
	22	19

	lb.	oz.
7.	236	12
	364	8
	228	6
	112	8

	ft.	in.
2.	37	10
	26	8
	45	3
	22	9

	sq. ft.	sq. in.
4.	29	28
	64	96
	32	32
	45	94

	ft.	in.
6.	38	10
	26	8
	36	11
	44	9

	lb.	oz.
8.	486	14
	324	13
	238	11
	117	12

Subtracting Compound Numbers. Suppose that a room is 18 ft. 3 in. square, and that we have picture molding on three sides and a piece 9 ft. 8 in. long for the fourth side. How much more must we have for this side?

Evidently we must subtract 9 ft. 8 in. from 18 ft. 3 in.

We cannot take 8 in. from 3 in., so we think of 18 ft. 3 in. as 17 ft. + 1 ft. + 3 in., or 17 ft. 15 in. Then 15 in. − 8 in. = 7 in.; 17 ft. − 9 ft. = 8 ft.

Therefore the difference is 8 ft. 7 in., and so we need 8 ft. 7 in. more molding.

ft.	in.
18	3
9	8
8	7

In practical life today we rarely use compound numbers of more than two denominations, and hence the examples given in this book are generally of not more than two denominations.

SUBTRACTING COMPOUND NUMBERS

1. A grocer has a piece of cheese weighing 7 lb. 9 oz. If he sells a piece weighing 1 lb. 12 oz., how much has he left?

2. If Bertha has 18 yd. 8 in. of cloth and uses 9 yd. 28 in., how much cloth has she left?

Subtract the following:

	lb.	oz.		bu.	qt.		ft.	in.		A.	sq. rd.
3.	48	14	6.	46	10	9.	62	10	12.	76	105
	22	8		29	16		28	14		38	118

	bu.	pk.		qt.	pt.		lb.	oz.		sq. ft.	sq. in.
4.	40	1	7.	96	0	10.	50	10	13.	98	102
	28	2		43	1		36	14		49	128

	gal.	qt.		ft.	in.		yd.	in.		cu. ft.	cu. in.
5.	22	2	8.	30	$5\frac{1}{2}$	11.	58	22	14.	76	272
	15	3		18	$9\frac{1}{4}$		46	28		48	564

Multiplying Compound Numbers. 1. A wholesale grocer is packing English walnuts in fancy boxes, 4 lb. 9 oz. to the box. He ships them in cases of a half dozen boxes. How many pounds of walnuts in a case?

lb.	oz.
4	9
	6
27	6

Here we have to multiply 4 lb. 9 oz. by 6.

As shown we have 6×9 oz. $= 54$ oz. $= 3$ lb. 6 oz.; 6×4 lb. $= 24$ lb.; and 24 lb. $+ 3$ lb. $= 27$ lb.

2. Multiply 243 ft. 7 in. by 27.

Multiplying as with other numbers, the result is first found to be 6561 ft. 189 in., which we reduce to 6576 ft. 9 in.

In this case it is not easy to reduce while we are multiplying, as we did in Ex. 1. We therefore multiply as with whole numbers and then reduce the product to a more simple form.

ft.	in.
243	7
	27
6561	189
6576	9

As teachers are aware, the tendency at present is to avoid the multiplication of compound numbers except in very simple cases, chiefly those involving feet and inches. Instead of multiplying 5 lb. 4 oz. by 8, we may multiply $5\frac{1}{4}$ lb. by 8, and similarly in other cases of this kind.

MULTIPLYING COMPOUND NUMBERS

1. If 1 cu. ft. of water weighs 62 lb. 8 oz., what is the weight of the water in a tank that contains 375 cu. ft. of water?

2. Each of 8 stories of a building is 11 ft. 9 in. high. What is the height of the building?

3. A man travels 6 da. in an automobile, averaging 9 gal. 3 qt. of gasoline a day. How much gasoline does he use in all?

4. A certain cereal is sold in packages of 1 lb. 10 oz. each. Find the weight of the cereal in a dozen packages.

Dividing Compound Numbers. If you have a board 7 ft. 6 in. long, and cut it into two equal lengths to make a top for a workshop bench, how long is each part?

Here we have to divide 7 ft. 6 in. by 2.

We see that 7 ft. ÷ 2 = 3 ft. with remainder 1 ft., or 12 in.

Then we have 12 in. + 6 in., or 18 in., still to be divided. We see that 18 in. ÷ 2 = 9 in.

Therefore the result is 3 ft. 9 in.

	ft.	in.
2)	7	6
	3	9

It is very rare to find a practical case in which we need to divide one compound number by another. If, for example, we are dividing 41 ft. 3 in. by 2 ft. 9 in., we reduce both to feet ($41\frac{1}{4}$ ft. ÷ $2\frac{3}{4}$ ft. = 15) or to inches (495 in. ÷ 33 in. = 15). In this book these unreal cases have no place.

DIVIDING COMPOUND NUMBERS

1. The perimeter of an equilateral (equal-sided) triangle is 17 ft. $9\frac{1}{2}$ in. Find the length of each side.

2. The perimeter of a square is 7 ft. 8 in. Find the length of each side.

3. A steamship changed its longitude 66° 44′ in 7 da. How many degrees and minutes of longitude did it change per day?

4. If you are in school 5 hr. 45 min. a day, and recite half of the time that you are in school, how many hours, minutes, and seconds do you recite in a day?

5. At the rate of a mile in 4 min., how far will an automobile go in 1 hr.? in $2\frac{1}{2}$ hr.? in $4\frac{1}{4}$ hr.?

6. How far will a horse go in $2\frac{1}{2}$ hr., traveling at the rate of a mile in 12 min.?

7. If a wagon wheel is 10 ft. 6 in. in circumference, how many times will it turn while the wagon is going 25 mi.?

REVIEW DRILL

Add the following:

	ft.	in.		ft.	in.		ft.	in.		ft.	in.
1.	36	5	3.	29	11	5.	38	10	7.	46	11
	42	3		86	6		42	6		28	8
	37	4		32	4		36	9		86	4
	68	2		17	3		70	11		49	10

	yd.	in.		gal.	qt.		bu.	pk.		bu.	pk.
2.	39	18	4.	38	3	6.	28	3	8.	81	3
	43	14		62	2		43	2		66	3
	76	13		68	1		37	1		73	0
	84	25		34	3		68	2		49	2

Subtract the following:

9. $8\frac{1}{2}$ lb. — 7 oz.
10. $4\frac{3}{8}$ ft. — 6 in.
11. 96 ft. — 7 yd.
12. $4\frac{3}{4}$ bu. — 90 qt.
13. 590 ft. 8 in. — 398 ft. 10 in.
14. 303 lb. 4 oz. — 109 lb. 15 oz.
15. 342 yd. 9 in. — 229 yd. 32 in.
16. 108 bu. 7 qt. — 102 bu. 24 qt.

Multiply the following:

17. 38 lb. 9 oz. by 8.
18. 46 lb. 7 oz. by 4.
19. 48 ft. 9 in. by 5.
20. 22 bu. 8 qt. by 8.
21. 74 lb. 15 oz. by 4; by 8.
22. 98 lb. 8 oz. by 6; by 4.
23. 628 ft. 6 in. by 4; by 5.
24. 212 bu. 18 qt. by 6; by 8.

Divide the following:

25. 91 lb. 8 oz. by 4.
26. 332 gal. by 4 gal.
27. 123 lb. 8 oz. by 2.
28. 395 lb. 1 oz. by 3.
29. 972 gal. by 9 gal.; by 2 qt.
30. 329 lb. 4 oz. by 4; by 8.

III. LITTLE EXAMINATIONS

i.
1. $85\frac{1}{2}$ ft. = (?) in.
2. 85 in. = (?) ft.
3. $7\frac{3}{4}$ lb. = (?) oz.
4. 44 qt. = (?) pk.
5. $9\frac{3}{4}$ ft. − 8 in.
6. $3\frac{1}{2} \times 7\frac{1}{2}$ ft.
7. 5 × 9 ft. 8 in.
8. $82\frac{1}{2}$ ft. ÷ 3.

ii.
1. $46\frac{3}{4}$ ft. = (?) in.
2. 108 in. = (?) yd.
3. $3\frac{4}{5} \times 6\frac{2}{3}$.
4. 61 sq. ft. = (?) sq. yd.
5. $9\frac{1}{2}$ yd. − 16 in.
6. $4\frac{7}{8} \times 8\frac{3}{4}$ yd.
7. 9 × 5 bu. 2 qt.
8. $103\frac{1}{3} \div 3\frac{1}{3}$.

iii.
1. $46\frac{1}{2}$ ft. = (?) yd.
2. 2560 A. = (?) sq. mi.
3. $17\frac{1}{8} - 9\frac{3}{4}$.
4. 9 sq. ft. = (?) sq. in.
5. $8\frac{1}{2}$ lb. + 14 oz.
6. $7\frac{3}{4} \times 9\frac{1}{2}$ lb.
7. 9 × 7 lb. 8 oz.
8. $16\frac{1}{2}$ yd. ÷ $8\frac{1}{4}$.

iv.
1. $61\frac{1}{2}$ ft. = (?) yd.
2. 24 A. = (?) sq. rd.
3. $2\frac{1}{4}$ T. = (?) lb.
4. 45 sq. ft. = (?) sq. yd.
5. $8\frac{1}{8}$ ft. − 6 in.
6. $3\frac{7}{8} \times 6\frac{2}{3}$ ft.
7. 8 × 5 ft. 4 in.
8. $31\frac{1}{2}$ ft. ÷ 6.

v.
1. 573 ft. = (?) yd.
2. $4\frac{1}{2}$ sq. mi. = (?) A.
3. $72\frac{1}{2}$ gal. = (?) qt.
4. 38 bu. = (?) pk.
5. $7\frac{1}{2}$ hr. + 45 min.
6. $9\frac{3}{4}$ bu. − 3 pk.
7. $7\frac{3}{8} \times 9\frac{1}{2}$ yd.
8. 12 × 8 yd. 6 in.

vi.
1. $42\frac{1}{2}$ yd. = (?) in.
2. 256 cu. ft. = (?) cd.
3. $8\frac{1}{2}$ gal. = (?) pt.
4. $3\frac{3}{4}$ hr. = (?) min.
5. $9\frac{1}{2}$ yd. + $9\frac{1}{2}$ ft.
6. 17 ft. − 10 in.
7. 8 × 9 gal. 3 qt.
8. 16 yd. 2 ft. ÷ 5.

IV. REVIEW AND DRILL

MINIMUM ESSENTIALS

Copy, add, and check the following:

1.	2.	3.	4.
$658.65	$390.45	$490.05	$783.60
398.38	207.98	232.68	55.62
502.98	700.00	329.87	454.08
879.26	56.50	762.45	384.96
563.42	862.73	82.74	672.00
802.75	426.37	538.73	586.49
508.72	758.42	457.07	754.62

Perform the following operations:

5. $9746.08 − $969.79.
6. $4012.12 − $834.47.
7. $2728.42 − $1954.39.
8. 36 × $92.30.
9. 248 × $3.12½.
10. 465 × $3.33⅓.
11. $17.40 ÷ 12.
12. $9736.40 ÷ 8.
13. $6975 ÷ 75.
14. $22,000 ÷ 125.
15. $2400 ÷ $375.
16. $33,156.25 ÷ 625.

Find the value of each of the following:

17. 7 × 34 ft. 8 in.
18. ½ of 37 ft. 11 in.
19. ¾ of 62 ft. 10 in.
20. 105 ft. 9 in. ÷ 3.
21. 233 ft. 10 in. ÷ 5.
22. 423 yd. 28 in. ÷ 4.

Perform the following operations:

23. ⅜ of ⅝.
24. ⅜ ÷ ⅝.
25. ⅞ + ¾.
26. ⅞ − ¾.
27. $6\frac{3}{8} \times 9\frac{3}{8}$.
28. $6\frac{3}{8} \div 9\frac{3}{8}$.
29. $2\frac{1}{2} + 1\frac{7}{8}$.
30. $4\frac{1}{8} - \frac{1}{4}$.
31. $3\frac{5}{8} + 2\frac{15}{16}$.
32. $3\frac{5}{8} - 2\frac{15}{16}$.
33. ⅖ of $\frac{15}{16}$.
34. ⅜ ÷ $\frac{9}{16}$.
35. $3 \times 12\frac{3}{4}$ in.
36. $12\frac{3}{4}$ in. ÷ 3.
37. $16\frac{3}{4}$ ft. ÷ 4.
38. $30\frac{3}{5}$ yd. ÷ 9.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If John picked a certain number of quarts of berries today and a certain part as many yesterday, how do you find how many quarts he picked yesterday? How do you find how many quarts he picked in the two days?

2. If Fred gathered a certain number of quarts of chestnuts and George gathered a certain part as many, how do you find how many quarts George gathered? How do you find how many quarts they both gathered?

3. If Ethel lives a certain distance from school and this distance is a certain number of times as far as Kate lives, how do you find how far from school Kate lives?

4. If you know the cost of a certain number of boxes of oranges, how do you find the cost of a certain other number?

5. If you know the length of a bookshelf filled with books that are all equally thick, and if you know the thickness of one of the books, how do you find the number of books?

6. If you know the dimensions of a rectangular bin, and the number of cubic inches in a bushel, how do you find the number of bushels in the bin?

7. If you know the number of rods in the length and width of a field, how do you find the number of acres in the field?

8. If you know the number of feet in the length, width, and depth of a cellar, how do you find the number of cubic yards in the cellar?

9. If you know that a field is a certain part of a mile long and a certain number of rods wide, how do you find the number of acres in the field?

PROBLEMS FOR COMPLETION

1. The daily route of a certain rural mail carrier is 36 mi. He goes over this route 305 times a year. Complete the problem and solve it.

2. In Ex. 1 the carrier on each trip uses 3 gal. of gasoline, which costs him 29¢ a gallon. Complete the problem and solve it.

As stated on page 70, the problem may be completed in more than one way. Any reasonable problem, if correctly solved, should be accepted.

3. In the 305 trips referred to in Ex. 1, the carrier delivered 12,256 letters, each bearing a two-cent stamp, and 9450 other pieces of mail, each bearing a one-cent stamp. Complete the problem and solve it.

Complete and solve the following problems:

4. The school expenses in a country school for four months were as follows: teacher's salary, $400; fuel, $42.80; text-books, $16.80; repairs, $18.28; janitor, $24.72. There were 20 pupils in the school.

5. Charles lives $1\frac{3}{4}$ mi. from his school. He walks the entire distance both ways 4 da. in the week, and rides the other school day. There are twenty school days in a certain month.

6. Mr. Gray gave his son John the use of a piece of land. John raised watermelons on it and sold the crop to the neighbors, selling 27 melons at an average price of $33\frac{1}{3}$¢ each. His father then gave him enough more to amount to $10.

7. Mr. Shaw had a crib of corn containing 900 bu. Because of his neglect to repair the crib the rats destroyed 25 bu. of corn. The corn was worth $1.20 a bushel.

Teachers should read the note after Ex. 1 on page 70 and also the one at the foot of that page.

ALERTNESS TEST

In the three lines given below, each number is obtained in a certain way from the numbers coming before it. If the teacher writes on the blackboard ten rows of numbers, four numbers in each row, the pupils should copy them all and write the fifth number in each row. The pupils will be allowed 30 sec. in which to copy the given numbers and insert the missing ones.

These three rows show the four given numbers in each case and also, in heavy type, the fifth number:

2,	4,	6,	8,	**10**
16,	8,	4,	2,	**1**
15,	13,	10,	6,	**1**

In the first row, 2 is added in each case; in the second, each number is divided by 2; in the third, 2 is subtracted from 15, 3 from 13, 4 from 10, and 5 from 6.

This is a specimen of the general-intelligence or alertness tests commonly given in such examinations. It may be used or not, at the teacher's discretion. It is, of course, undesirable to print the rows of numbers, for then the pupils would be studying them in advance. The numbers should be written by the teacher on the blackboard and covered with a map. They should be uncovered when the class is ready to begin, and no more than 30 sec. should be allowed for ten rows. Each paper should be marked by the number of rights, and each pupil graded by this number. The teacher should keep the record and compare it with the records secured in later tests. In such tests no pupil is expected to obtain all the results.

The following are typical arrangements:

9, 8, 7, 6	55, 61, 67, 73	3, 4, 3, 4
2, 4, 8, 16	39, 32, 25, 18	5, $7\frac{1}{2}$, 10, $12\frac{1}{2}$
3, 6, 9, 12	42, 51, 60, 69	1, 2, 4, 7
3, 6, 12, 24	10, 20, 40, 80	9, 10, 8, 9
4, 2, 1, $\frac{1}{2}$	60, 30, 15, $7\frac{1}{2}$	18, 17, 15, 12

Industrial Problems

MISCELLANEOUS PROBLEMS

1. How many tulip bulbs will it take for 4 rows 8 ft. long, the bulbs being placed 6 in. apart?

First draw a plan of the first row and then find exactly the number of bulbs needed for that row.

2. A farmer raises 1950 bu. of wheat on 95 acres. If he can increase the crop one half by proper fertilization, what will then be the yield per acre?

3. John, Ethel, and Charles picked strawberries for 2¢ a quart. Ethel picked twice as many as John, and Charles picked $1\frac{1}{2}$ times as many as Ethel. In one week John picked 63 qt. How much did Charles earn that week?

Such problems, although not very real, may occasionally be given for drill.

4. A suburban train makes ten round trips to the city each week day, and four round trips on Sunday. In 4 wk. the distance covered was 6144 mi. How far from the city is the suburban station from which the train starts?

5. A workman's time slip showed that at the end of the week he had worked 44 hr. He was paid at the rate of 62¢ an hour. How much money did he receive at the end of the week and how many eight-hour days did he work?

6. A factory employed 428 men who worked 8 hr. a day 5 da. in the week and 4 hr. on Saturday, their wages being 75¢ an hour. At the time of a great coal strike the number of working hours was reduced to 24 hr. per week. How much did the workmen lose in wages in one week?

7. A newsboy pays 3¢ for a weekly magazine that he sells for 5¢. How many magazines must he sell to make a profit of $8?

MISCELLANEOUS PROBLEMS

1. A dealer marked down some collars from 30¢ each to 25¢ each. The lower price is what part of the higher price?

2. In Ex. 1, the amount of the reduction is what part of the higher price? what part of the lower price?

3. A dealer marked down some neckties from 3 for $2 to 60¢ each. The lower price is what part of the higher price?

4. A man's salary in a shop was raised $\frac{1}{8}$ and was then $2700 a year. How much was the salary before it was raised?

After the salary was raised it was $\frac{8}{8}$ of what it was before. If $2700 is $\frac{8}{8}$, how much is $\frac{1}{8}$? how much is $\frac{8}{8}$?

5. The daily pay roll for 3600 men who work in a certain factory is $21,700. Of these men, 100 receive $7 each per day and the rest receive less but are paid equal wages. Find the daily wages of each man who receives the smaller amount.

6. On a certain elevated road the 2850 conductors received an increase in wages amounting to $4275 a day. What was the average increase per man?

7. A farmer shipped to a city dealer 800 barrels of apples. The dealer sold the apples for $4.40 a barrel, and charged the farmer 22¢ a barrel for selling, 8¢ a barrel for storage, $39.75 for freight, and $38.50 for cartage. How much did the farmer receive from the dealer?

8. When cotton was selling for 31¢ a pound a dealer sold 175 bales of cotton weighing on the average 500 lb. each. Find the amount the dealer received for the lot.

9. A man with a team earns $11.25 a day, working 6 da. a week. If during a certain week he pays $18.75 for groceries and $10.50 for rent, how much does he have left at the end of the week?

A FARM CLUB

1. A boy in a farm club husked all the corn grown on an acre of land. He saved out 7 bu. for seed and hauled the remainder, weighing 3225 lb., to market. Allowing 75 lb. to a bushel of new ear corn, how many bushels did the acre yield?

2. In Ex. 1, suppose that the seed corn was worth \$4 a bushel and the ordinary field corn was worth \$1.60 a bushel, and suppose that the stalks were worth a quarter as much as the corn itself, find the value of the crop.

3. The accounts kept by a club boy show the following items:

Rent of land,	\$7.00	Husking,	\$7.50
Plowing,	2.00	Loss of fertility,	5.00
Harrowing and fitting,	1.25	Hoeing, 12 hr. @ 44¢,	(?)
Seed,	.91	Cultivating, 3 times @ 60¢,	(?)

If the land yielded $72\frac{1}{2}$ bu., what was the cost per bushel?

4. A boy finds that if he uses stable fertilizer which he can get for nothing, a half acre produces corn that will sell for \$35. Without the fertilizer the land will produce only \$22. Find the gain in income per acre fertilized over that not fertilized.

5. An average ear of white dent seed corn weighs 12 oz. Allowing 75 lb. to a bushel of new ear corn, how many ears of seed corn are there in a bushel?

6. If there are 100 ears of ordinary field corn in a bushel, and if 15 ears will plant an acre, how many acres will 3 bu. plant?

7. The club found that the proceeds from 1 A. of land were \$64. How much would a tract $6\frac{1}{2}$ rd. by 14 rd. produce?

Pages of this type may be omitted in city schools or in sections of the country where conditions are such as not to make the problems seem real.

JOHN'S FIELD OF CORN

1. John decides to raise corn and so rents from his father, at $10 an acre, a field 32 rd. long and 10 rd. wide. How much should he charge to expenses as rent?

2. He estimates his labor at 40¢ an hour and the work of a horse at 60¢ an hour. In a certain month he puts in 42 hr., during 4 hr. of which he has a horse. Find the amount.

3. He needs for seed 16 ears of corn to the acre. At $3\frac{1}{2}$¢ an ear, how much does the seed cost for 2 A.?

4. If he uses 1 qt. of seed corn to 200 hills, how many hills can he plant with 1 pk. of seed?

5. If he plants the hills 3 ft. 4 in. apart, how many hills can he plant in one of the rows 32 rd. long, no hill to be nearer than 4 ft. to either end?

6. If the rows are 3 ft. 6 in. apart, how many rows can he plant, no row to be nearer to either side than 3 ft. 9 in.?

7. By carefully testing his seed corn John increases his crop by one ear of corn to every four hills. If we estimate 100 ears to a bushel, how many bushels does he increase the yield?

Explain that in problems like this, which deal in averages, meaningless fractions or remainders should be rejected. Thus, we should not say he raised 1805½ ears more, but 1805; and the remainder of 5 ears over the 18 bu. should be discarded and not called $\frac{1}{20}$ of a bushel.

8. If John sold his corn at $1.50 per bushel, how much more profit did he make by testing his seed?

9. On one acre of this land he used fertilizer costing $19 and produced 43 bushels. On the other acre without fertilizing he raised only 24 bushels. If he received $1.50 a bushel for his corn, how much did he gain by fertilizing?

CHAPTER III

I. DECIMAL FRACTIONS

Need for Decimals. You read in your geographies that the annual rainfall of Galveston, Texas, is 47.11 in., while at El Paso it is 9.8 in. In order to understand this we must know what a number like 47.11 means. We often see such numbers and we often need to work with them.

Meaning of Decimals. You have learned that \$3.25 means 3 dollars + 25 cents, or $\$3 + \$\frac{25}{100}$.

Just as \$3.25 means $\$3 + \$\frac{25}{100}$,

so 3.25 ft. means 3 ft. + $\frac{25}{100}$ ft., or $3\frac{1}{4}$ ft.

We might read \$3.25 "3 and $\frac{25}{100}$ dollars," but there are, as we know, more common ways of reading it, either "3 dollars and 25 cents," or "3 dollars and a quarter."

We read 3.25 ft. "3 and $\frac{25}{100}$ feet," but we know that $\frac{25}{100} = \frac{1}{4}$; so we may, if we wish, read it "3 and a quarter feet," or "3 feet and a quarter."

Similarly, 4.75 in. means 4 in. + $\frac{75}{100}$ in., or $4\frac{3}{4}$ in.;

2.5 means $2 + \frac{5}{10}$, or $2\frac{1}{2}$;

.5, or 0.5, means $\frac{5}{10}$, or $\frac{50}{100}$, or 0.50, or $\frac{1}{2}$;

.06, or 0.06, means $\frac{6}{100}$, or $\frac{3}{50}$;

and .125, or 0.125, means $\frac{125}{1000}$, or $\frac{1}{8}$.

Since 0.5 means $\frac{5}{10}$, we read it "5 tenths"; and since 0.125 means $\frac{125}{1000}$, we read it "125 thousandths."

Decimal Fraction. A fraction whose denominator is some power of 10 but is not written is called a *decimal fraction* or simply a *decimal.*

There was formerly some attempt in schools to distinguish between a decimal fraction and a decimal, but in practical life the two terms are synonymous. The word "decimal" is from the Latin *decem*, ten.

Thus 0.5 (5 tenths) is a decimal fraction. If written in the form $\frac{5}{10}$ it becomes a common fraction.

Teachers are sometimes asked if $\frac{5}{10}$ is not a decimal fraction. This is simply a question of definition. By the ordinary definition and by common usage it is not a decimal fraction when written in this form.

The expression 0.5 means the same as .5. The 0 may be written or not, as we please. Those who have much computing to do usually find it better to write the 0 before .5, except when the numbers are written in columns, so that there may be no misunderstanding. It is perfectly proper to omit it, however, as is always done when we have columns that are ruled for dollars and cents, and as is often the case in printed numbers. You should accustom yourself to both forms, for you will see both in newspapers and books.

The period placed at the left of the tenths in a decimal fraction is called the *decimal point.*

We now see what it means to say that the rainfall is 36.22 in. at a certain place; it means that in a year the rain falls to a total depth of $36\frac{22}{100}$ in., or $36\frac{11}{50}$ in., at that place.

We also see that we have long been using decimal fractions in writing dollars and cents, but we have not called them by this name. When we write \$2.25 we often speak of it as $2\frac{1}{4}$ dollars, because $\$2.25 = \$2\frac{25}{100} = \$2\frac{1}{4}$. In the same way the number 2.50 is often spoken of as $2\frac{1}{2}$, although it is more properly read "2 and 50 hundredths."

Reading Decimals. The names of the decimal places that are most frequently used are given in the following scheme:

Thousands	Hundreds	Tens	Units	(Decimal point)	Tenths	Hundredths	Thousandths	Ten-thousandths
8	2	0	5	.	3	7	0	5

This number is read "eight thousand two hundred five and three thousand seven hundred five ten-thousandths," or "eight-two-o-five point three-seven-o-five." The latter form, with "o" instead of "zero" or "naught," is rapidly coming into use. The orders beyond ten-thousandths are hundred-thousandths, millionths, ten-millionths, and so on.

Pupils should be given much exercise in reading such numbers from the blackboard, avoiding the use of "and" except between the units and tenths.

Writing Decimals. In a decimal the denominator is shown by the position of the decimal point. Thus 0.9 has no printed denominator, but we call the fraction "nine tenths" and thus indicate the fact that the denominator is ten.

Therefore, *write only the numerator of the decimal, inserting such zeros as may be necessary to indicate the denominator, and place the decimal point before the tenths.*

Thus, to write in figures the number two hundred four hundred-thousandths, we write 204 and prefix two zeros preceded by a decimal point. We then have 0.00204, decimal fractions not being separated into periods like whole numbers.

READING AND WRITING DECIMALS

Numbers 1 to 14, oral

Read the following statements and tell the meaning of each:

1. The annual rainfall at Phoenix, Arizona, is 7.9 in., and that at Montgomery, Alabama, is 51.2 in.

2. On a motor trip Ethel went 102.7 mi. in one day.

3. The average height of the boys in our class is 55.2 in.

4. The average strength of the forearm of the boys in our class is 52.9 lb. for the right arm and 48.5 lb. for the left.

5. Cork is 0.24 as heavy as water, granite is 2.78 times as heavy as water, and gold is 19.26 times as heavy as water.

Read the following numbers:

6. 28.5. 7. 8.34. 8. 0.375. 9. 14.025.

10. What is the name of the place to the right of tenths? to the right of that place? What is the name of the fourth decimal place? the fifth? the sixth?

Since we rarely read the names of numbers beyond three decimal places, even when we write them, the chief drill should be on reading decimals like those in Exs. 1–9.

Read the following numbers:

11. 0.0125. 12. 6.0081. 13. 2.87463. 14. 31.426395.

Write the following numbers in figures:

15. Twelve hundredths; fifty-five hundredths; nine hundredths; one hundredth; twenty hundredths.

16. One hundred twenty-five thousandths; forty-seven thousandths; ten thousandths; five thousandths.

Reducing a Decimal to a Common Fraction. James saw that there was to be a bargain sale and that he could buy a catcher's mask for 0.25 less than the marked price. Fred saw that he could buy one at another store for $\frac{1}{5}$ less than the same marked price. James and Fred then compared these prices.

Because 0.25 means $\frac{25}{100}$, which is equal to $\frac{1}{4}$, which price was the lower?

From this illustration we see that we need to know how to reduce a decimal to a common fraction.

To reduce a decimal to a common fraction, write the numerator and the denominator of the decimal as the corresponding terms of a common fraction and reduce to lowest terms.

In a case like that of $0.33\frac{1}{3}$ we proceed in the same way, thus,

$$0.33\tfrac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{300} = \frac{1}{3}.$$

A number like 3.75, consisting of a whole number and a decimal fraction, is called a *mixed decimal.*

The definition need not be memorized, but the term is a convenient one.

REDUCING DECIMALS TO COMMON FRACTIONS

1. Which is the better for the buyer, a reduction of $\frac{1}{6}$ from the marked price or one of 0.15?

Reduce to common fractions or to mixed numbers:

2. 0.8. 3. 0.35. 4. 0.375. 5. 3.75. 6. 0.625.

7. Express 0.125 as eighths. Find 0.125 of \$3200.

8. Express $0.33\frac{1}{3}$ as thirds. Find $0.33\frac{1}{3}$ of \$7110.75.

Express the following as thirds:

9. $0.66\frac{2}{3}$. 10. $1.33\frac{1}{3}$. 11. $1.66\frac{2}{3}$. 12. $2.66\frac{2}{3}$. 13. $3.33\frac{1}{3}$.

Annexing Zeros to a Decimal. If Frank is asked to write on the blackboard the cost of his bicycle, and he writes \$34.5, is this the usual way of writing \$34½? How should it be written?

Teachers should state that \$34.5 means \$34½, but that neither is the correct business form, since we usually write \$34.50.

Is there any difference in value between \$2.5 and \$2.50?

From this question and the answer we see that

Annexing a zero to a decimal does not change the value.

Decimals that have the same number of decimal places are called *similar decimals*, and decimals that have not the same number of decimal places are called *dissimilar decimals.*

Therefore, *to change dissimilar decimals to similar decimals, give them the same number of decimal places by annexing or cutting off zeros at the right.*

Since $0.25 = 0.250$, $2.125 = 2.125$, and $3.5 = 3.500$, we may write 0.25, 2.125, and 3.5 as similar decimals.

ANNEXING ZEROS TO DECIMALS

1. If you wish to subtract 1.75 from 2.6, it is easier to write the numbers as similar decimals. Write them in that way.

Express the following as similar decimals:

2. 0.4, 0.27.	5. 0.6, 0.548.	8. 0.13, 0.406.
3. 0.3, 0.03.	6. 0.8, 0.900.	9. 0.35, 0.008.
4. 0.7, 0.009.	7. 0.7, 0.080.	10. 0.69, 0.070.

11. Write 24.3 and 14.25 in forms more simple for the subtraction of the smaller from the larger.

12. Write 15.06 and 28.003 in forms more simple for the subtraction of the smaller from the larger.

Reducing a Common Fraction to a Decimal. The average height of the girls in a certain class was found to be $55\frac{4}{5}$ in. In reporting this fact the teacher used decimal fractions. How should she write $\frac{4}{5}$ as a decimal?

Because $\frac{4}{5}$ means the same as $4 \div 5$, or $4.0 \div 5$, we simply divide, placing the decimal point as we should in dividing United States money.

$$5\overline{)4.0}\quad 0.8$$

To reduce a common fraction to a decimal, place a decimal point after the numerator and divide by the denominator.

A common fraction cannot always be reduced to an exact decimal; thus, $\frac{1}{3} = 0.3\frac{1}{3} = 0.33\frac{1}{3} = 0.333\frac{1}{3}$. A common fraction or a series of dots or the plus sign is sometimes written to indicate that the work is not complete; thus, $\frac{1}{3} = 0.333\frac{1}{3}$, $0.333\cdots$, or $0.333+$. More often we write 0.333, discarding the fraction, after we have used as many places as may be necessary. In the case of $\frac{2}{3}$ we have $\frac{2}{3} = 0.6\frac{2}{3} = 0.66\frac{2}{3} = 0.666\frac{2}{3}$. Just as the storekeeper calls $\frac{1}{2}$ of a cent or more a full cent, we often write 0.667 for $0.666\frac{2}{3}$. Carry the division to one more place than is required in the answer, to determine whether to discard the remainder or increase by 1 the last figure.

REDUCING COMMON FRACTIONS TO DECIMALS

1. Lead is $11\frac{7}{20}$ times as heavy as water. Write $11\frac{7}{20}$ as a mixed decimal.

Reduce the following to decimals, not carrying the reduction beyond the nearest thousandth in any case:

2. $\frac{3}{4}$.	5. $\frac{4}{5}$.	8. $\frac{13}{16}$.	11. $\frac{5}{6}$.	14. $7.2\frac{5}{9}$.
3. $\frac{5}{8}$.	6. $\frac{1}{16}$.	9. $\frac{1}{32}$.	12. $\frac{8}{9}$.	15. $3.3\frac{3}{5}$.
4. $\frac{7}{8}$.	7. $\frac{8}{16}$.	10. $\frac{5}{32}$.	13. $\frac{9}{25}$.	16. $6.2\frac{3}{4}$.

Adding and Subtracting Decimals. 1. The class measured the sides of a large flower bed in a park. They found them to be 48.7 ft., 53.75 ft., and 62.9 ft. Find the perimeter; that is, the distance around the flower bed.

$$\begin{array}{r} 48.7 \\ 53.75 \\ 62.9 \\ \hline 165.35 \end{array}$$

If we write the numbers with the decimal points in a column, units will come under units, tenths under tenths, and so on. Adding these numbers, we find the sum to be 165.35, and so the perimeter of the flower bed is 165.35 ft.

We therefore see that we add decimal fractions just as we add numbers representing United States money.

2. A new trolley line is to be built a distance of 4.1 mi. After 0.26 mi. has been completed, how much more must be built?

$$\begin{array}{r} 4.10 \\ .26 \\ \hline 3.84 \end{array}$$

Arranging the numbers with the decimal points in a column and writing 4.1 as 4.10, the remainder is found to be 3.84, and so there are 3.84 mi. still to be built.

We therefore see that we subtract decimal fractions just as we subtract numbers representing United States money.

ADDING AND SUBTRACTING DECIMALS

1. If the three sides of a triangle are 3.75 in., 3.9 in., and 3.95 in., what is the perimeter?

2. On four successive days an automobile ran 94.8 mi., 127.6 mi., 98.8 mi., and 132.4 mi. What was the total distance?

3. From a city lot containing 7645.3 sq. ft. the owner sold 3426.5 sq. ft. and 2058.3 sq. ft. How much was left?

4. The perimeter of a triangle is 17.3 in. Two of the sides are 5.38 in. and 6.91 in. Find the third side.

5. The speedometer on my automobile registered 2134.8 mi. when I started for a ride and 2207.4 mi. when I returned. How many miles did I go?

6. The speedometer on an automobile registered 1872.9 mi. at the start of a trip of 298.8 mi. How much did it register at the end of the trip?

7. The normal temperature of the body when a person is well is 98.6 degrees. When Clara had the influenza her temperature rose to 102.1 degrees. How much was this above normal?

8. An airplane made 99.3 mi. in the first hour of a trip, 98.8 mi. in the second hour, and 94.9 mi. in the third hour. Find the distance covered in the 3 hr.

9. The weather bureau reported the rainfall in New York City for the successive days of a certain week to be 1.35 in., 1.8 in., 1.25 in., 0 in., 0.08 in., 0 in., 1.07 in. Find the total amount of rainfall for that week.

10. In Ex. 9 find the difference between the rainfall on the two days in which there was the most rain.

11. In cities it is the custom to measure building lots in feet and tenths of a foot. A lot having four unequal sides is measured, and the sides are found to be 32.8 ft., 48.3 ft., 30.9 ft., and 47.8 ft. Find the sum of the four sides.

12. The sum of the three sides of a triangular lot is 170.5 ft. The first side is 63.9 ft. and the second side is 57.8 ft. Find the third side.

13. The distance from Willimantic to Hartford is 31.55 mi., from Willimantic to Waterbury, which is on the same line and beyond Hartford, the distance is 62.59 mi. How far is it from Hartford to Waterbury?

DRILL WORK

Add the following:

1.	2.	3.	4.	5.
2.7	14.6	17.68	28.30	7.832
3.9	7.8	9.07	49.96	2.978

The above five examples should be copied and added in 1 min.

6. From the sum of 6.83 and 4.07 subtract 9.68.

Add the following:

7.	9.	11.	13.	15.
3.4	18.2	34.43	38.82	9.826
9.6	17.8	9.8	4.09	3.498
2.9	6.6	16.07	.69	.697
8.	**10.**	**12.**	**14.**	**16.**
5.7	32.5	14.69	63.81	7.009
6.8	17.9	8.58	.96	6.89
3.5	6.	7.46	45.	.008

Exs. 7–16 should be copied and added in 3 min.

Add the following:

17.	19.	21.	23.	25.
98.72	39.04	53.72	29.79	8.769
64.39	7.68	16.87	8.23	4.38
41.08	34.	9.08	19.68	5.007
27.62	6.7	63.5	72.	.62
18.	**20.**	**22.**	**24.**	**26.**
53.47	32.46	41.68	82.96	5.758
62.68	.7	37.98	3.48	2.752
91.89	75.	62.75	47.	9.683
.73	3.98	4.28	6.93	3.429
63.08	52.09	37.6	43.98	8.9

Exs. 17–26 should be copied and added in 6 min.

DRILL WORK

Add the following:

1.	2.	3.	4.	5.
\$3.49	\$43.29	38.96	38.26	52.701
6.62	16.42	46.89	.09	31.026
5.88	18.00	7.08	9.5	8.879

Subtract the following:

6.	7.	8.	9.	10.
90.24	70.26	79.62	65.3	87.03
30.08	41.9	32.98	28.96	7.6

Add the following:

11.	12.	13.	14.	15.
92.85	62.38	2.728	712.3	9.288
72.97	93.97	5.683	528.2	.396
61.82	75.63	3.929	887.7	.486
32.09	76.06	6.635	771.1	.567
43.58	29.37	4.004	928.6	8.06
21.23	82.05	4.8	671.8	9.
3.89	40.63	7.303	466.5	.079

Subtract the following:

16. $3 - 0.27$.
17. $14 - 2.09$.
18. $37 - 14.127$.
19. $123{,}489.3 - 76{,}879.478$.
20. $342{,}008.1 - 148{,}936.3275$.
21. $40{,}000 - 27{,}623.4856$.

22. The average height of the girls in each of three groups was found to be 55.8 in., 58.1 in., and 60.3 in. respectively. Find the difference between the first and each of the other two.

Find the sum of each of the following:

23. 8.7 ft., 6.9 ft., 7.2 ft.
24. 7.9 in., 8.3 in., 8.4 in.

Multiplying a Decimal by a Whole Number. 1. In a history of the World War, Kate read of one enemy trench that was only 35 meters from our front line, and she wanted to know how many feet this was. She knew that a meter is 39.37 in., so she found that she must first reduce 35 meters to inches and then reduce this to feet. This meant that she must first multiply 39.37 in. by 35.

$$\begin{array}{r} 39.37 \text{ in.} \\ 35 \\ \hline 19685 \\ 11811 \\ \hline 1377.95 \text{ in.} \end{array}$$

She multiplied just as she would multiply $39.37 by 35, and found that the distance was 1377.95 in. This is practically 1378 in., and she reduced this number of inches to feet by multiplying as follows:

$$1378 \times \tfrac{1}{12} \text{ ft.} = 114\tfrac{10}{12} \text{ ft.}$$
$$= 114 \text{ ft. } 10 \text{ in.}$$

2. John reads in a book on machines that the distance around a certain wheel is 7×3.142 in. Find this distance.

Omitting the inches in this case, we multiply as in Ex. 1. When we multiply 0.002 by 7 we see that the result is thousandths, and so we know that there are three decimal places in the product.

$$\begin{array}{r} 3.142 \\ 7 \\ \hline 21.994 \end{array}$$

3. Multiply 2.346 by 40.

If we were using only whole numbers, the product would have been 93,840; but since we have multiplied thousandths by a whole number, the result is thousandths, and so we write 93.840, or simply 93.84.

$$\begin{array}{r} 2.346 \\ 40 \\ \hline 93.840 \end{array}$$

Multiplication of a decimal by a common fraction or a mixed number is too rarely met to demand special treatment. It is easily understood from the above cases.

DRILL WORK

1. If a nine-story city building averages 10.75 ft. to a story, how high is the building?

2. The measure called the meter, mentioned on page 144, is 39.37 in. long. How many inches in 75 meters?

3. If each side of a square city lot is 66.6 ft., how long is the perimeter of the lot?

Multiply the following:

4. 3×7.04 ft.
5. 8×6.38 in.
6. 4×5.32 mi.
7. 6×3.08 in.
8. 9×8.75 ft.
9. 7×0.96 oz.
10. 8×0.73 ft.
11. 5×3.77 yd.
12. 7×8.08 ft.
13. 2×4.26 in.
14. 3×5.08 ft.
15. 7×6.35 lb.
16. 9×4.77 oz.
17. 6×5.32 mi.
18. 8×6.41 yd.
19. 5×4.43 in.
20. 9×9.96 ft.
21. 5×3.40 mi.
22. 12×3.26.
23. 56×8.22.
24. 94×3.28.
25. 81×4.48.
26. 85×8.23.
27. 92×3.32.
28. 34×5.81.
29. 78×8.22.
30. 66×4.92.
31. 62×12.48.
32. 28×20.09.
33. 94×66.82.
34. 73×81.62.
35. 55×70.04.
36. 239×32.01.
37. 161×83.406.
38. 325×31.124.
39. 884×42.385.
40. $5\frac{1}{2} \times 78.36$.
41. $4\frac{1}{2} \times 36.62$.
42. $2\frac{1}{2} \times 41.14$.
43. $6\frac{1}{2} \times 51.10$.
44. $2\frac{1}{4} \times 88.60$.
45. $4\frac{3}{4} \times 70.08$.
46. $2\frac{3}{4} \times 86.04$.
47. $7\frac{1}{4} \times 92.36$.
48. $5\frac{2}{3} \times 8.763$.
49. $7\frac{1}{3} \times 9.645$.
50. $4\frac{5}{8} \times 7.096$.
51. $3\frac{7}{8} \times 3.496$.
52. $5\frac{3}{8} \times 2.736$.
53. $8\frac{1}{12} \times 17.28$.
54. $3\frac{5}{12} \times 3.456$.
55. $2\frac{7}{12} \times 6.912$.
56. $3\frac{11}{12} \times 6.912$.
57. $1\frac{1}{16} \times 33.76$.

Multiplying by a Decimal. 1. Albert's father is building a cylindric silo for feed for the cattle. It is 15 ft. $3\frac{1}{2}$ in. across, or practically 15.3 ft. He asks Albert to find the length of an iron band that will just go round the silo, telling him that he can do this by multiplying 15.3 ft. by 3.1. What is the length of the band?

Explain the meaning of "silo" if necessary.

15.3 ft.
3.1
153
459
47.43 ft.

Multiplying in the ordinary way, the figures of the product beginning at the left are 4, 7, 4, and 3. Because $\frac{1}{10} \times \frac{3}{10} = \frac{3}{100}$, we see that $0.1 \times 0.3 = 0.03$; that is, the 3 in the product is hundredths. This tells us where to place the decimal point.

So Albert found that the length of the band was 47.43 ft., or nearly 47 ft. 6 in.

2. Multiply 0.42 by 0.35.

If we multiply 42 by 35 the result is 1470. Since hundredths multiplied by hundredths is equal to ten-thousandths, the result is 0.1470; that is, $0.42 \times 0.35 = \frac{42}{100} \times \frac{35}{100} = \frac{1470}{10000} = 0.1470 = 0.147$.

0.42
0.35
210
126
0.1470

Decimal Places in the Product. If we multiply 8.96 by 3.142, there are five decimal places in the product, because $\frac{2}{1000} \times \frac{6}{100} = \frac{12}{100000}$, so that the product is hundred thousandths.

From this example and from the examples above we have the following rule for multiplying by a decimal:

Multiply as with whole numbers, and then, beginning at the right, point off as many decimal places in the product as there are decimal places in both the factors together.

USING WHAT YOU HAVE LEARNED

1. The speedometer on an automobile shows that the car has gone 27.8 mi. This is how many feet more than 27 mi.?

2. A railway time-table gives the distance between two stations as 7.4 mi. This is how many feet more than 7 mi.?

3. Taking the rod as 16.5 ft., how many feet in 19 rd.?

4. At $36.50 per front foot, how much is the cost of a piece of land with a frontage of 33.3 ft.?

In city surveying, decimal parts of a foot are frequently used. A "front foot" is a foot measured along the street on which the land fronts.

5. A machinist makes a certain steel plate 0.75 as wide as it is long. If the length is 8.2 in., what is the width?

6. A certain rectangle is 2.5 times as long as it is wide. If the rectangle is 5.7 ft. wide, what is the length?

7. A city lot is 38.6 ft. wide and 68.3 ft. deep. Find the number of square feet in the lot.

8. The circumference of a circle is 3.1416 times the diameter. If the diameter is 13.4 ft., what is the circumference?

The teacher should explain the terms if necessary. The pupils probably know them, however. The mensuration of the circle is considered later.

9. A cubic foot of water weighs 62.5 lb., and ice is 0.92 times as heavy as water. How much does 1 cu. ft. of ice weigh?

10. Oak is 1.17 times as heavy as water. How much does 1 cu. ft. of oak weigh?

Use Ex. 9 for the weight of 1 cu. ft. of water.

Teachers are again reminded that in all applied problems of this kind, as well as in all abstract work, the pupils should estimate the results in advance, for the reason that this affords a check upon their work. Every business man does this so as to avoid any absurd result that may be due to an error in computation.

DRILL WORK IN MULTIPLICATION

Numbers 1 to 17, oral

Multiply the following:

1. 0.8×2.	6. 0.9×0.3.	11. 0.8×0.7.
2. 0.2×6.	7. 0.4×0.7.	12. 0.9×0.9.
3. 0.1×5.	8. 0.1×0.2.	13. 0.2×0.3.
4. 0.3×4.	9. 0.3×0.5.	14. 0.4×0.4.
5. 0.6×7.	10. 0.6×0.8.	15. 0.7×0.8.

16. Why will there be three decimal places in the product of 0.25 multiplied by 0.7? of 3.375 multiplied by 9?

17. How do you know the number of decimal places in the product of 0.78 and 3.74? of 5.759 and 4.126?

Multiply the following:

18. 3×0.49.	31. 0.8×9.9.	44. 0.6×47.
19. 0.42×7.	32. 7.7×85.	45. 0.6×4.7.
20. 4×2.75.	33. 3.1×8.1.	46. 6.3×4.86.
21. 3×0.97.	34. 58×0.8.	47. 96×9.65.
22. 0.87×8.	35. 6.7×7.8.	48. 5.4×3.53.
23. 7×2.42.	36. 0.6×96.	49. 73.2×98.6.
24. 3×0.78.	37. 0.7×7.8.	50. 0.423×3940.
25. 0.83×6.	38. 1.8×67.	51. 4.49×70.23.
26. 8×7.23.	39. 2.9×3.4.	52. 3.99×228.63.
27. 9×0.93.	40. 0.6×74.	53. 79.1×39.3.
28. 0.27×9.	41. 0.8×4.9.	54. 32.9×86.09.
29. 6×9.23.	42. 1.4×35.	55. 46.4×79.82.
30. 0.9×74.	43. 1.4×2.6.	56. 0.700×0.600.

Multiplying by Multiples of 10. On page 26 we found how to multiply by 10, 100, 1000, and so on for other powers of 10. We also found how to multiply by such numbers as 40, 700, and so on. To multiply decimals by such numbers is substantially the same as to multiply dollars and cents. Therefore, to multiply 487.7 by 600 we may multiply by 6 and move the decimal point two places to the right.

To multiply 89.07 by 740 we may multiply by 74 and move the decimal point one place to the right.

DRILL WORK IN MULTIPLICATION

Numbers 1 to 16, oral

State the value of each of the following:

1. $5 \times \frac{1}{5}$.
2. $4 \times \frac{1}{4}$.
3. $7 \times \frac{1}{7}$.
4. $10 \times \frac{1}{10}$.
5. 10×0.1.
6. 10×0.6.
7. 20×6.
8. 20×0.6.
9. $100 \times \frac{1}{100}$.
10. 100×0.01.
11. 100×0.02.
12. 100×0.65.
13. $1000 \times \frac{1}{1000}$.
14. 1000×0.001.
15. 1000×0.125.

16. How many places do you move the decimal point when you multiply a decimal by 10? by 100? by 1000? by 10,000?

Multiply the following:

17. 50×8.74.
18. 40×6.89.
19. 70×7.81.
20. 30×86.4.
21. 40×36.6.
22. 80×45.49.
23. 60×725.8.
24. 700×67.09.
25. 500×61.6.
26. 1000×2.25.
27. 2000×3.725.
28. 4970×83.2.
29. 7250×67.28.
30. 6220×750.4.
31. 5600×895.7.
32. 3500×774.66.
33. 4500×508.07.
34. $25{,}000 \times 1.25$.

Dividing by a Whole Number. 1. Ralph rode 925.6 mi. in his automobile in 13 da. Find the average distance per day.

We see that 92 tens ÷ 13 = 7 tens, and there remains 15.6 to be divided.

We see that 15 ÷ 13 = 1, and there remains 2.6 to be divided.

We see that $2.6 \div 13 = 2\frac{6}{10} \div 13 = \frac{26}{10} \div 13 = \frac{2}{10} = 0.2$, so that the quotient is 71.2.

Therefore the average per day was 71.2 mi.

$$\begin{array}{r} 71.2 \\ 13\overline{)925.6} \\ \underline{91} \\ 15 \\ \underline{13} \\ 26 \\ \underline{26} \end{array}$$

Do not expect the pupils to give an elaborate explanation. Proceed as with whole numbers, and then ask how they know where to place the decimal point.

2. If Ralph rode 814 mi. in his automobile in 12 da., what was the average distance per day?

Since 814 = 814.0 = 814.00, we may place a decimal point after the 814, followed by as many zeros as we please. Dividing, we have the quotient 67.8, with remainder 0.4.

We may write the quotient $67.8\frac{1}{3}$ or we may annex more zeros and continue to divide as far as we please. For practical purposes the result may be written 67.8.

$$\begin{array}{r} 67.8 \\ 12\overline{)814.0} \\ \underline{72} \\ 94 \\ \underline{84} \\ 100 \\ \underline{96} \\ 4 \end{array}$$

The pupil should use his common sense as to how far to carry a division. In ordinary measurement of distance traveled a result to tenths of a mile is sufficient. It is rare in a practical problem in this part of arithmetic that a result is needed beyond hundredths.

If we wish to show that the result is greater than 67.8 but less than 67.85, we may write 67.8+; if the result is 67.85 or greater, but less than 67.9, we may write 67.9−. More often we write simply 67.8 or 67.9, omitting the signs. To find a quotient to two decimal places we should find the third decimal figure and write in the second decimal place the figure which gives the closer approximation.

DIVIDING BY A WHOLE NUMBER

1. The perimeter of a triangle with sides all equal is 43.2 in. What is the length of each side?

2. The perimeter of a six-sided figure with equal sides is 15.24 in. What is the length of each side?

3. Four boys are camping on the shore of a lake and pay $21.48 for the outfit. How much is each boy's share?

4. The distance between two places is 21.7 mi. How many miles from one of the places to a point halfway between them?

5. The distance from second base directly through the pitcher's box to home plate is 127.3 ft. Find the distance from home plate to a point one fourth of the way to second base.

Divide, carrying each quotient to one decimal place:

6. $4276.8 \div 6$.	9. $802.7 \div 69$.	12. $34.1 \div 128$.
7. $3748.1 \div 9$.	10. $534.8 \div 92$.	13. $46.7 \div 293$.
8. $5262.7 \div 8$.	11. $417.6 \div 87$.	14. $99.8 \div 487$.

Divide, carrying each quotient to two decimal places:

15. $428.3 \div 41$.	22. $73.42 \div 68$.	29. $2909 \div 426$.
16. $720.6 \div 84$.	23. $82.37 \div 49$.	30. $4702 \div 328$.
17. $293.4 \div 73$.	24. $64.86 \div 38$.	31. $5309 \div 772$.
18. $820.9 \div 29$.	25. $42.73 \div 19$.	32. $6027 \div 428$.
19. $873.4 \div 37$.	26. $68.42 \div 37$.	33. $5009 \div 762$.
20. $607.3 \div 49$.	27. $72.26 \div 35$.	34. $8124 \div 406$.
21. $700.6 \div 58$.	28. $90.71 \div 54$.	35. $9303 \div 500$.

Divide, carrying each quotient to three decimal places:

36. $427.1 \div 93$.	37. $63.85 \div 47$.	38. $8030 \div 237$.

Dividing by Powers of 10. From the work studied on page 33 we obtain the following:

To divide a number by 10, move the decimal point one place to the left.

To divide a number by 100, move the decimal point two places to the left.

Thus, $8 \div 100 = 0.08$, or .08.

To divide a number by 1000, move the decimal point three places to the left.

As we learned on page 138, any whole number may be thought of as having a decimal point after units. That is, 9 is the same as 9., or 9.0, or 9.00.

Dividing by Multiples of Powers of 10. *To divide a number by a number that ends in zeros, move the decimal point in the dividend as many places to the left as there are zeros in the divisor and divide by the number preceding the zeros.*

Thus, to divide 216 by 600, we should divide 2.16 by 6.

$$6\text{ø}\text{ø})\overline{2.16}$$
$$0.36$$

DIVIDING BY TENS

Divide the following:

1. $203 \div 70$.	8. $5191 \div 290$.	15. $7434 \div 9000$.
2. $70.8 \div 60$.	9. $296.1 \div 470$.	16. $625 \div 25{,}000$.
3. $39.9 \div 70$.	10. $273.6 \div 120$.	17. $961 \div 31{,}000$.
4. $6745 \div 500$.	11. $2984 \div 4000$.	18. $882 \div 21{,}000$.
5. $4956 \div 300$.	12. $3415 \div 5000$.	19. $888 \div 37{,}000$.
6. $2345 \div 500$.	13. $8436 \div 3000$.	20. $644 \div 28{,}000$.
7. $9036 \div 600$.	14. $2928 \div 4000$.	21. $902 \div 22{,}000$.

Dividing by a Decimal. If a swift scouting-plane can make 2.5 mi. a minute, how long will it require to go 6.25 mi.?

It will take as many minutes as the 1-minute distance is contained in 6.25 mi.

If necessary, the teacher should illustrate this on the blackboard, using a line to represent the total distance and a part of the line to represent the distance made in 1 min.

That is, we must find the quotient of $6.25 \div 2.5$.

Since this quotient is found to be 2.5, it will take the plane 2.5 min. to go 6.25 mi.

If we divide 60 by 20, we get the same result as if we divide 6 by 2. That is, $60 \div 20$ gives the same quotient as $6 \div 2$, and therefore $62.5 \div 25$ will give the same quotient as $6.25 \div 2.5$.

So we may divide 62.5 by 25 instead of dividing 6.25 by 2.5, and the work will be found to be easier.

$$\begin{array}{r} 2.5 \\ 25\overline{)62.5} \\ 50 \\ \hline 125 \\ 125 \\ \hline \end{array}$$

The teacher should take up this work at the blackboard and show the pupils that they already know how to divide 62.5 by 25.

In general, the pupils should see that they should multiply both dividend and divisor by such a power of 10 as makes the divisor a whole number, after which they should divide in the usual way. Thus, if they move the decimal point two places to the right in the divisor, they must move it two places to the right in the dividend, and so on.

In division by a decimal multiply both divisor and dividend by such a power of 10 as makes the divisor a whole number, and then divide as with whole numbers, placing the decimal point in the quotient above the decimal point in the dividend.

There are several variations of this rule, but none is so simple as the rule itself. Teachers may, however, write $6.\overset{\vee}{2}5 \div 2.5$, if they prefer, placing a small check ($\vee$) to indicate the position of the decimal point in the quotient.

USING WHAT YOU HAVE LEARNED

1. How long will it take a railway train traveling 0.9 mi. a minute to go 20.7 mi.? to go 28 mi.?

2. How long will it take a railway train traveling 0.7 mi. a minute to go 36.4 mi.? to go 49 mi.?

3. A drawing is 10.8 in. long and 0.8 in. wide. The drawing is how many times as long as it is wide?

4. How long will it take an airplane traveling at the rate of 1.2 mi. a minute to go 19.2 mi.?

5. The speedometer of an automobile shows that the car has gone 37.4 mi. on a trip of 336.6 mi. What part of the total distance has the car gone?

6. A manufacturer finds that a pile of 500 sheets of a certain metal is 12.5 in. high. He wishes to know the average thickness per sheet. How many inches in this average thickness?

Divide the following, first estimating each result and writing down the estimate:

7. $17.25 \div 500.$	**9.** $817.6 \div 700.$	**11.** $0.7364 \div 7000.$
8. $3.248 \div 200.$	**10.** $82.48 \div 8000.$	**12.** $74.34 \div 9000.$

Divide the following, first estimating each result and then carrying the quotient to three decimal places:

13. $17.78 \div 1.5.$	**16.** $19.26 \div 3.1.$	**19.** $33.43 \div 6.8.$
14. $62.23 \div 0.23.$	**17.** $21.43 \div 3.7.$	**20.** $423.6 \div 0.27.$
15. $5.423 \div 0.38.$	**18.** $584.3 \div 5.9.$	**21.** $8.297 \div 7.9.$

22. Traveling at the rate of 42.4 mi. an hour, how long will it take to travel 161.12 mi.?

23. Divide 0.7 by 0.91 to four decimal places.

Canceling with Decimals. A dealer sells candy in boxes of two sizes. The larger box is 4.2 in. by 8.6 in. by 2.1 in., and the smaller is 4.2 in. by 4.3 in. by 1.4 in. The larger holds how many times as much as the smaller?

We must divide $4.2 \times 8.6 \times 2.1$ by $4.2 \times 4.3 \times 1.4$.

We write the work like a fraction and then cancel common factors.

$$\frac{\overset{2}{\cancel{4.2}} \times \overset{3}{\cancel{8.6}} \times \cancel{2.1}}{\cancel{4.2} \times \cancel{4.3} \times \underset{2}{\cancel{1.4}}} = 3$$

Evidently 4.2 may be canceled.

We also see that $8.6 = 2 \times 4.3$, and so we cancel 4.3, placing 2 over 8.6 in the dividend.

We have left 2×2.1 in the dividend and 1.4 in the divisor. We now cancel 0.7 from 1.4 and 2.1, and then cancel 2.

We find that the larger box is 3 times the smaller.

If we wish to do so, in this example we may multiply both dividend and divisor by $10 \times 10 \times 10$, thus having $42 \times 86 \times 21$ to be divided by $42 \times 43 \times 14$, and avoiding decimals. This is often less confusing to pupils.

The subject is not of great importance, and teachers may omit it if they wish. It is, however, required in certain courses of study.

DRILL IN CANCELLATION

1. A block of marble 3.5 ft. by $4\frac{1}{4}$ ft. by 2.2 ft. weighs how many times as much as one that is $2\frac{1}{2}$ ft. by 7.7 ft. by 0.85 ft.?

Find the value of each of the following:

2. $\dfrac{3.9 \times 0.84}{15.6}$.

3. $\dfrac{1.4 \times 0.6}{0.63}$.

4. $\dfrac{2.2 \times 2.4}{4 \times 7.7}$.

5. $\dfrac{4.5 \times 6.3}{8.1}$.

6. $\dfrac{5.6 \times 0.3}{0.63 \times 0.8}$.

7. $\dfrac{2.1 \times 0.4}{0.015}$.

8. $\dfrac{0.8 \times 1.5 \times 0.6}{3.6 \times 0.2 \times 10}$.

9. $\dfrac{1.2 \times 0.39 \times 1.4}{2.6 \times 0.42 \times 2}$.

10. $\dfrac{0.2 \times 3.6 \times 1.43}{9.9 \times 0.4 \times 2.6}$.

Bill. If goods are not paid for by cash when they are purchased, the dealer usually sends a *bill* showing the purchases, the prices, and the amount due. The bill is generally paid at some later date. You probably studied such bills when you were in the primary grades. The following is a model bill, receipted by the dealers to show that it has been paid:

Atlanta, Ga., May 9, 1922

Mr. S. R. Samson, 2798 Dewey Street

Bought of

MONTGOMERY & HILL

Dealers in Sporting Goods

8296 Main St.

May	*9*	*2 tennis rackets*	*8.25*	*16*	*50*
		6 tennis balls	*.50*	*3*	*00*
				19	*50*
		Received payment			
		Montgomery & Hill			

MAKING OUT BILLS

Make out bills for the following items, inserting the names of the purchasers and the dealers and receipting the bills:

1. 1 baseball cap, 80¢; 1 catcher's mitt, $2.80; 1 baseball mask, $2.75; 1 catcher's body protector, $3.85.

2. 1 league ball, $1.35; 3 bats @ $1.20; 2 fielder's mitts @ $1.53; 3 canvas bases @ $2.40; 1 pitcher's toe plate, 45¢.

3. 4 hunting caps @ 80¢; 1 hunting vest, $2.25; 2 pr. corduroy hunting pants @ $4.45; 2 hunting coats @ $4.80.

Price List. The following is a price list of articles interesting to boys and girls. It is used in the problems on this page and may be used in other problems that the class may make up.

Boy's storm-proof boots, $4.75	Bicycle, $32.75
Girl's school shoes, $4.25	Scout camera, $5.25
Girl's skating cap, $2	Hammock, $4.50
Boy's sweater, $2.90	Small typewriter, $12.50
Boy's nickel watch, $4.25	School drawing set, 60¢
Girl's nickel watch, $4.50	Schoolbook satchel, $1.25
Rubber-stamp outfit, $1.75	Catcher's mitt, $2.50
Cartridge belt, $2.85	Catcher's mask, $3.50
Baseball bat, $1.25	Basket ball, $5.25
Football pants, $3.25	Guitar, $16.50

USING A PRICE LIST

Find the cost of the following:

1. 2 sweaters, 3 skating caps, and a boy's watch.

2. A bicycle, a sweater, a guitar, and 2 catcher's mitts.

3. A rubber-stamp outfit, a hammock, and a cartridge belt.

4. 4 pairs of football pants and 2 basket balls.

5. A school drawing set, a pair of boy's storm-proof boots, a schoolbook satchel, and 2 pairs of girl's shoes.

6. 4 baseball bats, 2 catcher's masks, a basket ball, a catcher's mitt, a scout camera, and a typewriter.

7. Make out three different lists of things you would like to buy, six items in each, and find the cost of each list.

The purpose of this work at this particular time is to show that the chief application of decimal fractions is to United States money.

Marketing. The following is a price list of household supplies. It may be used in the problems given below and in others that the class may make up, or the current prices may be used.

White flour,	\$1.65 per bag
Yellow corn meal,	7¢ per pound
Rye flour,	\$12.25 per barrel
Potato flour,	18¢ per package
Cornstarch,	16¢ per package
Hominy,	8¢ per pound
Alaska salmon,	32¢ per can
Corn sirup,	28¢ per 1½-pound can
Graham flour,	25¢ per package

MARKETING

Find the cost of the following:

1. 1 bag of white flour and 3 packages of potato flour.

2. 1 bbl. of rye flour and 6 lb. of yellow corn meal.

3. 8 packages of cornstarch and 12 lb. of hominy.

4. 2 packages of cornstarch, 6 cans of Alaska salmon, 8 packages of potato flour, and a bag of white flour.

5. 5 cans of corn sirup, 1 bbl. of rye flour, 10 lb. of hominy, and 9 lb. of yellow corn meal.

6. 11 lb. of hominy, 7 packages of potato flour, 14 cans of Alaska salmon, and a bag of white flour.

7. 3 packages of graham flour, a bag of white flour, and 4 packages of cornstarch.

On account of fluctuations of price and in order to make the work seem more real, the teacher is advised to use local price lists, if convenient, and to have the pupils make up problems similar to those given above.

USING WHAT YOU HAVE LEARNED

1. How much will 25 phonographs cost at $52.40 each?

2. How much will 38 typewriters cost at $72.50 each?

3. How much will 240 whalebones cost at 35¢ each?

4. How much will 96 yd. of silk cost at $2.25 a yard?

5. How much will 75 yd. of voile cost at $1.60 a yard?

6. How much will 96 yd. of cloth cost at $1.25 a yard?

7. How much will 24 yd. of cloth cost at $87\frac{1}{2}$¢ a yard?

8. How much must a school pay for 64 manual-training benches at $18.50 each?

9. How much must a school pay for 20 jack planes at $2.60 each? at $2.75 each? at $3.25 each?

10. A dealer pays $360 for some gingham at 90¢ a yard. How many yards of gingham does he buy?

11. A dealer pays $330 for some percaline at 75¢ a yard. How many yards of percaline does he buy?

12. If 4 tennis rackets cost $17.20, how much will 2 rackets cost? Solve by one division.

13. If 12 fishing reels cost $31.20, how much will 4 fishing reels cost? How much will 6 fishing reels cost? In each case solve by one division.

14. If 3 shotguns cost $117, how much will 12 shotguns cost? Solve by one multiplication.

15. If 3 cameras cost $27.75, how much will 4 cameras cost? Solve by one division and one addition, or else by one multiplication by a mixed number.

In all such problems as these the pupils should use short methods, employing aliquot parts if possible.

USING WHAT YOU HAVE LEARNED

1. A man has two farms, one containing 168.9 A. and the other 147.8 A. How many acres of land has he in both farms?

2. At $125 an acre, how much will 128.4 A. of land cost?

3. An automobile travels 27.6 mi. the first hour and 23.4 mi. the second hour. How far does it travel in the two hours?

4. In going from Denver to Pueblo, a distance of 119.4 mi., the train passes through Colorado Springs, 74.9 mi. from Denver. How far is it from Colorado Springs to Pueblo?

5. A train leaving Denver at 7 45 P.M. reaches Santa Fe the next day at 4 25 P.M., a distance of 405.9 mi. Find the average rate per hour. Remember that 4 25 P.M. means 4 hr. 25 min. P.M.

6. In traveling from Chicago to Winona, a distance of 296.1 mi., we pass through Madison, 129.7 mi. from Chicago. The distance from Chicago to Madison lacks how much of being half the distance from Chicago to Winona?

7. One train leaves Chicago at 10 30 A.M. and arrives at Omaha at 11 59 P.M. Another train leaves Chicago at 7 P.M. and arrives at Omaha at 7 20 the next morning. The distance being 491.3 mi., find the average rate of each train per hour.

8. A man on an automobile trip stops for luncheon when the speedometer on his car shows that he has traveled 46.9 mi. At the end of the trip he finds he has traveled 120.4 mi. What part of the trip had he made when he stopped for luncheon?

9. Two cousins, Frank and Tom, visit their uncle in Denver. Frank lives in Colorado Springs, 74.9 mi. from Denver, and Tom lives in Carbondale, 373 mi. from Denver. Tom tells his uncle that Carbondale is exactly five times as far from Denver as Colorado Springs is, but Frank says it is not. Which is right?

USING WHAT YOU HAVE LEARNED

1. Two airplanes leave the same aviation field at the same moment, the first flying 75 mi. an hour and the second 60 mi. an hour. If they fly continuously until the first plane has gone 600 mi., how far has the second gone?

2. Henry lives in the country 3 mi. from the town school. The school is in session 5 da. a week for 18 wk. The fare by trolley is 5¢ each way, but Henry rode a second-hand bicycle which cost him only $1 more than his car fare would have amounted to if he had gone to school every day that it was in session. How much did he pay for the bicycle?

3. The owner of a farm is building a new house. The excavation for the cellar is 28 ft. long, 27 ft. wide, and 6 ft. deep. How many cubic yards are excavated?

4. In a certain city the schools are in session 5 da. a week, except for 6 holidays, for 39 wk. 4 da. If a boy goes to school every day the schools are in session, how many days is he in school? How many days in the year does he have for rest and play?

5. During the summer vacation a boy worked 10 wk. in an office which he reached by a street car. He made one round trip daily, and the car fare was 7¢ each way. How much did his car fares amount to if he worked every week day except the Fourth of July? The previous summer the car fare was 5¢ each way. How much did the increase in car fare cost him?

6. A boy earns $8.50 a week and pays each week day 12¢ for car fares and 30¢ for lunch. He pays 15¢ each week for a ticket to the moving-picture theater, and 50¢ for a ticket to the ball game on Saturday afternoon. His other expenses are 60¢ each week. How much does he save in 8 wk.?

7. A certain cow whose milk contains 4.6 lb. of butter fat to every 100 lb. of milk produces 4375 lb. of milk in a year. How many pounds of butter fat does the cow produce in a year?

8. A certain village decides to pave its main street for a distance of 1275 ft. The roadway as paved is 32.7 ft. wide. How many square yards of paving are there?

9. If a cubic foot of water weighs 62.5 lb., how many ounces does it weigh?

10. Using the result of Ex. 9, find the weight of 1 cu. in. of water, carrying the result to hundredths of an ounce and then writing it to the nearest tenth of an ounce.

11. Tin is 7.29 times as heavy as water. Using the first result in Ex. 10, find the weight of 1 cu. in. of tin.

12. Gold being 19.26 times as heavy as water, and 1 cu. ft. of water weighing 62.5 lb., how much does 1 cu. ft. of gold weigh?

Since the weight of water is given only to the nearest tenth of a pound, the result of Ex. 12 need be given only to the nearest tenth of a pound. This applies to all such problems, but pupils are hardly ready at this stage to appreciate this fact. The pupils may therefore multiply 62.5 lb. by 19.26 in the usual way, unless the teacher directs otherwise. In all such problems the influence of temperature is not considered, the pupil not being sufficiently advanced to undertake the solution of problems involving this feature.

13. If 1 cu. ft. of copper weighs 8950 oz. and 1 cu. ft. of granite weighs 2780 oz., copper is how many times as heavy as granite?

The teacher may direct that the results in Exs. 13–15 be carried only to the nearest tenth, which requires the division to be carried to hundredths in order to see what is the nearest tenth.

14. Oak is 1.17 times and maple is 0.75 times as heavy as water. Oak is how many times as heavy as maple?

15. Copper is 8.95 times and silver is 10.47 times as heavy as water. Silver is how many times as heavy as copper?

II. INTRODUCTION TO PERCENTAGE

Need for Per Cents. Fred Roberts is captain of the baseball team of the sixth grade of our school. He finds that he can buy at the store a catcher's mitt for $2. The next day he hears that this same mitt has been marked down 10 per cent. He now begins to think that it is necessary that he should know what per cent means.

If any one in the class knows, he may tell the meaning, otherwise it may be taken up after studying the next page. This page is merely a kind of preparatory reading lesson.

Mrs. Roberts, Fred's mother, wishes to buy a suit of clothes for Fred. The price was $15, but the suit has been marked down 20 per cent. If she wishes to buy the suit, she should know what this means. Do you know what it means?

The teacher has to make a report of the number of pupils tardy or absent last week and says in the report that 5 per cent of the pupils were tardy and 2 per cent were absent. Do you know what this means?

A man wishes to buy an automobile. He can buy a new one of the kind he likes for $1160, but one that is nearly new is offered for 25 per cent less than this price. Do you know how to find what the man would have to pay for the second car?

It is not necessary that any pupil should answer these questions. What is of importance is that all the class should see that per cents are frequently met and that everyone should know what they mean.

The subject of percentage is taken up more extensively in Book Three. It is a mistake, however, not to show the pupil that per cents are merely decimal fractions, and that no different principle is involved in finding 15 per cent of $50 from that involved in finding 0.15 of $50. The only difference is one of symbolism, and hence a brief introduction to percentage should be looked upon as part of the study of decimal fractions.

Per Cent. Another name for "hundredths" is *per cent.* For example, instead of saying "ten hundredths" we may say "ten per cent." The two expressions mean the same.

If Jack finds that a catcher's mitt which is marked \$2 can be bought for 10 per cent less than the marked price, this means that it can be bought for $\frac{10}{100}$, or $\frac{1}{10}$, less than \$2; that is, it can be bought for \$2 less $\frac{1}{10}$ of \$2, or \$2 − \$0.20, or \$1.80.

Symbol for Per Cent. There is a special symbol for per cent, %. Thus we write **40%** for 40 per cent, or 0.40.

Hundredths written as Per Cents. Because hundredths and per cents are the same, any fraction with denominator 100 may be written in the form of per cent. Thus:

$$\frac{3}{100} = 3\% \qquad \frac{1}{100} = 1\% \qquad \frac{21}{100} = 21\% \qquad \frac{200}{100} = 200\%$$

READING PER CENTS

All work oral

Read the following as per cents:

1. $\frac{3}{100}$.	4. $\frac{8}{100}$.	7. $\frac{28}{100}$.	10. $\frac{50}{100}$.	13. $\frac{82}{100}$.
2. $\frac{5}{100}$.	5. $\frac{10}{100}$.	8. $\frac{37}{100}$.	11. $\frac{65}{100}$.	14. $\frac{95}{100}$.
3. $\frac{6}{100}$.	6. $\frac{18}{100}$.	9. $\frac{44}{100}$.	12. $\frac{75}{100}$.	15. $\frac{100}{100}$.

Read the following as hundredths:

16. 10%.	18. 38%.	20. 60%.	22. 82%.	24. 200%.
17. 30%.	19. 45%.	21. 75%.	23. 98%.	25. 300%.

26. Read $\frac{225}{100}$ as per cent, and 275% as hundredths.

27. Read as per cents: $\frac{12\frac{1}{2}}{100}$, $\frac{62\frac{1}{2}}{100}$, $\frac{\frac{1}{4}}{100}$, $\frac{33\frac{1}{3}}{100}$, $\frac{\frac{1}{8}}{100}$, $\frac{66\frac{2}{3}}{100}$.

Halves and Fourths as Per Cents. Mr. Simpson says that he sold an automobile for 50% of its cost. What does this mean? He sold the car for how many hundredths of what it cost him? Express this as a common fraction in lowest terms.

If a baseball team plays 48 games and loses only 25% of them, how many games does it lose? How many hundredths of the games does the team lose? How many fourths?

From these two examples we see that

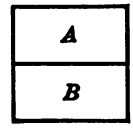
$$50\% = \frac{50}{100} = \frac{1}{2} \qquad 25\% = \frac{25}{100} = \frac{1}{4}$$

Other simple examples having local interest to the pupils should be given. The following exercise emphasizes the relations of $\frac{1}{2}$ and $\frac{1}{4}$ to per cents.

HALVES AND FOURTHS AS PER CENTS

All work oral

1. If this square represents a floor containing 100 sq. ft., how many square feet in B? Then B is how many hundredths of the square? What per cent of the square is B? Then A is what per cent of the square?

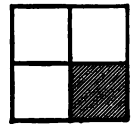


2. If asked to find 50% of a number, you may find what fraction of the number instead? Illustrate your answer.

Find 50% of each of the following numbers:

3. 30. 4. 36. 5. 48. 6. 60. 7. 300.

8. If this square represents a box cover containing 100 sq. in., how many square inches are shaded? How many fourths are shaded? How many hundredths are shaded? What per cent is shaded?



Find 25% of each of the following numbers:

9. 36. 10. 44. 11. 60. 12. 2. 13. 4000.

Important Per Cents. If your school team plays 8 games and loses 50% of them, how many games does it lose? Suppose that it loses 25%, how many games does it lose?

How much is $\frac{1}{2}$ of 25? $\frac{1}{2}$ of 25%?

Suppose that your team loses $12\frac{1}{2}\%$ of 8 games, how many games does it lose?

We have now found that

$$50\% = \frac{1}{2} \qquad 25\% = \frac{1}{4} \qquad 12\frac{1}{2}\% = \frac{1}{8}$$

USING WHAT YOU HAVE LEARNED

1. A school has 284 pupils, and 50% of them are girls. How many girls are there in the school?

2. A boy had 68 marbles and lost 25% of them. How many did he lose, and how many did he have left?

3. Fred has made 50 runs in this season's games, and Jack has made 8% more. How many runs has Jack made?

4. A school has 488 pupils and $12\frac{1}{2}\%$ were tardy last week. How many were not tardy?

5. Of 968 sheep owned by a farmer $12\frac{1}{2}\%$ were sold. How many were sold? How many remained?

Find the values of the following:

6. 50% of 2078.
7. 25% of 3572.
8. 50% of 16,390.
9. 25% of \$4416.72.
10. 50% of 1276 bu.
11. 25% of 84 ft. 4 in.
12. $12\frac{1}{2}\%$ of \$99.28.
13. $12\frac{1}{2}\%$ of \$97.76.

14. A man owned 128 A. of land and sold $12\frac{1}{2}\%$ of it at \$125 an acre. How many acres did he sell? How much did he receive for what he sold? How many acres had he left?

Other Important Per Cents. Gertrude finds that some handkerchiefs have been marked down $33\frac{1}{3}\%$ at a bargain sale. She wishes to think of this per cent as a common fraction. How shall she do it?

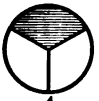
How much is $\frac{1}{3}$ of 100? How much is $\frac{2}{3}$ of 100?
How much is $\frac{1}{2}$ of $\frac{1}{3}$? How much is $\frac{1}{6}$ of 100?
Can you answer Gertrude's question about $33\frac{1}{3}\%$?
We have found that

$$16\tfrac{2}{3}\% = \tfrac{1}{6} \qquad 33\tfrac{1}{3}\% = \tfrac{1}{3} \qquad 66\tfrac{2}{3}\% = \tfrac{2}{3}$$

USING WHAT YOU HAVE LEARNED

Numbers 1 to 4, oral

1. If circle *A* is called 100, how much is the shaded part? This is what per cent of the circle?


A

2. Read $0.33\frac{1}{3}$, using the words "per cent"; using the word "hundredths." How many thirds in 1? How many times is $33\frac{1}{3}\%$ contained in 1?

3. If circle *B* is called 100, the shaded part is what part of $33\frac{1}{3}$? How much is this? Then the shaded part is what per cent of the circle?

B

4. If circle *C* is called 100, the shaded part is what part of $16\frac{2}{3}$? How much is this? Then the shaded part of the circle is what per cent of the whole circle? Then $\frac{1}{12}$ is what per cent?

C

Find the values of the following:

5. $33\frac{1}{3}\%$ of 894.
6. $66\frac{2}{3}\%$ of \$3714.30.
7. $16\frac{2}{3}\%$ of \$1234.56.
8. $66\frac{2}{3}\%$ of 1047 yd.
9. $16\frac{2}{3}\%$ of 7386 bu.
10. $33\frac{1}{3}\%$ of 6528 mi.

Other Important Per Cents. It is easily seen that

$20\% = \frac{1}{5}$	$60\% = \frac{3}{5}$	$75\% = \frac{3}{4}$	$62\frac{1}{2}\% = \frac{5}{8}$
$40\% = \frac{2}{5}$	$80\% = \frac{4}{5}$	$37\frac{1}{2}\% = \frac{3}{8}$	$87\frac{1}{2}\% = \frac{7}{8}$

Such per-cent equivalents should be drilled upon so thoroughly that the mention of one form suggests the other automatically.

USING WHAT YOU HAVE LEARNED

Numbers 1 to 9, oral

1. In a school of 450 pupils 20% of them are in the fifth and sixth grades. How many pupils are in these grades?

2. An automobile is making a trip of 320 mi. After going $37\frac{1}{2}\%$ of the distance how many miles has it gone?

3. In a school of 500 pupils 60% are in the first four grades. How many pupils are in those grades?

Find the values of the following:

4. 20% of 45.
5. 40% of 35.
6. 60% of 50.
7. 80% of 50.
8. 75% of 400.
9. $87\frac{1}{2}\%$ of 1600.

10. A man has to take a motor trip of 344 mi. After finishing $37\frac{1}{2}\%$ of the trip how many miles has he gone?

11. If a carpenter has completed $62\frac{1}{2}\%$ of a job that is to bring him in \$2720, and is paid for the amount completed, how much does he receive?

Find the value of each of the following:

12. $37\frac{1}{2}\%$ of 248 mi.
13. $62\frac{1}{2}\%$ of 360 ft.
14. $87\frac{1}{2}\%$ of 424 lb.
15. 20% of $62\frac{1}{2}\%$.
16. 40% of $87\frac{1}{2}\%$.
17. $66\frac{2}{3}\%$ of $37\frac{1}{2}\%$.

Per Cents as Common Fractions. We have learned that

$$62\tfrac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}.$$

To express per cent as a common fraction, write the number indicating the per cent as the numerator of a fraction and 100 as the denominator, and reduce to lowest terms.

In computing with per cents it is often convenient to use one form, and often another. Thus, if we are multiplying by 13.7% it is easier to think of the multiplier as 0.137; but if we are multiplying by $33\frac{1}{3}\%$ it is easier to think of it as $\frac{1}{3}$.

We know that $\frac{2}{3}$ may be reduced to hundredths by multiplying each term by $33\frac{1}{3}$. We then have

$$\frac{2}{3} = \frac{66\frac{2}{3}}{100} = 66\tfrac{2}{3}\%.$$

To express a common fraction as per cent, reduce it to hundredths, and then write the numerator followed by the symbol for per cent.

The teacher should show that we may also reduce $\frac{2}{3}$ to hundredths by dividing 2 by 3; thus:

$$\tfrac{2}{3} = 2 \div 3 = 2.00 \div 3 = 0.66\tfrac{2}{3} = 66\tfrac{2}{3}\%.$$

PER CENTS AND COMMON FRACTIONS

Express as common fractions in lowest terms:

1. 36%.	3. 25%.	5. 30%.	7. 18%.	9. $82\frac{1}{2}\%$.
2. 64%.	4. $6\frac{1}{4}\%$.	6. 125%.	8. 375%.	10. $33.3\frac{1}{3}\%$.

Express the following as per cents:

11. $\frac{3}{8}$.	13. $\frac{7}{8}$.	15. $\frac{2}{5}$.	17. $\frac{4}{5}$.	19. $\frac{3}{16}$.	21. 1.
12. $\frac{5}{8}$.	14. $\frac{1}{5}$.	16. $\frac{3}{5}$.	18. $\frac{1}{16}$.	20. $\frac{17}{16}$.	22. $3\frac{1}{2}$.

Finding Per Cents. We have now found that certain important fractions like $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{2}{3}$ are easily expressed as per cents; that any fraction can be expressed as per cent if it is first reduced to hundredths; and that per cents can easily be expressed as common fractions. We shall now learn how to find a per cent of a number.

Illustrative Problems. 1. The price of a dress which Mary wishes is \$16, but tomorrow the dress is to be marked down 15%. How much will it then cost?

Since 15% = 0.15, we multiply \$16 by 0.15, to find the amount the dress is to be marked down. We then have

15% of \$16 = \$2.40;

and \$16 − \$2.40 = \$13.60, cost.

\$16	\$16.
0.15	2.40
80	\$13.60
16	
\$2.40	

To find a per cent of a number, express the per cent as a decimal and multiply the number by this decimal.

2. John bought a camping suit that had been marked \$12 but was marked down $12\frac{1}{2}$%. How much did the suit cost?

Since $12\frac{1}{2}\% = \frac{1}{8}$, we take $\frac{1}{8}$ of \$12, to find the amount the suit was marked down.

Then we have $\frac{1}{8}$ of \$12 = \$1.50;

and \$12 − \$1.50 = \$10.50, cost.

8)\$12.	
\$1.50	
	\$12.
	1.50
	\$10.50

We see that if we wish to find 50%, 25%, $12\frac{1}{2}$%, $16\frac{2}{3}$%, $33\frac{1}{3}$%, or $66\frac{2}{3}$% of a number, it is easier to use a common fraction. By so doing we can frequently solve mentally a problem that would otherwise require written work. When we wish to find such a per cent as 15% it is better to use the decimal form, as in Ex. 1.

USING WHAT YOU HAVE LEARNED

Numbers 1 to 7, oral

1. If you have 50 words in a spelling test and are marked 90%, how many words did you spell correctly?

2. If you have 10 problems in arithmetic and are marked 90%, how many problems did you solve correctly?

3. If you have 20 questions in geography and are marked 75%, how many questions did you answer correctly?

4. If 80% of a class of 20 have not been absent a day this year, how many have been present every day?

5. If a merchant pays 4¢ apiece for a lot of pencils and sells the pencils so as to gain 50% on what they cost him, what is the selling price?

6. If a milk dealer pays 15¢ a quart for milk and sells it so as to gain 20% on what it costs him, how much does he receive for the milk per quart?

7. If a butcher buys meat at 24¢ a pound and sells it so as to gain $16\frac{2}{3}\%$ on what it costs him, at what price does he sell it per pound?

8. In the world's championship series in a certain year the players' share was $135,000, of which the Giants received 60%. How much money did they receive?

9. The St. Louis team was at bat 4812 times in one season, and the base hits were 25% of the times at bat. How many base hits did they make?

10. In a school of 350 pupils 8% were in the sixth grade. How many pupils were there in that grade?

11. If a dealer pays 10¢ apiece for paper pads and sells them so as to gain 60% on this price, what is the selling price?

PROBLEMS ABOUT SCHOOLS

1. In a certain school 250 pupils are enrolled. Of these 8% are in the sixth grade and 12% are in the first. How many pupils are there in each of these grades?

2. Of 340 pupils in a certain school 5% have been tardy or absent this month. How many have been tardy or absent?

3. If there is an increased enrollment of $3\frac{1}{3}\%$ this year in a school which had 360 pupils last year, how many pupils are there this year?

4. In a school district of 160,000 inhabitants 20% are school children. How many school children are there?

5. In a school of 250 pupils, if there is an increase of 4% next year, how many pupils will there then be?

6. In a school of 300 pupils $4\frac{2}{3}\%$ are absent today. How many pupils are absent?

7. If 15% of the 240 pupils of a school are new pupils this year, how many new pupils are there?

8. The class in cooking found that some food costing \$8.84 could be bought for 12% less at another grocery. How much could be saved by buying the food at this store?

Have the pupils state the result to the nearest cent.

9. If there are 50 pupils in a class and 40% are boys, what fractional part of the class are boys? How many boys are there? If 60% are girls, how many girls are there?

10. A class in industrial arts bought 50 yd. of cloth, and when the cloth was washed the shrinkage was $\frac{1}{2}\%$. How much did the cloth shrink? What was the length after it was washed?

Show the class that, since per cent means hundredths, $\frac{1}{2}\%$ means $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$. This shows that we can find $\frac{1}{2}\%$ of a number by taking $\frac{1}{200}$ of it.

PROBLEMS ABOUT GEOGRAPHY

1. When the population of Texas was 4,500,000, it is said that 25% lived within 100 mi. of Dallas. How many lived within that distance?

2. The population of Alabama at one time was 2,140,000, and ten years later it had increased 9.3%. Find its population then.

3. If the receipts at the New Orleans docks were 125 carloads of freight on one day and 20% less the next day, what were the receipts on the second day? on both days together?

4. If 575 tons of ore are shipped to Seattle on a certain day, and 15% of it is loaded on a steamer, how many tons are loaded on the steamer?

5. The area of Illinois is 56,600 sq. mi. approximately. Of this 63% is underlain with coal beds. How many square miles of coal lands are there in the state?

6. The area of the United States is 3,600,000 sq. mi. approximately, and of this about $7\frac{1}{2}$% is in Texas and about 3.4% is in New Mexico. Find the approximate number of square miles in each of these states.

7. The total productive land of the United States is about 880,000,000 acres, of which 13% is in Texas. Find the number of acres of productive land in Texas.

8. Mt. McKinley, the highest peak in Alaska, is 20,300 ft. high and Blue Mountain, the highest elevation in Arkansas, is approximately 13.8% as high. How high is Blue Mountain?

Give the result to the nearest foot.

9. North Truchas Peak in New Mexico is 13,306 ft. high and East Peak in Nevada is approximately 1.2% lower. Find the height of East Peak to the nearest foot.

USING WHAT YOU HAVE LEARNED

1. A certain water tank holds 85,000 gal. A large pump at the water works pumps 4% of this amount in a single stroke. How many gallons is this?

2. A manufacturing company has certain engines of 18,600 horse power and certain others of $66\frac{2}{3}\%$ as much horse power. Find the horse power of the latter engines.

3. A pound of candy is put into a box which weighs 10% as much as the candy itself. Find in pounds and ounces the total weight of 1 doz. such boxes filled with candy.

4. A man rents a store for $1400 the first year, 10% more the second year, and 10% more the third year than the second. How much rent does he pay in 3 yr.?

5. A rectangle that was $8\frac{1}{2}$ in. long and 7.2 in. wide has been increased 40% in length and $33\frac{1}{3}\%$ in width. What are the length and width now?

6. The man who owns a shoe-shining place in a city says that the money he receives in a month is divided in this way: rent, 30%; help, 50%; light and fuel, 3%; supplies, 1%. The balance is profit. If he takes in $620 a month, how much of this is profit?

7. A farmer who sends butter to the city finds that the commission and railroad charges are 32% of the sale price of the butter. How much is left for him on a shipment of 200 lb. of butter if it sells at 60¢ a pound?

8. If a baker in making a certain quantity of bread uses 732 lb. of flour and $33\frac{1}{3}\%$ as much liquid as flour, what is the weight of the liquid used?

9. In Ex. 8 if the baker wished to make 25% more bread, what would be the weight of the flour and liquid when mixed?

Per Cents of Profit and Loss. A class in a school has a baseball team. It paid \$70 for suits, bats, and other equipment. At the end of the season the team found that it had in its treasury not only \$70 but 42% more.

That is, 42% of \$70 = \$29.40, profit.

One use of per cents is to express profit or loss in business. The profit or loss is commonly expressed as some per cent of the total cost of goods bought (the cost including freight, share of store rent, insurance, cost of selling, and the like). The per cent of profit or loss is often called the *rate of profit or loss.*

The computing of profit on the selling price, in large business affairs, is considered later.

PER CENTS OF PROFIT AND LOSS

1. A merchant paid \$4920 for some goods. He estimated the cost of freight, share of rent and selling expenses, and other charges as \$480. He sold the goods at a price that brought him a profit of 20%. Find the profit.

2. A man paid \$11,520 for a house. He spent for repairs \$300 and estimated all other expenses at \$180. He sold the house at a loss of 4%. What was the selling price?

3. A grocer bought \$3500 worth of goods and his expenses were \$500. If his rate of profit was 19%, find the profit.

4. A hardware dealer paid \$1937.50 for a lot of tools. He estimated all other expenses as \$98.50. Through carelessness the tools rusted so that he had to sell them at a loss. The rate of loss was 8%. How much was lost through carelessness?

5. A dealer pays \$4.32 a gross for some penholders and sells them at a gain of $66\frac{2}{3}$% on this cost. Find the price at which he sells each penholder.

Per Cents Off. If you have a catcher's glove or a tennis racket that you have used for a year, you may be glad to sell it at half price so as to get part of the money that it cost and have this much toward buying a new one. In that case you would sell it at a *discount* of 50%.

Sometimes merchants have bargain sales to dispose of odd pieces or of goods that are going out of style. At such times they usually sell goods at a certain "per cent off."

For example, goods which are marked \$3.50 but are sold at 10% off are sold at \$3.50 less 10% of \$3.50, or \$3.15.

\$3.50	\$3.50
0.10	.35
\$0.35	\$3.15

Merchants who sell at wholesale, that is, in large quantities to dealers, often sell at a certain per cent off the price as stated in their catalogues, or off the *list price*, as it is called.

Discount. A reduction from a price or amount is called a *discount*. If you should offer to sell for \$2 a tennis racket that cost \$3, you would be offering to sell it at a discount of \$1, or at a discount of $\frac{1}{3}$ of its cost, or at a discount of $33\frac{1}{3}\%$.

Rate of Discount. The per cent of discount is called the *rate of discount*. The amount of a price after the discount has been taken off is called the *net price* or *selling price*, and similarly the amount of a bill after the discount has been taken off is called the *net amount* of the bill.

For example, find the discount and the net price when the list price of a library table is \$245 and the rate of discount is 12%.

We have 12% of \$245 = \$29.40, discount.

Then \$245 − \$29.40 = \$215.60, net price.

\$245	
0.12	
\$29.40	
\$245.	
29.40	
\$215.60	

PER CENTS USED IN DISCOUNTS

Numbers 1 to 8, oral

1. If you can buy at a reduction in price of 10% a baseball mitt that is marked \$2.60, what is the amount of the reduction? What is the net price?

2. If you can buy at 20% off, that is, at a reduction of 20%, a tennis racket that was marked \$4.50, what is the net price?

Find the net price in each case, given the marked price and the rate of discount as follows:

3. \$8, 10%.
4. \$12, 25%.
5. \$60, $33\frac{1}{3}$%.
6. \$80, 25%.
7. \$300, 10%.
8. \$500, 20%.

9. A man can buy a \$1500 automobile at a reduction of 12%. Find the net price.

10. A suit of clothes marked \$33 is advertised to be sold at $33\frac{1}{3}$% off. What is the net price?

11. A dealer offers Boy Scout suits which were marked \$17.85 each at $33\frac{1}{3}$% off. Find the net price.

12. If you buy a \$28 suit of clothes at 15% discount, how much do you save on the marked price?

13. How much will a set of Scott's works, marked \$14, cost at 20% discount? Check by taking 80% of \$14.

Explain informally that $100\% - 20\% = 80\%$, and hence that any price less 20% of that price is 80% of the price.

14. At a bargain sale a \$12 chair was sold at 25% discount. For how much did it sell? Check by taking 75% of \$12.

15. If the cost of the books, pencils, and paper which you use in a year is \$8.50 at list prices, how much will they cost at 8% discount? Check by taking 92% of \$8.50.

Order for Goods. The following is a model order for goods:

WOOD & ROBERTS
Booksellers

Jacksonville, Fla., Feb. 15, 1922

Messrs. Mason and Company, Publishers
Atlanta, Ga.

Dear Sirs: Please send at once, by express,
50 Abridged Star French Dictionaries,
6 Complete Star French Dictionaries.

Yours truly,
WOOD & ROBERTS

Model Bill. The following is a form of the discounted bill that Mason and Company would send in reply:

Feb. 16, 1922

Messrs. Wood & Roberts
Jacksonville, Fla.,

Bought of

MASON AND COMPANY
Educational Publishers
ATLANTA, GA.

50 Abridged Star French Dictionaries	$.50	$25.00
6 Complete Star French Dictionaries	4.25	25.50
		$50.50
Less $\frac{1}{6}$		8.42
		$42.08

It is customary to state such per cents as $16\frac{2}{3}\%$, $12\frac{1}{2}\%$, 20%, 25%, and $33\frac{1}{3}\%$ in the common-fraction form, as above.

PER CENTS USED IN DISCOUNTS

Write the orders and the discounted bills for the following goods:

1. 8 chairs @ \$6.00; 12 armchairs @ \$9.25; 8 rocking-chairs @ \$14.50; 24 tables @ \$42.75. Discount, $16\frac{2}{3}\%$.

The item "8 chairs @ \$6.00" means 8 chairs at \$6.00 each.

2. 420 yd. satin @ \$3.50; 340 yd. flannel @ \$1.60; 400 yd. canvas @ 90¢. Discount, $12\frac{1}{2}\%$.

3. 750 yd. lawn @ 90¢; 450 yd. percale @ \$1.10; 380 yd. satin @ \$4.50. Discount, 8%.

4. 250 yd. serge @ \$3.25; 300 yd. cheviot @ \$2.75; 175 yd. velvet @ \$3.40; 420 yd. tweed @ \$5.50; 240 yd. broadcloth @ \$7.25. Discount, 9%.

5. 75 doz. plates @ \$9.25; 60 doz. cups and saucers @ \$11.00; 40 doz. tumblers @ \$2.40; 12 doz. individual butter plates @ \$1.75; 16 doz. bowls @ \$4.30. Discount, 8%.

The item "75 doz. plates @ \$9.25" means 75 doz. plates at \$9.25 a dozen.

Find the discounts on the following amounts at the rates specified:

6. \$530, 15%.	10. \$2235, 18%.	14. \$72.64, 25%.
7. \$380, 20%.	11. \$3350, 15%.	15. \$79.36, $6\frac{1}{4}\%$.
8. \$575, 10%.	12. \$6336, $33\frac{1}{3}\%$.	16. \$94.54, $16\frac{2}{3}\%$.
9. \$785, 12%.	13. \$7328, $37\frac{1}{2}\%$.	17. \$67.20, $8\frac{1}{3}\%$.

Find the net amounts of bills with discounts as follows:

18. \$375, 20%.	23. \$925, 10%.	28. \$58.84, $16\frac{2}{3}\%$.
19. \$475, 5%.	24. \$620, 14%.	29. \$76.32, 50%.
20. \$663, 3%.	25. \$32.75, 20%.	30. \$3235, 22%.
21. \$850, $12\frac{1}{2}\%$.	26. \$4202, 14%.	31. \$44.73, $33\frac{1}{3}\%$.
22. \$886, 16%.	27. \$5672, $12\frac{1}{2}\%$.	32. \$67.28, $37\frac{1}{2}\%$.

Per Cents used in Commission. George earns some money by selling apples for Mr. Hilton. Mr. Hilton pays him $\frac{1}{4}¢$ for each apple he sells. This $\frac{1}{4}¢$ is George's *commission.*

A man sells automobiles. He is paid $150 for selling each car of a certain kind. This is his commission.

Very often commissions are stated as per cents, and the per cent is the *rate of commission.*

For example, an agent sells a house for the owner for $8000 and receives $2\frac{1}{2}\%$ commission. Find the commission.

Here we have $2\frac{1}{2}\%$ of $8000 = $200.

$80
$2\frac{1}{2}$
40
160
$200

In such cases have the pupils avoid using unnecessary figures. They should think of 1% of $8000 as $80, and multiply this by $2\frac{1}{2}$. There is a short way of multiplying by $2\frac{1}{2}\%$, or 0.025. Since $\frac{2\frac{1}{2}}{100} = \frac{1}{40}$, simply divide $8000 by 40, or $800 by 4. Such short cuts are interesting and valuable.

PER CENTS USED IN COMMISSION

All work oral

1. An agent sells a house for $8000 and receives 2% commission. Find the commission.

2. An agent sells some trolley cars to a company for $50,000 and receives 4% commission. Find the commission.

3. An agent sells to a hotel $12,000 worth of electric-light fixtures, at 2% commission. Find his commission.

Find the commissions on the following sales at the rates specified:

4. $300, 6%.
5. $200, 3%.
6. $750, 8%.
7. $4000, 5%.
8. $3000, 3%.
9. $2500, 9%.
10. $3000, 2%.
11. $5000, 4%.
12. $15,750, 3%.

USING WHAT YOU HAVE LEARNED

1. Find the commission on the sale of some furniture for \$1250, the rate being $2\frac{1}{2}\%$.

2. How much is the commission on the sale of some wagons for \$2400, the rate being $5\frac{1}{2}\%$?

3. A commission merchant sells for a factory 6850 lb. of cheese at $32\frac{1}{2}¢$ a pound, deducting 5% commission. What sum does he remit to the factory?

4. A man sells at an auction \$385 worth of property for another man and receives 10% commission. How much is the commission? How much is left for the other man?

5. A city dealer sends an agent to the fruit region to buy for him and pays him 2% commission. The agent purchases \$3500 worth of fruit in one place, \$4200 worth in another, and \$5275 worth in another. What is his total commission? What is the total cost of the fruit, including the commission?

6. A real-estate dealer sells 175 acres of land at \$80 an acre. How much is his commission at $2\frac{1}{2}\%$?

7. An agent sells our school \$85 worth of maps, and his firm pays him $7\frac{1}{2}\%$ commission. How much is this commission?

8. A farmer sends some butter and eggs to the city, and they are sold for \$84.50. He is charged 8% by his agent. What is the net amount received by the farmer?

9. How much is an auctioneer's commission for selling \$975 worth of furniture, at 6%?

Commissions are often paid for other services than buying and selling. This is seen in Ex. 10.

10. How much is an architect's commission at 5% on a house costing \$8400?

REVIEW OF PER CENTS

Write the following as common fractions, and also as per cents:

1. 0.10.	4. 0.02.	7. 0.06.	10. 0.37.	13. 0.87.
2. 0.20.	5. 0.38.	8. 0.33.	11. 0.60.	14. $0.87\frac{1}{2}$.
3. 0.30.	6. $0.37\frac{1}{2}$.	9. $0.33\frac{1}{3}$.	12. $0.66\frac{2}{3}$.	15. $0.16\frac{2}{3}$.

Write the following as per cents:

16. 100. 17. 200. 18. 275. 19. 325. 20. 178.

21. A man agreed to build a stone wall 315 ft. long. He has already built $33\frac{1}{3}\%$ of it. How many feet has he built?

22. A trolley car has a run of 84 mi. a day. At 2 P.M. it has made 65% of its run. How many miles has it still to go?

Find the values of the following:

23. 25% of 184.	27. 5% of 380.	31. 0.25×884.
24. 75% of 164.	28. 5% of 450.	32. 25% of 936.
25. 30% of 145.	29. 100% of 250.	33. 0.75×564.
26. 40% of 190.	30. 200% of 320.	34. 75% of 960.

35. How much will $37\frac{1}{2}\%$ of a cubic foot of steel weigh if 1 cu. ft. of steel weighs 490 lb.?

36. If a steel car when full carries 96,000 lb. of coal, how many tons does it carry when it carries 75% of its capacity?

37. If you solve without errors $87\frac{1}{2}\%$ of 160 problems in arithmetic, how many problems do you solve correctly?

38. If a locomotive weighing 124 tons can exert a pull equal to $22\frac{1}{2}\%$ of its weight, how many tons of pull can it exert?

39. A baseball team has played 64 games this season and has won $62\frac{1}{2}\%$ of them. How many games has it lost?

DRILL ON PER CENTS

1. A man buys a house for $8750 and pays 30% down. How much does he pay down? How much does he still owe?

Find the values of the following:

2. 30% of $9250.
3. 35% of $8640.
4. 40% of $3480.
5. 55% of $6230.
6. 85% of $5280.
7. $1\frac{1}{4}$% of $84.
8. $2\frac{1}{2}$% of $60.
9. $3\frac{3}{4}$% of $80.
10. $4\frac{1}{4}$% of $60.
11. $5\frac{5}{8}$% of $64.
12. 3% of $175.
13. 6% of $350.
14. 9% of $475.
15. 4% of $985.
16. 5% of $675.

17. A man agrees to buy a building lot for $1575 and pays $33\frac{1}{3}$% down. How much has he still to pay?

Whenever possible the pupil should use the short methods on page 170.

Find the values of the following:

18. $12\frac{1}{2}$% of $960.
19. $16\frac{2}{3}$% of $960.
20. 25% of $1680.
21. $33\frac{1}{3}$% of 888.
22. $37\frac{1}{2}$% of 888.
23. 50% of 770.
24. $62\frac{1}{2}$% of 96.
25. $66\frac{2}{3}$% of 96.
26. 75% of 480.
27. $87\frac{1}{2}$% of 480.
28. $2\frac{1}{2}$% of $480.
29. $6\frac{1}{4}$% of $480.
30. $3\frac{1}{8}$% of $675.
31. $3\frac{3}{4}$% of $800.
32. $6\frac{2}{3}$% of $720.

33. A man bought a house for $7500 and immediately sold it for 125% of what he paid for it. How much did he gain?

Find the values of the following:

34. $\frac{1}{2}$% of $1200.
35. 50% of $1200.
36. $\frac{1}{2}$ of $1200.
37. $\frac{1}{200}$ of $1200.
38. 23.8% of 69.
39. $3\frac{1}{8}$% of $7200.
40. $1\frac{2}{3}$% of $7200.
41. $83\frac{1}{3}$% of $960.
42. 60% of 0.60.
43. 5% of 1275.
44. $8\frac{1}{3}$% of 9675.
45. 15% of 8175.

REVIEW OF PER CENTS

Find the values of the following:

1. 50% of 275.
2. 30% of 945.
3. 80% of 440.
4. 3% of $628.
5. 8% of $448.
6. 9% of $868.
7. $\frac{3}{4}$ of 1236.
8. 75% of 1236.
9. 0.75×1236.

Find the net amounts of the following bills on which discounts are allowed at the rate specified in each case:

10. $750, 20%.
11. $475, 15%.
12. $880, 25%.
13. $975, 8%.
14. $1640, $12\frac{1}{2}$%.
15. $1775, 24%.
16. $1560, $16\frac{2}{3}$%.
17. $9768, $12\frac{1}{2}$%.
18. $4578, $33\frac{1}{3}$%.

19. A typewriter was marked $90 but was sold at 15% discount. What was the net price?

20. An automobile was marked $1275 but was sold at 12% discount. What was the net price?

21. A merchant bought some goods for $5062.50 less 20%. What was the net amount of the bill for the goods?

22. On which will the net amount after deducting the commission be greater, a sale of $3500 less 2% commission or a sale of $3560 less 3% commission? How much greater?

23. A clerk receives $32 a week and 3% commission on goods sold. If he sells $900 worth of goods in a certain week, what is his income for the week?

24. How much is the commission on a sale amounting to $2750, the rate being $4\frac{1}{2}$%?

25. A dealer bought a bill of goods amounting to $3475. By paying promptly he was allowed a discount of 8% on this bill. How much was the discount?

REVIEW OF PER CENTS

1. The diameter of a circle is about $31\frac{4}{5}\%$ as long as the circumference. What is the diameter of a circle whose circumference is 60 ft.?

2. A meter is about $9\frac{3}{8}\%$ longer than a yard. How many inches are there in a meter?

3. A knot, used in measuring distances at sea, is about $15\frac{1}{3}\%$ longer than a mile. How many feet are there in a knot?

Find the values of the following:

4. 40% of 280.
5. 50% of 380.
6. 60% of 450.
7. 70% of 920.
8. 5% of 320.
9. 8% of 730.
10. 9% of 925.
11. $4\frac{1}{2}\%$ of 735.
12. $\frac{1}{5}$ of 8275.
13. 20% of 295.
14. $\frac{3}{4}$ of 964.
15. 75% of 964.

16. A man makes a profit of $12\frac{1}{2}\%$ on a sale of property which cost him \$3200. How much is the profit?

Find the values of the following:

17. 50% of \$748.
18. 25% of \$864.
19. $12\frac{1}{2}\%$ of \$1296.
20. $6\frac{1}{4}\%$ of \$6752.
21. $37\frac{1}{2}\%$ of \$57.76.
22. $62\frac{1}{2}\%$ of \$99.92.
23. $87\frac{1}{2}\%$ of \$50.48.
24. $3\frac{1}{8}\%$ of \$1478.40.
25. $66\frac{2}{3}\%$ of \$972.60.
26. $83\frac{1}{3}\%$ of \$84.72.
27. 20% of \$728.45.
28. 40% of \$936.25.
29. 100% of \$750.
30. 200% of \$3.50.
31. 125% of \$300.
32. 284% of \$500.

33. Find $1\frac{1}{4}$ times 880; 1.25 times 880; 125% of 880.

34. Find $1\frac{3}{8}$ times 328; $1.37\frac{1}{2}$ times 328; $137\frac{1}{2}\%$ of 328.

Per Cents used in Interest. Ordinarily Mr. Wood pays his rent on the first day of the month. On May 1 of a certain year he has just bought a stock of goods for his store and finds that he has no money on hand. So he borrows enough money for a month or two and pays the rent.

When he borrows this money, he has to pay for the use of it. In finding how much he must pay, we need to use per cents.

Some schools prefer not to take up the topic of simple interest in this school year. In that case pages 186–191 may be omitted.

Interest. The money paid for the use of money is called *interest.* For example, if I borrow \$50 for a year and pay \$3 for the use of it, the \$3 is interest.

Principal. The sum of money on which interest is charged is called the *principal.* For example, if you borrow \$100 for a year and pay \$6 for the use of it, the \$100 is the principal.

Reckoning Interest. Interest is ordinarily reckoned as a certain per cent of the principal. For example, if the interest for 1 yr. is 6% of the principal, and the principal is \$200, the interest for 1 yr. is 6% of \$200, or \$12.

If the interest for 1 yr. is \$12, the interest for 6 mo., which is $\frac{1}{2}$ yr., is $\frac{1}{2}$ of \$12, or \$6.

Rate of Interest. The per cent of the principal paid yearly as interest is called the *rate.* That is, if the interest for 1 yr. is 6% of the principal, the rate is 6%.

Illustrative Problem. Find the interest on \$250 for 1 yr. at 6%.

Since the interest is 6% for 1 yr., we take 6% of \$250. Since 6% of \$250 is \$15, this is the interest for 1 yr.

If we wish to find the interest for 6 mo., we take $\frac{1}{2}$ of \$15, which is \$7.50.

\$250
0.06
\$15.00

FINDING INTEREST

Numbers 1 to 24, oral

Find the interest on the following sums for 1 yr.:

1. \$100 at 6%.
2. \$200 at 5%.
3. \$300 at 4%.
4. \$400 at 6%.
5. \$500 at 4%.
6. \$600 at 5%.
7. \$700 at 4%.
8. \$800 at 5%.
9. \$900 at 4%.
10. \$1000 at 5%.
11. \$2000 at 6%.
12. \$3000 at 5%.

Find the interest on the following sums for 6 mo., first finding the interest for 1 yr. and then dividing by 2:

13. \$100 at 6%.
14. \$100 at 4%.
15. \$100 at 5%.
16. \$100 at 3%.
17. \$200 at 6%.
18. \$200 at 4%.
19. \$300 at 6%.
20. \$400 at 5%.
21. \$10 at 6%.
22. \$20 at 5%.
23. \$30 at 4%.
24. \$2000 at 5%.

Find the interest on the following sums for 1 yr.:

25. \$125 at 6%.
26. \$175 at 6%.
27. \$225 at 5%.
28. \$275 at 5%.
29. \$375 at 4%.
30. \$425 at 6%.
31. \$675 at 6%.
32. \$750 at 5%.
33. \$850 at 5%.
34. \$350 at 4%.
35. \$50 at 7%.
36. \$50 at 8%.
37. \$80 at $5\frac{1}{2}$%.
38. \$80 at $4\frac{1}{2}$%.
39. \$120 at $6\frac{1}{2}$%.

Find the interest on the following sums for 6 mo.:

40. \$250 at 6%.
41. \$350 at 5%.
42. \$450 at 4%.
43. \$640 at 6%.
44. \$820 at 5%.
45. \$200 at $5\frac{1}{2}$%.
46. \$360 at $4\frac{1}{2}$%.
47. \$800 at $3\frac{1}{2}$%.
48. \$900 at $5\frac{1}{2}$%.
49. \$800 at $5\frac{1}{4}$%.
50. \$2750 at 7%.
51. \$1875 at 8%.
52. \$3400 at 5%.
53. \$5600 at 6%.
54. \$4875 at 4%.

Interest for Months. On pages 186 and 187 we found interest for years and for half of a year. However, when Mr. Wood borrowed the money as stated on page 186, he wished it for only one or two months. How do we find the interest when the number of months is not six?

For example, what is the interest on \$75 for 3 mo. at 6%?

Since the interest for 1 yr. is 6% of \$75, or \$4.50, the interest for 3 mo., which is $\frac{3}{12}$ of a year, or $\frac{1}{4}$ yr., is $\frac{1}{4}$ of \$4.50.

$$\tfrac{1}{4} \text{ of } \$4.50 = \$1.12\tfrac{1}{2}.$$

Since $\frac{1}{2}\text{¢}$ is usually counted as a cent, the result should be given as \$1.13.

\$75
0.06
4)\$4.50
\$1.12½

Convenience of Cancellation. Ordinarily it saves time and work if we use cancellation in finding interest as well as in solving other kinds of problems.

For example, what is the interest on \$1620 for 7 mo. at 4%?

It is more convenient to write $\frac{4}{100}$ instead of 4% if we wish to cancel. Since the interest for 1 yr. is $\frac{4}{100}$ of \$1620, the interest for 7 mo. is $\frac{7}{12}$ of this. We then have to find

$$\tfrac{7}{12} \text{ of } \tfrac{4}{100} \text{ of } \$1620.$$

This may be more conveniently written as a single fraction, the work then appearing as follows:

$$\frac{7 \times \overset{1}{\cancel{4}} \times \overset{540}{\cancel{\$1620}}}{\underset{3}{\cancel{12}} \times \underset{25}{\cancel{100}}} = \frac{\$3780}{100} = \$37.80.$$

We could also cancel 10 and 2 from 540 and 100, but when we are to divide by 100 there is no gain in doing so. The pupil should use his common sense as to whether to cancel or not in such a case.

INTEREST FOR MONTHS

Numbers 1 to 18, oral

1. What is the interest on \$100 for 6 mo. at 6%?

2. What is the interest on \$100 for 3 mo. at 6%?

3. What is the interest on \$200 at 4% for 1 yr.? for 3 mo.?

4. What is the interest on \$200 at 6% for 1 yr.? for 4 mo.?

5. If you had \$300 in a bank that paid 4% a year, how much interest would you receive for 6 mo.?

6. How much are the interest and principal together on \$100 at the end of 6 mo., the rate of interest being 6%?

Find the interest on the following sums for 3 mo.:

7. \$400 at 6%.	11. \$200 at 5%.	15. \$1000 at 6%.
8. \$600 at 4%.	12. \$300 at 4%.	16. \$4000 at 5%.
9. \$800 at 5%.	13. \$500 at 4%.	17. \$2000 at 5%.
10. \$800 at 6%.	14. \$800 at 3%.	18. \$3000 at 4%.

Find the interest on the following sums at 6%:

19. \$175, 2 mo.	24. \$680, 7 mo.	29. \$1200, 5 mo.
20. \$250, 2 mo.	25. \$950, 8 mo.	30. \$1875, 7 mo.
21. \$380, 3 mo.	26. \$780, 9 mo.	31. \$2630, 8 mo.
22. \$420, 4 mo.	27. \$600, 10 mo.	32. \$4250, 9 mo.
23. \$570, 5 mo.	28. \$840, 11 mo.	33. \$7500, 10 mo.

34. Find the interest on \$1750 for 2 mo. at 5%; at 7%.

35. If the interest on a certain sum is \$187.68 for 1 yr., how much is it for 3 mo.? for 5 mo.? for 7 mo.?

36. How much interest is due on \$375 for 8 mo. at $5\frac{1}{2}$%?

37. Find the interest on \$6750 at 8% for 1 yr.; for 3 mo.

Interest for Years and Months. Usually the interest is paid at least as often as every 6 mo., but this is not always the case.

For example, Mr. Wood may borrow $250 and not be able to pay the money back, or even the interest, for more than a year, say for 1 yr. 5 mo. At 6%, how much interest would he then have to pay?

Since the interest for 1 yr. is 6% of $250, or $15, it follows that the interest for 1 yr. 5 mo., or $1\frac{5}{12}$ yr., is $1\frac{5}{12} \times \$15$, or $21.25.

The pupil will gain in time by using cancellation. Taking $1\frac{5}{12}$ as $\frac{17}{12}$, we have to find

$$\tfrac{17}{12} \text{ of } \tfrac{6}{100} \text{ of } \$250.$$

$$\begin{array}{r} \$250 \\ 0.06 \\ \hline \$15.00 \\ 1\frac{5}{12} \\ \hline \$21.25 \end{array}$$

The work will then appear as follows:

$$\frac{17 \times \cancel{6} \times \overset{125}{\cancel{\$250}}}{\underset{2}{\cancel{12}} \times \cancel{100}} = \$21\tfrac{1}{4} = \$21.25.$$

Since this is merely a brief introduction to interest, the finding of interest for days is postponed until the pupil takes up Book Three.

INTEREST FOR YEARS AND MONTHS

Find the interest on the following:

1. $450, at 5%, for 6 mo.
2. $400, at 6%, for 8 mo.
3. $4200, at 6%, for $1\frac{1}{4}$ yr.
4. $825, at 5%, for $2\frac{1}{2}$ yr.
5. $450, at 6%, for $1\frac{3}{4}$ yr.
6. $375, at 5%, for $2\frac{1}{3}$ yr.
7. $85.50, at 6%, for 1 yr. 5 mo.
8. $3250, at $4\frac{1}{2}$%, for 2 yr. 9 mo.
9. $2400, at $5\frac{1}{2}$%, for 1 yr. 6 mo.
10. $3200, at $6\frac{1}{2}$%, for 1 yr. 8 mo.
11. $4800, at 7%, for 2 yr. 6 mo.
12. $6500, at 8%, for 2 yr. 6 mo.

REVIEW OF INTEREST

Find the interest on $450, at 5%, for:

1. 1 yr. 6 mo.
2. 2 yr. 3 mo.
3. 1 yr. 8 mo.
4. 2 yr. 4 mo.
5. 3 yr. 1 mo.
6. 3 yr. 2 mo.

Find the interest on $650, for 1 yr. 3 mo., at:

7. 5%.
8. 4%.
9. 6%.
10. $5\frac{1}{2}$%.
11. $4\frac{1}{2}$%.

Find the interest for 1 yr. 5 mo., at 6%, on:

12. $255.
13. $475.
14. $580.
15. $775.
16. $1200.

17. A man borrows $600, agreeing to pay 5% interest. He pays the principal and interest in 1 yr. 2 mo. How much money does he pay?

18. A man buys 26 head of cattle at $125 a head, and agrees to pay for them in 7 mo., with interest at 6%. How much must he pay in all, principal and interest?

19. Mr. Jacobs agrees to pay Mr. Simpson $240 for some work, payment to be made at the end of 5 mo. with interest at 6%. How much is due at the end of the 5 mo.?

20. Woodhull & Co. are dealers in jewelry. They need $3500 with which to buy holiday goods. They borrow it on Dec. 1 and agree to pay it back on Feb. 1 with interest at 6%. How much do they owe on Feb. 1?

21. Mr. Baker buys $3200 worth of goods from a manufacturer, agreeing to pay for the goods in 7 mo., with interest at 6%. How much does he owe at the end of the 7 mo.?

22. On May 1, Mr. Stevens buys a second-hand automobile from Mr. Robinson, agreeing to pay $525 for it on Aug. 1, with interest at 6%. How much interest is due on Aug. 1?

III. LITTLE EXAMINATIONS

i.

1. Reduce $37\frac{1}{2}$ to halves.
2. $\frac{2}{3} + \frac{3}{4} + \frac{1}{2}$.
3. $175\frac{1}{2} - 156\frac{7}{8}$.
4. $27 \times \$1.66\frac{2}{3}$.
5. $\frac{3}{4}$ of \$164.48.
6. $3\frac{3}{4} \times 164$.
7. $8\frac{3}{4} \div 3\frac{1}{3}$.
8. $6 \times \$1.12\frac{1}{2}$.

ii.

1. 0.28×47.6.
2. $5 - 0.378$.
3. $17.28 \div 1.44$.
4. $16\frac{2}{3}\%$ of \$8.46.
5. 103% of $84\frac{1}{2}$ ft.
6. 5% of \$7876.
7. 1.8% of \$675.
8. 412% of \$990.

iii.

1. $7 + 0.968$.
2. $7 - 0.968$.
3. $7 \div 0.05$.
4. 0.8×0.08.
5. $16\frac{2}{3}\%$ of \$78.60.
6. $3\frac{3}{4}\%$ of 4 ft. 6 in.
7. $2\frac{7}{8}\%$ of 8 yd. 6 in.
8. 8% of \$85.50.
9. 200% of \$75.
10. 275% of \$800.

iv.

1. $12\frac{1}{2}\%$ of \$96.40.
2. $37\frac{1}{2}\%$ of \$97.60.
3. $62\frac{1}{2}\%$ of \$41.60.
4. $87\frac{1}{2}\%$ of \$43.20.
5. $66\frac{2}{3}\%$ of \$54.90.
6. $2\frac{1}{2}\%$ of \$650.
7. $3\frac{1}{3}\%$ of \$4.20.
8. 9% of \$84.40.
9. $2.3\frac{1}{3}\%$ of \$75.
10. 250% of 95 ft.

v.

1. How much is \$475 less 8% of itself?
2. How much is \$850 increased by 16% of itself?
3. Make out a bill of six items and receipt it.
4. How much is the commission at 9% on a sale of goods amounting to \$4780? at $9\frac{1}{2}\%$? at $10\frac{1}{2}\%$?
5. Find 325% of \$17,600; of \$28,720.

IV. REVIEW AND DRILL

MINIMUM ESSENTIALS

1. Write 1924 and 1930 in Roman numerals, and LXIX and LXXXVIII in common numerals.

Find the sum, difference, product, and quotient of each of the following pairs of numbers, dividing the greater by the less:

2. 6531, 2177.	5. 30,195, 6039.	8. 31,104, 3456.
3. 18,846, 3141.	6. 56,080, 7010.	9. 24,258, 4043.
4. 29,547, 4221.	7. 28,404, 7101.	10. 69,993, 7777.

Find the sum, difference, product, and both quotients of each of the following pairs of fractions:

By "both quotients" is meant that we should divide $\frac{3}{8}$ by $\frac{3}{4}$ and $\frac{3}{4}$ by $\frac{3}{8}$.

11. $\frac{2}{3}$, $\frac{3}{4}$.	13. $\frac{2}{3}$, $\frac{5}{8}$.	15. $\frac{3}{4}$, $\frac{1}{8}$.	17. $\frac{1}{2}$, $\frac{3}{16}$.
12. $\frac{1}{3}$, $\frac{3}{8}$.	14. $\frac{2}{3}$, $\frac{1}{8}$.	16. $\frac{3}{4}$, $\frac{5}{8}$.	18. $\frac{1}{3}$, $\frac{5}{16}$.

Find the values of the following:

19. 7% of 48.	24. $2\frac{1}{2}$% of 38.	29. 50% of 9 ft. 8 in.
20. 9% of 75.	25. $3\frac{1}{3}$% of 69.	30. $33\frac{1}{3}$% of 10 ft. 9 in.
21. 6% of 97.	26. $1\frac{1}{4}$% of 64.	31. 25% of 8 ft. 6 in.
22. 9% of 7.4.	27. $6\frac{1}{4}$% of 7.2.	32. 75% of 3 lb. 4 oz.
23. 8% of 6.8.	28. $7\frac{1}{2}$% of 8.4.	33. $62\frac{1}{2}$% of 3 bu. 8 qt.

Multiply or divide as indicated, carrying the quotients to four figures only:

34. 587 × \$265.25.	37. 488.3 ÷ 27.4.	40. 2.93 × 52.3.
35. 417 × \$865.75.	38. 69.58 ÷ 3.54.	41. 77.6 × 3.19.
36. 409 × \$628.57.	39. 86.49 ÷ 9.47.	42. 87.4 × 9.86.

MINIMUM ESSENTIALS

DRILL ON FRACTIONS

Add as stated:

1. $3\frac{1}{2} + 2\frac{1}{8}$.
2. $4\frac{1}{4} + 3\frac{5}{8}$.
3. $2\frac{3}{4} + 5\frac{3}{8}$.
4. $4\frac{1}{2} + 2\frac{1}{4}$.
5. $5\frac{1}{2} + 3\frac{3}{4}$.
6. $2\frac{1}{3} + 3\frac{1}{2}$.
7. $2\frac{2}{3} + 4\frac{1}{2}$.
8. $5\frac{1}{3} + 2\frac{1}{4}$.
9. $5\frac{2}{3} + 3\frac{1}{4}$.
10. $7\frac{2}{3} + 5\frac{3}{4}$.
11. $2\frac{1}{5} + 3\frac{1}{2}$.
12. $3\frac{2}{5} + 4\frac{1}{2}$.
13. $5\frac{3}{5} + 3\frac{1}{4}$.
14. $7\frac{5}{6} + 3\frac{1}{2}$.
15. $8\frac{4}{5} + 2\frac{2}{3}$.

Subtract as stated:

16. $15 - 2\frac{1}{2}$.
17. $17 - 3\frac{7}{8}$.
18. $15\frac{1}{2} - 3\frac{3}{4}$.
19. $17\frac{1}{2} - 4\frac{7}{8}$.
20. $10\frac{1}{2} - 3\frac{2}{3}$.
21. $8\frac{3}{4} - 4\frac{5}{8}$.
22. $8\frac{1}{4} - 2\frac{7}{8}$.
23. $6\frac{3}{4} - 4\frac{1}{2}$.
24. $5\frac{1}{2} - 2\frac{3}{4}$.
25. $6\frac{1}{2} - 2\frac{1}{3}$.
26. $8\frac{1}{3} - 5\frac{3}{8}$.
27. $9\frac{2}{3} - 3\frac{7}{8}$.
28. $7\frac{1}{6} - 3\frac{3}{4}$.
29. $8\frac{5}{6} - 2\frac{7}{8}$.
30. $3\frac{1}{8} - 1\frac{1}{12}$.

Multiply as stated:

31. $3\frac{1}{2} \times 12$.
32. $4\frac{1}{4} \times 13$.
33. $5\frac{3}{4} \times 15\frac{1}{4}$.
34. $9\frac{1}{8} \times 12\frac{1}{2}$.
35. $6\frac{5}{8} \times 14\frac{1}{2}$.
36. $7 \times 9\frac{1}{2}$.
37. $8 \times 9\frac{1}{2}$.
38. $4\frac{1}{4} \times 8\frac{1}{3}$.
39. $5\frac{3}{4} \times 7\frac{2}{3}$.
40. $6\frac{1}{6} \times 8\frac{2}{3}$.
41. $7\frac{1}{5} \times 9\frac{1}{3}$.
42. $6\frac{1}{5} \times 8\frac{3}{5}$.
43. $5\frac{3}{5} \times 7\frac{2}{3}$.
44. $7\frac{4}{5} \times 9\frac{3}{4}$.
45. $6\frac{3}{8} \times 8\frac{7}{8}$.

Divide as stated:

46. $3\frac{1}{3} \div \frac{1}{2}$.
47. $6\frac{2}{3} \div \frac{1}{2}$.
48. $6\frac{2}{3} \div 3\frac{1}{3}$.
49. $3\frac{1}{3} \div 6\frac{2}{3}$.
50. $8\frac{1}{2} \div 2\frac{5}{6}$.
51. $8\frac{1}{4} \div 2$.
52. $8\frac{1}{4} \div \frac{2}{3}$.
53. $8\frac{1}{4} \div 1\frac{1}{2}$.
54. $7\frac{1}{2} \div 2\frac{1}{2}$.
55. $2\frac{1}{2} \div 7\frac{1}{2}$.
56. $16\frac{2}{3} \div 8\frac{1}{3}$.
57. $8\frac{1}{3} \div 16\frac{2}{3}$.
58. $9\frac{1}{3} \div 3\frac{1}{9}$.
59. $7\frac{1}{8} \div 2\frac{1}{3}$.
60. $2\frac{1}{3} \div 7\frac{1}{8}$.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If you have a given per cent, how do you express it as a common fraction?

2. If you are to find a certain fractional part of a number, how do you express this as a per cent of the number?

3. If you have a certain rate per cent expressed with the per cent sign, how do you express it as a decimal?

4. If you have a decimal fraction, how do you express the same number with a per cent sign?

5. If you know what a given fraction of a number is, how do you find the number?

6. If you know the amount paid for a certain property and the rate of commission, how do you find the commission?

7. If you know the amount of a bill and the rate of discount, how do you find the net amount of the bill?

8. If you know the number of decimal places in each of two numbers, how do you find the number of decimal places in their product?

9. Given the number of decimal places in the product of two numbers, and the number of decimal places in one of them, how do you find the number of decimal places in the other?

10. A man's salary two years ago was increased a certain per cent. Last year it was decreased the same per cent. Is his salary at present greater than or less than it was two years ago?

11. A merchant marks his goods a certain per cent above cost and sells them at the same per cent below the marked price. Does he sell them for more than or less than cost?

PROBLEMS FOR COMPLETION

1. The three sides of a triangle are respectively 3.7 in., 4.25 in., and 5.6 in. Complete the problem and solve it.

As stated on page 70, the problem may be completed in more than one way. Any reasonable problem, if correctly solved, should be accepted.

2. An automobile goes 28.1 mi. the first hour on a trip and 26.8 mi. the second hour. Complete the problem and solve it.

Complete and solve the following problems:

3. A cubic foot of water weighs 62.5 lb., and ice is 92% as heavy as water. A block of ice is 4 ft. long, 2 ft. wide, and 1 ft. thick.

4. Clay is 1.2 times as heavy as water, and 1 cu. ft. of water weighs $62\frac{1}{2}$ lb.

5. Tin is 7.29 times as heavy as water, and 1 cu. in. of water weighs 0.58 oz.

6. A cubic foot of cast iron weighs 7210 oz., and a cubic foot of marble weighs 2700 oz. One is a certain number of times heavier than the other.

7. A circular race track is 190.98 ft. across, and the circumference is 3.1416 times the distance across.

8. A man used 72,600 cu. ft. of gas in 4 mo. when gas was selling at 90¢ per 1000 cu. ft.

9. Kate has $45 in the bank. Her father gives her 20% more to put in the bank.

10. Richard buys $18.40 worth of books and pays 75% of this amount, his mother paying the rest.

11. A postal airplane recently went from Washington to New York in 1 hr. 34 min., flying at the rate of 138 mi. per hour.

Industrial Problems

THRIFT IN THE HOME

1. Mrs. Cobb finds that she can save at least 70¢ a week by going to the market and buying in person. By doing this, how much a year, at least, will she save?

2. Fred helps to cut down the high cost of living. He says he will raise vegetables. Allowing $1.80 for plowing, $3.25 for fertilizer, $1.50 for seed, and 85¢ a week for 22 wk. for wages, which his mother pays him, what is the entire cost of the vegetables for the season?

3. Since Fred's mother no longer buys over the telephone, she decides to cut off all telephone expenses. These expenses amount to $2.40 a month. How much does she save in a year on the rent of the telephone?

4. By buying at a bargain sale a $30 suit of clothes at 12% off, three $2.50 shirts at 10% off, a pair of $6.25 shoes at 20% off, a dozen 35-cent handkerchiefs at $16\frac{2}{3}$% off, and a hat, a pair of gloves, and some ties amounting to $9 at 20% off, how much does Fred save in all?

5. Mrs. Cobb can buy rolled oats, 20 oz. to the package, for 13¢ a package, or she can buy rolled oats of the same quality in bulk at 9¢ a pound. If she wishes to buy 20 lb., how much does she save if she buys it in bulk?

6. Mrs. Cobb can buy salad oil at 68¢ a quart or at 38¢ a pint. If she wishes to buy 1 gal., how much does she save if she buys it by the quart?

7. Mrs. Cobb can buy granulated sugar in packages at $15\frac{1}{2}$¢ a pound or in bulk at 15¢ a pound. If she needs 10 lb. in a month, how much does she save if she buys it in bulk?

PER CENTS USED IN AGRICULTURE

1. Corn meal, used in feeding cattle, is 15% water. How much water is there by weight in 75 lb. of corn meal?

2. There is only 0.9% of digestible fat in alfalfa hay. How many pounds of digestible fat are there in a ton of alfalfa hay?

3. For fattening sheep a farmer used a ration of 2 lb. corn, 11.3% water; $\frac{1}{2}$ lb. cottonseed meal, 7% water; and $1\frac{1}{2}$ lb. of hay, 6% water. Find the weight of the water in this ration.

4. For fattening hogs the Florida authorities recommend a ration of 10 lb. corn, 11.3% water; 10 lb. sorghum seed, 12.8% water; and 7.75 lb. cowpeas, 14.6% water. Find the weight of the water in this ration.

5. It is estimated that a wagon depreciates in value 8% a year through ordinary use. If a farmer pays $110 for a wagon and gives it ordinary use for a year, how much is it fairly worth at the end of the year?

6. If corn fodder loses 60% of its weight in curing, how much will 32 T. of green fodder weigh after it is cured?

7. If the cream from a model dairy contains 25.9% butter fat, what is the weight of butter fat in 1240 lb. of cream?

8. A bushel of corn requires for its production 60 lb. of stalk. One of the most important foods that plants take from the soil is nitrogen, and a ton of corn stalks takes 10 lb. of nitrogen. How many pounds of nitrogen are taken from the soil to produce 100 bu. of corn?

Teachers will see that this is a very important question. Vegetables require food just as animals do. They take it from the soil, and unless it is replaced by fertilizers the soil will not have the food to give to the plants. In agricultural communities such problems are particularly valuable. In cities their use is optional with the teacher.

SCHOOL-AND-HOME GARDENING

1. Some boys in a high school rented, at $10 an acre, a piece of land 40 rd. long by 20 rd. wide, the length running north and south. How much rent did they pay?

2. Starting from the northwest corner, the boys in Ex. 1 first laid out a strip 4 rd. wide the entire length of the field. This they planted to early potatoes. How many acres of early potatoes did they plant?

3. Adjoining the strip in Ex. 2 they laid out a strip 33 yd. wide, the length of the field, and allotted 55 yd. of it to early sweet corn. What part of an acre did they allow for early corn?

4. They allotted the rest of the strip in Ex. 3 to a later variety of corn. How many acres did they allot to this later variety?

5. Make a plan of the field to the scale $\frac{1}{8}$ in. to 1 rd., putting in all the measurements and the areas of the various pieces, showing how the land has thus far been used.

6. They planted the strip of early corn in Ex. 3 in rows 3 ft. 2 in. apart, the rows running north and south. The first and last rows were each 2 ft. from the edges of the strip. They planted the corn in hills 2 ft. 9 in. apart, leaving a space equal to the distance between the hills at each end of the strip. How many hills of early corn did they plant?

7. The strip of late corn in Ex. 4 was planted in exactly the same way as the early corn in Ex. 6. The hills in the strips of early and late corn produced an average of 3 marketable ears per hill, and the boys sold the corn to local hotels at 20¢ per dozen. How much did they receive for their corn?

It may be well in Exs. 6 and 7 to have drawn on the blackboard a large-scale plan in which the rows and the hills in one row are indicated.

8. The boys planted to late potatoes a strip 115½ ft. wide the entire length of the field, and planted to sugar beets all the remainder of the piece of land. How many acres did they plant to each of these crops? Complete the plan in Ex. 5 and check the areas of the different strips by adding them together and comparing the result with the number of acres which you have already found in Ex. 1.

9. It takes 7 bu. of seed potatoes to plant an acre. If seed potatoes cost $3.20 per 100 lb., and 1 bu. weighs 60 lb., how much did the boys pay for the seed potatoes needed to plant the strip in Ex. 8?

10. The strip of late potatoes in Ex. 8 yielded 3 pk. to each square rod of land, and the boys sold the potatoes for $1.50 per 100 lb. Taking the weight of 1 bu. as 60 lb., how much did they receive for their crop of potatoes?

11. Land such as was used for the sugar beets in Ex. 8 yields 18 tons per acre, and the crop is worth $10 per ton. The expenses per acre of raising sugar beets are as follows: rent, $10; seed, 12 lb. at 70¢ per pound; labor, $21. Find the profit the boys made on their crop of beets.

12. The farmer whose land adjoined the field the boys rented was careless. He planted 82½ acres of corn without giving the seed an ear-to-ear test, the result being that four ears out of every bushel of seed corn failed to grow. If we estimate 100 ears of seed corn to a bushel, and the yield at 50 bu. to the acre, what was the farmer's loss at $1.25 per bushel, from failing to test the seed?

13. In Ex. 12, if the total work of testing the seed corn would have required 7 da., what wages per day would the increased yield of corn have paid the farmer?

CHAPTER IV

I. PRACTICAL MEASUREMENTS

Common Plane Figures. We are already familiar with many of the plane figures that we commonly measure. We shall now review these and shall learn how to measure others.

Triangle

Quadrilateral

Rectangle

Square

Triangle. A plane figure which is bounded by three straight lines is called a *triangle.*

Quadrilateral. A plane figure which is bounded by four straight lines is called a *quadrilateral.*

Rectangle. A quadrilateral all the angles of which are right angles is called a *rectangle.*

If necessary the teacher should explain the meaning of *right angle,* referring to the illustration and definitions on page 114.

Square. A rectangle all the sides of which are equal is called a *square.*

Polygon. A plane figure which is bounded by straight lines is called a *polygon.*

The teacher should draw on the blackboard polygons of various shapes.

As a preliminary to this chapter the tables on pages 106–114 should be reviewed; and in order that this review shall be thorough, much oral work is provided on pages 202 and 203.

REVIEW OF THE TABLES

All work oral

1. Express as miles: 320 rd., 640 rd., 1280 rd.
2. Express as pounds: 32 oz., 48 oz., 64 oz.
3. Express as square rods: 1 A., 2 A., 3 A.
4. Express 33 ft. as rods; as yards.
5. Express as cubic yards: 27 cu. ft., 54 cu. ft.
6. Express 9 ft. as inches; as yards.
7. Express 10 da. as hours; as weeks.
8. Express 17 qt. as pints; as gallons.
9. Express 9 pk. as quarts; as bushels.
10. How many cubic inches in 1 cu. ft.? in 10 cu. ft.?
11. How many rods in 1 mi.? in 2 mi.? in 10 mi.?
12. How many yards in 1 rd.? in 10 rd.? in 2 rd.?
13. How many cubic feet in 1 cd. of wood? in 2 cd.?
14. Express 300 min. as hours; as seconds.
15. How many quarts in a barrel of $31\frac{1}{2}$ gal.?
16. How many quarts in a 60-gallon hogshead?
17. Express as a fraction of an acre: 80 sq. rd., 40 sq. rd.
18. How many right angles in 270°? in 180°? in 360°?
19. How many days from May 23 to June 25?
20. How many days from June 11 to July 17?
21. What part of a 16-carat ring is pure gold?
22. Express 11 yd. as rods; as feet; as inches.
23. Express 30 pt. as quarts; as gills; as gallons.
24. How many feet in 2 rd.? in 2 rd. 10 ft.? in 2 rd. 8 ft.?
25. How many feet in 10 rd.? in 5 rd.? in 2 rd.? in $\frac{1}{2}$ rd.?

REVIEW OF THE TABLES

Numbers 1 to 6, oral

1. How many acres in a tract of land 10 mi. square?

2. How many days from Jan. 19 to Feb. 19?

3. How many square feet in 10 sq. yd.? in 5 sq. yd.?

4. Using 6 qt. a day, how long will 3 bu. of oats last a horse?

5. What is the perimeter of a square that is $4\frac{1}{2}$ ft. on a side?

6. What is the side of a square whose perimeter is 14 ft. 4 in.?

7. Since a gallon contains 231 cu. in., how many gallons in 693 cu. in.? in 1617 cu. in.? in 2541 cu. in.?

8. How many square feet in a surface having an area of 720 sq. in.? in a surface having an area of 5040 sq. in.?

9. Express a long ton (2240 lb.) as a short ton and hundredths of a short ton.

10. What is the perimeter of a triangle that is 3 ft. 8 in. on each side?

11. If school closes June 21, how many days from noon of the day that school closes to noon of the Fourth of July?

12. How many quarts will a 10-bushel bin hold? a 15-bushel bin? How many pecks will each bin hold?

13. A man buys a building lot that has a frontage of 2 rd. 8 ft. How many square feet in his sidewalk if it is $5\frac{1}{2}$ ft. wide?

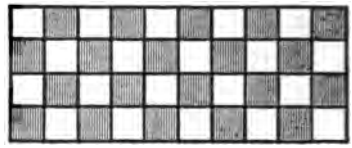
14. A cellar 16 ft. by 22 ft. is to be excavated to a depth of 6 ft. How many loads (cubic yards) of earth must be removed?

15. If a man buys 13 T. 1800 lb. of coal, how many pounds does he buy? Express this weight in tons and tenths of a ton.

16. A farmer has 3 A. of celery, and the yield is 1550 doz. heads to the acre. How many heads of celery did he raise?

Area of a Rectangle. A school building has an entrance hall 10 ft. long and 4 ft. wide. The floor is made of marble squares 1 ft. on a side. What is the number of squares?

There are 10 squares in each row, and there are 4 rows. Hence there are 4×10 squares.



In the same way we see that the area of the floor is

$$4 \times 10 \text{ sq. ft.} = 40 \text{ sq. ft.}$$

The length, 10 ft., and the width, 4 ft., are called the *dimensions* of the floor.

Base and Height. The line on which a figure appears to stand is called the *base*. The distance from the highest point above the base to the base is called the *height* or *altitude* of the figure.

Finding the Area of a Rectangle. From the above example and definitions and from page 107 we see that

The area of a rectangle is equal to the product of the base and height.

Such a rule refers to abstract numbers. We do not multiply 10 ft. by 4 ft., but we say that $4 \times 10 = 40$, the *number* of square feet.

AREA OF A RECTANGLE

Find the areas of the following rectangles, using short methods:

1. $37\frac{1}{2}$ ft. by 40 ft.
2. $33\frac{1}{3}$ yd. by 45 yd.
3. 75 ft. by 88 ft.
4. $66\frac{2}{3}$ yd. by 60 yd.
5. 125 ft. by 240 ft.
6. 250 ft. by 424 ft.
7. 625 ft. by 640 ft.
8. $87\frac{1}{2}$ in. by 88 in.

The teacher should give plenty of practical work in finding the areas of floors, platforms, desks, the school grounds, the sidewalk, and the like.

SCHOOL-AND-HOME PROJECTS

1. In a certain school district the pupils take a school-and-home project; that is, they do a piece of work, keep track of expenses, and report the profit. Sometimes this work is gardening. The standard size of a garden plot for this purpose is 8 rd. long and 2 rd. wide. What part of an acre is this?

2. Sometimes a pupil cultivates a piece of land 10 rd. long and 4 rd. wide. What part of an acre is this?

3. Frank cultivates a garden 5 rd. long and $2\frac{1}{2}$ rd. wide. He puts a fence around it, using four wires nailed to posts. If a spool of 330 yd. of wire costs \$4, find the cost of the wire for the fence.

4. Richard Heath rented a piece of land from his father. He measured it and found it 39 ft. wide and 66 ft. long. He said that it contained a little over $5\frac{1}{2}$ sq. rd. Was he right? How much over $5\frac{1}{2}$ sq. rd. was there in the piece of land?

5. On the piece of land described in Ex. 4 Richard raised 29 pk. of potatoes. Find to the nearest thousandth of a bushel the amount of potatoes he raised to the square yard.

6. At the number of bushels to the square yard found in Ex. 5, how many bushels should Richard raise to an acre?

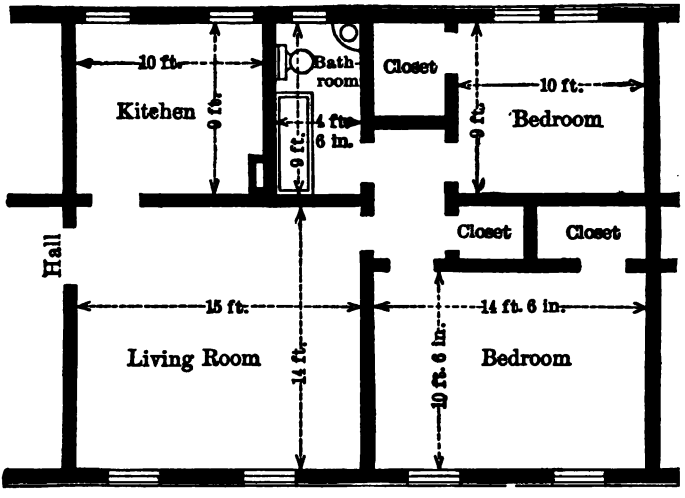
7. Richard sold the potatoes mentioned in Ex. 5 to his father at \$1.30 a bushel. How much money did he receive?

In many parts of the country the schools encourage school-and-home projects. These include school-and-community gardens, canning, poultry-raising, sewing, business, and cooking projects. Such projects encourage thrift, dignify labor, instill a love of nature, and furnish excellent practical applications of arithmetic. A reasonable number of projects are suggested in this book, but the complete carrying out of such projects often results in a waste of time and a loss of interest. Their use must depend on the needs of the school.

DRAWING TO SCALE

1. The plan of a certain apartment is here shown. The plan is drawn to the scale of 1 in. to 10 ft.

As the pupils learned on page 108, to draw to the scale of 1 in. to 10 ft., we represent 10 ft. by 1 in. To draw to the scale $\frac{1}{12}$, we represent 12 in. by 1 in. We frequently write 2′ 7″ for 2 ft. 7 in.



2. What is the area of the floor of the living room?

3. Measure the plan and find the size of the inner hall into which open both bedrooms and the living room.

4. What is the width of the windows in the living room and in the kitchen?

5. What is the area of the floor of the kitchen and living room together?

6. What is the area of the floor of each bedroom?

Teachers should show that this is related to map drawing.

SCHOOL-AND-HOME PROJECTS

1. Mabel Wagner was a pupil in a school where the pupils took as their projects in arithmetic the growing of garden produce. She rented a plot of land $16\frac{1}{2}$ ft. wide and 66 ft. long. Draw a map to the scale of 1 in. to 1 rd.

2. Ralph Baker rented a plot of land $82\frac{1}{2}$ ft. long and 33 ft. wide. Draw a map to the scale of 2 in. to 1 rd. and find what part of an acre Ralph rented.

3. Mary Worden rented a piece of land on which she raised garden vegetables. She had $\frac{1}{10}$ of an acre, and the strip was 2 rd. wide. Draw a map to the scale of $\frac{1}{2}$ in. to 1 rd.

4. Fred and Walter took as their project the laying of a gravel walk from the schoolhouse to the road. The walk was $8\frac{1}{4}$ ft. wide and 7 rd. long. Draw a plan of the walk to the scale of 1 in. to 1 rd.

5. Kate and Mary took as their project the making of a garden 2 rd. long and $1\frac{1}{2}$ rd. wide. Draw a plan of the garden to the scale of $\frac{1}{2}$ in. to 1 rd.

6. The boys of our school wanted some land for a baseball field, and they asked a farmer whose land adjoined the school grounds if he would rent them enough for this purpose. He asked how much they needed, and one boy thought they should have a piece having ten times the area of the diamond, which is a square 90 ft. on a side. The farmer said that he could rent them a piece 9 rd. wide and 10 rd. long. Was that as large as they wished? Another boy suggested that they draw a plan of the piece of land to the scale of $\frac{1}{2}$ in. to 1 rd., and on the plan show how the baseball diamond could best be placed. Draw this plan and decide whether you think the field the farmer would rent to the boys was large enough.

DRAWING TO SCALE

1. Draw a plan of a book cover that the boys are making. The cover is to be 6 in. by 9 in. Use the scale $\frac{1}{2}$.

2. A school corridor is 8 ft. by 80 ft. Draw a floor plan of this corridor, using the scale of $\frac{1}{16}$ in. to 1 ft.

3. A playground is 64 ft. by 96 ft. Draw a plan of the playground, using the scale of $\frac{1}{16}$ in to 1 ft.

4. A schoolroom is 30 ft. long and 25 ft. wide. Draw a plan of the floor, using the scale $\frac{1}{120}$.

5. Draw a plan of a floor 24 ft. long and 16 ft. wide, using the scale of $\frac{1}{8}$ in. to 1 ft.

6. Draw a plan of a school garden 64 ft. long by 40 ft. wide to the scale of $\frac{1}{8}$ in. to 1 ft.

7. Draw a plan of a rug 16 in. by 28 in. to the scale $\frac{1}{4}$.

8. A plan for a playhouse is drawn to the scale of 1 in. to $2\frac{1}{2}$ ft. The plan is $6\frac{2}{5}$ in. long and 4 in. wide, and the playhouse is rectangular. Find the area of the playhouse floor.

9. The class has considered the question of a model kitchen in an apartment house. The pupils drew a plan to the scale $\frac{1}{60}$, representing the kitchen as 3 in. long and 2 in. wide. What is the floor space of the proposed kitchen?

10. The class has made a drawing of a rectangle. The drawing has an area of 6 sq. in. If a scale twice as large were used, what would be the area of the drawing? Draw two plans of the rectangle, so as to show that the answer is correct.

11. Measure the top of your desk to the nearest inch. Make a drawing of the top, as seen from above, using the scale $\frac{1}{8}$.

12. Make two drawings of the blackboard, one to the scale $\frac{1}{120}$ and the other to the scale $\frac{1}{64}$.

PROBLEMS ABOUT THE SCHOOL

1. A schoolroom is 36 ft. long and 24 ft. wide. Draw a plan of the floor to the scale of 1 in. to 6 ft.

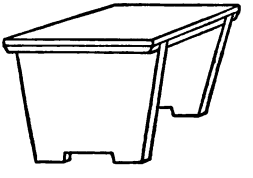
2. Measure the cover of your geography and draw a plan to the scale $\frac{1}{4}$.

3. On one side of a schoolroom is a blackboard 3 ft. high. The chalk rack is 3 ft. from the floor. The room is 30 ft. long and 15 ft. high. Draw a plan of the side of the room to the scale of 1 in. to 1 yd.

4. A door in a schoolhouse is 3 ft. 6 in. wide and 7 ft. 3 in. high. Draw two plans of the door, one to the scale of 1 in. to 1 ft. and the other to half this scale.

5. A school building is 120 ft. long and 60 ft. wide. Draw a plan to the scale of 1 in. to 40 ft. A street 40 ft. wide runs in front of the building and 60 ft. from it. Show on the plan the position of the street.

6. This is a drawing of a doll's table made by a class. The top of the table is to be 10 in. by 12 in. The end supports are to be 10 in. at the top, 7 in. at the bottom, and 8 in. high. Draw plans for the top and ends, using the scale $\frac{1}{4}$. The end supports are cut in 1 in. at the distance of 2 in. from each bottom corner.



7. The boys in the school play baseball on a field that is 480 ft. long and 320 ft. wide. Draw a plan of the field to the scale of 1 in. to 80 ft.

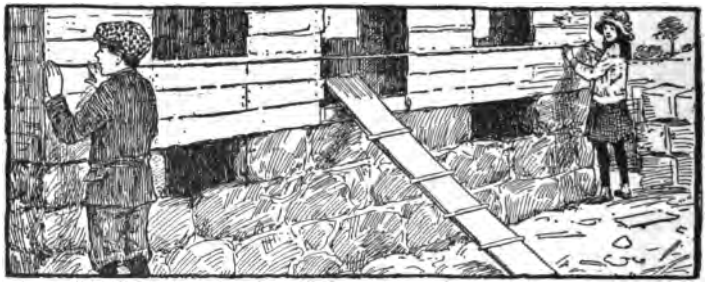
8. The inside dimensions of a small loom made by a class in manual training are 12 in. by 15 in. The frame of the loom is $\frac{3}{4}$ in. wide. Find the outside dimensions and draw a plan of the loom to the scale $\frac{1}{2}$.

OUTDOOR MEASUREMENTS

1. A baseball diamond, or infield, is a square 90 ft. on a side. Draw a plan to the scale of 1 in. to 20 ft.

2. On the infield the pitcher's plate is 60.5 ft. from the point where two sides of the diamond meet at the home base. Draw another plan of the infield, using the scale of $\frac{1}{16}$ in. to 1 ft., and mark the position of the pitcher's plate.

3. The first, second, and third bases are placed in squares 15 in. on a side. Draw one of these squares to the scale $\frac{1}{5}$.



4. George and Kate measure a house that is being built near the school. They find it to be 20 ft. by 30 ft. Draw a ground plan of the house to the scale of 1 in. to 5 ft.

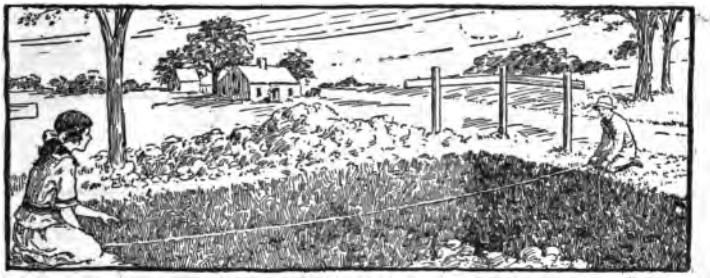
5. They find that the opening for the front door is 4 ft. by 7 ft. 6 in. Draw a plan of the opening to the scale $\frac{1}{12}$.

6. They find that each of the openings for the front windows of the house is 3 ft. by 5 ft. Draw a plan of one of these openings to the scale $\frac{1}{24}$.

7. Each of the openings for the cellar windows is 2 ft. wide and 15 in. high. Draw a plan of one of these openings to the scale $\frac{1}{12}$.

OUTDOOR MEASUREMENTS

1. Frank and Lena measure this cellar that they see being dug for a house in the country on land recently sold by their father. They use a tape measure that is 100 ft. long. How many yards are there in the tape? How many rods?



2. They find the excavation to be 22 ft. 6 in. long. Express this in yards and a fraction.

3. The distance from where Frank is sitting to the house is 412 ft. 6 in. Express this distance in yards.

4. If the building lot is 96 ft. 6 in. long and 72 ft. 6 in. wide, what length of fence is needed to inclose it?

5. Frank paces off 120 steps of 27 in. each in walking from where he is to a spring which is to supply the house with water. How many feet does he walk?

The pupils should measure the lengths of their ordinary steps, and should learn to pace such distances as the length of the room and the width of the playground. They should become perfectly familiar with the divisions of the ruler, and should have practice in estimating the height of the rooms, of trees, and of the schoolhouse; the width of the street; the dimensions of pieces of land; and the distances they travel in coming to school. In order to be real, this work must apply to local conditions and must therefore be given by the teacher.

Parallelogram. It is often convenient and necessary to find the area of a parallelogram. In the picture below, the heavy lines show the shape of a parallelogram.

A quadrilateral which has its opposite sides parallel is called a *parallelogram.*

The teacher should explain *parallel* informally, and illustrate the nature of a parallelogram by a drawing.

Area of a Parallelogram. If we cut the triangle T from this parallelogram and place it where X is, what kind of figure do we have? What does this tell us about the area of a parallelogram?

Because the area of the parallelogram is equal to the area of a rectangle of the same base and the same height, we see that

The area of a parallelogram is equal to the product of the base and height.

AREA OF A PARALLELOGRAM

1. On paper draw and cut out three parallelograms with different bases and heights. Show, by cutting off triangles and placing them as explained above, that each parallelogram can be made into a rectangle of the same area.

2. Draw a parallelogram with the following dimensions to the scale $\frac{1}{4}$: base, 12 in.; other side, 8 in.; height, 6 in. Find the area of the parallelogram.

Find the areas of parallelograms with these bases and heights:

3. 14 in., 12 in.
4. 28 ft., 18 ft.
5. 22 yd., 13 yd.
6. 24.8 ft., 18.5 ft.
7. 4.75 in., $3\frac{1}{8}$ in.
8. 18.5 ft., 9.25 ft.
9. 52.6 ft., 32.5 ft.
10. 82.7 yd., 14.6 yd.
11. 54.3 mi., 28.6 mi.

Height of a Parallelogram. What is the height of a parallelogram if the base is 10 ft. and the area 60 sq. ft.?

If the base is 10 ft. and the height 1 ft., the area is 10 sq. ft. Therefore the number of feet in the height of the parallelogram is 60 sq. ft. ÷ 10 sq. ft., or 6. In practical computation we do not label the numbers; we simply divide 60 by 10.

Teachers should encourage the pupils to find the area of a piece of land in the form of a parallelogram that may be in the vicinity of the school. Only part of the examples on this page need be taken by any one class.

MEASURING PARALLELOGRAMS

1. A man has a piece of land in the form of a parallelogram. The base of the parallelogram is 36 rd. and the height is 24 rd. How many acres are there in the piece of land?

2. A woman cuts a piece of velvet on the bias, cutting off a parallelogram of base 28 in. and height 9 in. How many square inches of velvet are there in the piece?

Find the areas of parallelograms with these bases and heights:

3. 8.7 in., 7.9 in.
4. 32.5 ft., 27.8 ft.
5. 26.9 ft., 17.2 ft.
6. 5 ft., 3 ft. 8 in.
7. 6 rd. 6 ft., $3\frac{1}{2}$ rd.
8. 27 ft. 9 in., 1 yd.
9. 8 ft. 6 in., 7 ft. 8 in.
10. 445 ft., 228 ft.
11. 3 yd. 17 in., 3 ft. 3 in.
12. 27 yd. 18 in., 15 yd. 9 in.

Find the heights of parallelograms with these bases and areas:

13. $15\frac{1}{2}$ in., 108.5 sq. in.
14. 25 ft., 275 sq. ft.
15. 22 ft. 4 in., 268 sq. ft.
16. 24 ft. 6 in., 588 sq. ft.
17. 129 ft., 10,320 sq. ft.
18. 31 ft. 2 in., 187 sq. ft.
19. 16 ft. 8 in., 100 sq. ft.
20. 17 ft. 6 in., 105 sq. ft.

Equilateral Triangle. A triangle in which the three sides are equal is called an *equilateral triangle.*

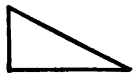
Equilateral

Isosceles

Scalene

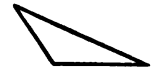
Isosceles Triangle. A triangle in which two of the sides are equal is called an *isosceles triangle.*

A triangle in which no two sides are equal is sometimes called a *scalene triangle.* It is illustrated above. The term is, however, going out of use.



Right

Acute



Obtuse

Right Triangle. A triangle in which one of the angles is a right angle is called a *right triangle.* The side opposite the right angle is called the *hypotenuse.*

Acute Triangle. A triangle in which all the angles are acute angles is called an *acute triangle.*

Obtuse Triangle. A triangle in which one of the angles is an obtuse angle is called an *obtuse triangle.*

If necessary the teacher should explain informally the terms *acute angle* and *obtuse angle,* which are used in the definitions.

The terms *right-angled triangle, acute-angled triangle,* and *obtuse-angled triangle* are also used, but the terms given above are preferred.

The teacher should discuss a page of this kind, having the pupils draw the figures on the blackboard, but it is not necessary that the formal definitions should be memorized. In addition to the classification of triangles given, the teacher should ascertain that the pupils understand, and are able to use intelligently, the following terms relating to triangles: right angle, acute angle, obtuse angle, side, height or altitude, and perimeter.

SIDES AND PERIMETERS

1. How long is the perimeter of an equilateral triangle 19 ft. 7 in. on a side?

Have the pupils draw approximate figures in all examples on this page.

2. A triangle has a perimeter of 114 ft. 2 in. One side is 30 ft. and another is half as much longer; how long is the third side of the triangle?

3. The base of an isosceles triangle is 25 ft., and each of the equal sides is $\frac{1}{5}$ longer. How long is the perimeter?

4. Cut off the three angles of a triangle and place them together. The sum is equal to how many right angles?

Find the sides of equilateral triangles of which the perimeters are as follows:

5. 113 ft. 3 in. 6. 127 yd. 7. 88 ft. 3 in.

Find the sides of squares of which the perimeters are as follows:

8. 110 ft. 4 in. 10. 61 yd. 2 ft. 12. 118 ft. 8 in.

9. 42 ft. $3\frac{1}{2}$ in. 11. 2 ft. 4 in. 13. 227 mi. 40 rd.

14. The perimeter of an isosceles triangle is 100 ft., and the unequal side is 33 ft. Find the length of one of the equal sides.

15. The perimeter of an isosceles triangle is 88 ft., and the unequal side is $\frac{1}{5}$ of the perimeter. Find the sides.

16. The perimeter of a triangle is 24 ft. One side is $\frac{1}{3}$ of the perimeter and another is $\frac{3}{8}$ of the perimeter. How many feet are there in each of the three sides?

17. The perimeter of a triangle is 120 ft. One side is $\frac{1}{4}$ of the perimeter and another is $\frac{1}{3}$ of the perimeter. How many feet are there in each of the three sides?

AREA OF A TRIANGLE

All work oral

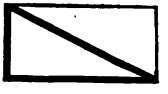
1. How is the area of a parallelogram found?

2. In the parallelogram here shown how does the area of the triangle indicated by the heavy lines compare with the area of the other triangle? A triangle is what part of a parallelogram of the same base and height?

3. If the parallelogram in Ex. 2 is 6 in. wide and 3 in. high, what is its area? What is the area of each of the triangles?

4. In this parallelogram how do the areas of the triangles compare? Each of the triangles is what part of the parallelogram?

5. In the rectangle here shown how does the area of the triangle indicated by the heavy lines compare with the area of the other triangle? A triangle is what part of a rectangle of the same base and height?



6. If the rectangle in Ex. 5 is 10 ft. wide and 5 ft. high, what is its area? What is the area of each of the triangles?

The teacher should call attention to the fact that we have thus found the area of an acute triangle, an obtuse triangle, and a right triangle.

7. How do you find the area of a triangle?

8. If you wished to find the area of some triangular piece of ground, what measurements would you take?

Find the areas of triangles with bases and heights as follows:

9. 5 in., 4 in.	11. 6 in., 4 in.	13. 9 in., 10 in.
10. 7 in., 6 in.	12. 8 in., 6 in.	14. 10 ft., 20 ft.

Area of a Triangle. As you learned on the preceding page, *The area of a triangle is equal to half the product of the base and height.*

For example, what is the area of a triangle of base 18 in. and height 7 in.?

$\frac{1}{2}$ of 18 × 7 sq. in. = 63 sq. in.

$$\frac{\overset{9}{\cancel{18}} \times 7}{\underset{1}{\cancel{2}}} = 63$$

AREA OF A TRIANGLE

Numbers 1 to 6, oral

Using aliquot parts, state the areas of triangles with bases and heights as follows:

1. 40 in., 25 in.
2. 160 ft., 125 ft.
3. 90 in., 33⅓ in.
4. 160 in., 25 in.
5. 64 in., 50 in.
6. 60 in., 16⅔ in.

Find the areas of triangles with bases and heights as follows:

7. 43 ft., 32 ft. 6 in.
8. 27.9 in., 18.7 in.
9. 28.7 in., 36.3 in.
10. 25.2 in., 19.4 in.
11. 36½ ft., 44½ ft.
12. 3 yd. 32 in., 1 ft.
13. 4 ft. 8 in., 3 ft. 6 in.
14. 3 ft. 4½ in., 2 ft. 6½ in.

15. What is the area of an equilateral triangle with perimeter 37 ft. 6 in. and height 10.8 ft.?

16. What is the area of a triangle with height 6.4 rd. and base 17 rd.? with height 7.2 in. and base 17 in.?

17. The two sides forming the right angle of a right triangle are 30 rd. and 40 rd. What is the area in square rods? in acres?

18. What is the number of square feet of slating required for a steeple the surface of which is made up of 6 triangles, each with base 20 ft. and height 65 ft.?

Trapezoid. A quadrilateral which has one pair of opposite sides parallel is called a *trapezoid.*

Area of a Trapezoid. If we cut from paper the exact double, D, of a trapezoid, T, the two may be fitted together to form a parallelogram as here shown. Therefore

The area of a trapezoid is equal to half the area of a parallelogram with the same height and with a base equal to the sum of the two parallel sides of the trapezoid.

For example, what is the area of a trapezoid of height 4 in. and parallel sides 8 in. and 10 in.?

The area is $\frac{1}{2}$ of $4 \times (8+10)$ sq. in.,

or $\frac{1}{2}$ of 4×18 sq. in. $= 36$ sq. in.

$$\frac{\overset{2}{\cancel{4}} \times \overset{9}{\cancel{18}}}{\cancel{2}} = 36$$

Teachers should call attention to the fact that the ability to find the area of a trapezoid is important, since fields and building lots are often in the shape of a trapezoid.

AREA OF A TRAPEZOID

1. A farmer has a field in the form of a trapezoid. The height of the trapezoid is 10 rd., and the parallel sides are $31\frac{1}{4}$ rd. and $16\frac{3}{4}$ rd. What is the area of the field?

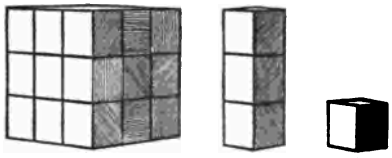
2. A building lot in a village is in the form of a trapezoid. The height of the trapezoid is 66 ft., and the parallel sides are 78 ft. 6 in. and 86 ft. 8 in. What is the area of the lot?

Find the areas of the following trapezoids, the height in each case being given first and then the two parallel sides:

3. $16\frac{1}{2}$ in.; $8\frac{1}{4}$ in. and 12 in.
4. 3.5 in.; 3.5 in. and 5.4 in.
5. $4\frac{1}{2}$ ft.; 3 ft. and $4\frac{1}{2}$ ft.
6. 16 rd.; 18 rd. and 24 rd.

Rectangular Solid. A solid having six sides, each side being a rectangle, is called a *rectangular solid.* If the sides are all squares, the solid is called a *cube.*

In the figures here shown *A*, *B*, and *C* are all rectangular solids, *A* and *C* being cubes.



Volume of a Rectangular Solid. As was shown on page 109,

The volume of a rectangular solid is equal to the product of the length, width, and height.

In finding volumes we do not write 3 in. × 4 in. × 2 in., but 3 × 4 × 2 cu. in. It is customary, however, in giving dimensions to write 3 in. × 4 in. × 2 in., or 3″ × 4″ × 2″, reading either of these expressions "3 inches by 4 inches by 2 inches."

VOLUME OF A RECTANGULAR SOLID

All work oral

Find the volumes of solids of the following dimensions:

1. 3″ × 4″ × 7″.
2. 6′ × 8′ × 10′.
3. 3′ × 8′ × 11′.
4. 2″ × 8″ × 9″.
5. 2′ × 3′ × 6′.
6. 5′ × 6′ × 10′.
7. 4′ × 7′ × 10′.
8. 6′ × 9′ × 10′.
9. 20′ × 30′ × 5′.
10. 11″ × 11″ × 10″.
11. 6 yd. × 8 yd. × 2 yd.
12. 6 ft. × 8 ft. × 11 ft.

13. How many cubic inches in a box that is 10 in. long, 8 in. wide, and 5 in. deep?

VOLUME OF A RECTANGULAR SOLID

1. How many cubic inches in an aquarium 14 in. long, $9\frac{1}{2}$ in. wide, and $8\frac{1}{2}$ in. deep?

2. How many cubic feet of masonry in a wall 60 ft. long, 6 ft. high, and $1\frac{1}{2}$ ft. thick?

3. How many loads of earth must be taken out in excavating a cellar 34 ft. by 16 ft., and 9 ft. deep?

State to the class, if necessary, that a load is a cubic yard.

4. A cellar 28 ft. by 38 ft. by 6 ft. is to be excavated. How much will it cost at 50¢ a load?

5. How much will it cost to excavate a ditch 240 ft. by 2 ft. by 5 ft., at 50¢ per cubic yard? at $62\frac{1}{2}$¢ per cubic yard?

6. How many cords in a pile of 4-foot wood 5 ft. high and 68 ft. long? 6 ft. high and 64 ft. long?

7. A schoolroom is 36 ft. 8 in. long, $24\frac{1}{2}$ ft. wide, and $12\frac{1}{2}$ ft. high. What is the number of cubic feet in the room?

8. A tunnel 940 ft. long has a cross-section area of 180 sq. ft. How much earth and rock were excavated?

The pupils may imagine that the cross section is 10 ft. by 18 ft.

9. The dimensions of a room are 16 ft., 14 ft., and 10 ft. How many square feet of floor? of walls? of ceiling?

10. In preparing a flower bed in a park it was necessary to fill in a space 80 ft. by 120 ft. to an average depth of $1\frac{1}{2}$ ft. The earth cost 50¢ a load. What was the total cost of the earth used?

11. The interior of a freight car is 8 ft. wide, 34 ft. long, and 7 ft. high. How many cubic feet does it contain? If it is filled with grain to a height of 5 ft., what is the weight of the grain at 60 lb. to the bushel, allowing $1\frac{1}{4}$ cu. ft. to the bushel?

Practical Equivalents. In solving problems you may refer to these measurements without memorizing them:

Exact values	*Approximate values*
$31\frac{1}{2}$ gal. = 1 bbl.	$4\frac{1}{5}$ cu. ft. = 1 bbl.
231 cu. in. = 1 gal.	$7\frac{1}{2}$ gal. = 1 cu. ft.
2150.42 cu. in. = 1 bu.	$1\frac{1}{4}$ cu. ft. = 1 bu.

The values on the right are the approximations commonly used, while those on the left are exact. While 1 bu. = 2150.42 cu. in., the value commonly taken is $1\frac{1}{4}$ cu. ft. or, less often, 2150 cu. in.

Since 1 bu. = $1\frac{1}{4}$ cu. ft., we can find the number of bushels in a bin by dividing the number of cubic feet by $1\frac{1}{4}$, or by $\frac{5}{4}$; still more conveniently, we may multiply by $\frac{4}{5}$. We then have

$$\frac{4 \times \text{length} \times \text{width} \times \text{height}}{5} = \text{number of bushels},$$

the dimensions in every case being expressed in feet.

Illustrative Problems. 1. Find the number of bushels which a bin 20 ft. long, 16 ft. wide, and 7 ft. deep will hold.

Using the method of solving given above, and canceling, we have the work as here shown.

$$\frac{\overset{4}{\cancel{4}} \times \overset{}{\cancel{20}} \times 16 \times 7}{\underset{}{\cancel{5}}} = 1792$$

2. Find the cost of the wheat that is required to fill the bin in Ex. 1, at \$2.20 a bushel.

We may simply multiply \$2.20 by the result found in Ex. 1, but the complete statement is as here shown.

$$\frac{\overset{4}{\cancel{4}} \times \overset{}{\cancel{20}} \times 16 \times 7 \times \$2.20}{\underset{}{\cancel{5}}} = \$3942.40$$

Further Equivalents. A ton of hay varies in volume from about 450 cu. ft. to 550 cu. ft., depending on the quality and on how much the hay has settled, but 500 cu. ft. may be taken as the volume in solving the problems in this book.

The volume of a ton of hard coal may be taken as 35 cu. ft. and the volume of a ton of soft coal as 42 cu. ft.

In brickwork 22 bricks with the mortar may be taken as occupying a volume of 1 cu. ft.

In the examples all measures refer to inside dimensions.

Illustrative Problems. 1. A hay-mow is approximately 50 ft. long and 30 ft. wide, and is filled to a depth of 20 ft. What is the approximate weight of the hay in the mow?

$$\frac{50 \times 30 \times \overset{2}{\cancel{20}}}{\underset{1}{\cancel{500}}} = 60$$

We have the work as here shown, the result being 60, the number of tons.

2. A coal bin 20 ft. long and 16 ft. wide is filled to a depth of 7 ft. with hard coal worth $11.35 a ton. How much is the coal worth?

$$\frac{20 \times 16 \times 7 \times \overset{2.27}{\cancel{\$11.35}}}{\underset{5}{\cancel{35}}} = \$726.40$$

We have the work as here shown, the result being $726.40.

3. A mason is building a wall 25 ft. long, 8 ft. high, and 1 ft. thick. How much will the bricks cost at $15 per thousand?

$$\frac{\overset{3}{\cancel{25}} \times 8 \times 1 \times 22 \times \overset{\$15}{\cancel{\$15}}}{\underset{5}{\cancel{1000}}} = \$66$$

We have the work as here shown, the result being $66.

PRACTICAL EQUIVALENTS

1. How many bushels in a bin 6 ft. by 7 ft. by 4 ft.?

2. How many bushels in a bin 5 ft. by 9 ft. by 3 ft.?

3. How many bushels in a box 18 in. by 3 ft. by 2 ft.?

4. How many gallons in a tank 5 ft. by 6 ft. by 4 ft.?

5. How many gallons in an aquarium 9 in. by 14 in. by 11 in.?

6. How many gallons in a tank $3\frac{1}{2}$ ft. $\times$ $6\frac{1}{4}$ ft. $\times$ $2\frac{3}{8}$ ft.?

7. A cistern is 6 ft. square at the bottom and $4\frac{2}{3}$ ft. deep. How many barrels will it hold?

8. An iron tank is $7\frac{1}{2}$ ft. long, $6\frac{3}{4}$ ft. wide, and $4\frac{2}{3}$ ft. deep. How many barrels will it contain?

Find the number of bushels of grain in bins of the following dimensions, filled to the depths stated:

9. 12 ft. by 16 ft., filled to a depth of 6 ft.

10. 14 ft. by 18 ft., filled to a depth of 9 ft.

11. 16 ft. by 24 ft., filled to a depth of 10 ft.

Find the number of bushels of grain in freight cars of the following dimensions, filled to the depths stated:

12. $7\frac{1}{2}$ ft. by 28 ft., filled to a depth of 7 ft.

13. $7\frac{1}{2}$ ft. by 30 ft., filled to a depth of $5\frac{1}{2}$ ft.

Find the value of the wheat in each of the following bins:

14. 8 ft. by 18 ft. by 6 ft., at \$2.20 per bushel.

15. 9 ft. by 20 ft. by 6 ft., at \$2.10 per bushel.

16. 9 ft. by 16 ft. by 5 ft., at \$1.90 per bushel.

17. Find the value of the corn in a car 8 ft. by $26\frac{3}{4}$ ft., filled to a depth of 6 ft., the corn being worth \$1.25 per bushel.

PRACTICAL EQUIVALENTS

1. A coal bin is 21 ft. long, 13 ft. wide, and 8 ft. deep. About how many tons of soft coal will it hold?

2. A coal bin is 14 ft. long, $8\frac{1}{2}$ ft. wide, and $6\frac{1}{4}$ ft. deep. About how many tons of soft coal will it hold?

3. A haymow is 22 ft. long, 14 ft. wide, and 9 ft. high. About how many tons of hay will it hold?

4. A barn has two haymows, each 24 ft. long, 15 ft. 6 in. wide, and 10 ft. high. About how many tons of hay will it hold?

5. How many cords of wood in a pile 4 ft. wide, 8 ft. high, and $32\frac{1}{2}$ ft. long?

6. How many cords of wood in a pile 4 ft. wide, $6\frac{1}{2}$ ft. high, and 44 ft. long?

7. If a schoolroom is 30 ft. long, 15 ft. wide, and 12 ft. high, and 180 cu. ft. of air should be allowed to each person, how many persons will the room accommodate?

8. A tank holds 82,500 gal. About how many cubic feet does it contain?

9. An aquarium contains $3\frac{4}{5}$ cu. ft. About how many gallons does it hold?

10. An aquarium is 18 in. long, $9\frac{1}{4}$ in. wide, and $10\frac{1}{8}$ in. deep. How many gallons does it hold?

11. A can containing 1732.5 cu. in. is full of milk. What is the milk worth at 12¢ a quart?

12. A wall is $32\frac{1}{2}$ ft. long, 2 ft. thick, and $32\frac{1}{2}$ ft. high. How many perches of masonry does it contain?

13. A brick wall is to be built 42 ft. long, $1\frac{1}{2}$ ft. thick, and $6\frac{1}{4}$ ft. high. About how many bricks will be needed?

SHIPPING FRUIT

1. The standard box for shipping Florida oranges is 12 in. by 12 in. by 33 in. How many cubic inches does it contain?

2. In shipping Georgia peaches a crate 8 in. by $12\frac{1}{2}$ in. by 22 in. is used. This contains how much more than 1 bu.?

3. In New Jersey a bushel cranberry box is $8\frac{3}{4}$ in. by 12 in. by 22 in. How does this differ from the standard bushel?

Have the pupils use cancellation in all such cases, thus:

$$8\tfrac{3}{4} \times 12 \times 22 \text{ cu. in.} = \frac{35 \times \overset{3}{\cancel{12}} \times 22}{\cancel{4}} \text{ cu. in.} = 2310 \text{ cu. in.}$$

4. California apples and pears for local use are shipped in a 50-pound box 10 in. by 22 in. by 12 in. By how much does one of these boxes differ from the standard bushel?

5. In California the orange box is sometimes $11\frac{1}{4}$ in. by $26\frac{1}{2}$ in. by $11\frac{1}{4}$ in., and sometimes $7\frac{3}{4}$ in. by 22 in. by $17\frac{1}{2}$ in. Which holds the more, and how many cubic inches more?

6. Dried fruits are shipped in 50-pound or 25-pound boxes. The 50-pound box is 9 in. by $15\frac{1}{4}$ in. by 9 in. Find the volume.

7. The 25-pound box mentioned in Ex. 6 is sometimes $9\frac{3}{4}$ in. by $13\frac{3}{4}$ in. by $5\frac{3}{4}$ in., and sometimes 9 in. by 16 in. by 5 in. Which box holds the more, and how much more?

8. For shipping fruit out of the state the sizes of some of the boxes used in California are as follows:

Plums, $11\frac{3}{4}$ in. by $18\frac{1}{2}$ in. by 4 in.
Cherries, $10\frac{3}{8}$ in. by $16\frac{1}{2}$ in. by $2\frac{1}{2}$ in.
Pears, $11\frac{3}{4}$ in. by $18\frac{1}{2}$ in. by $8\frac{1}{2}$ in.
Peaches, $11\frac{3}{4}$ in. by $18\frac{1}{2}$ in. by $4\frac{7}{8}$ in.

Find the number of cubic inches in each of these boxes.

Other Problems relating to Volumes. We may also have the volume and certain dimensions given to find other dimensions.

1. How deep is a box 5 ft. long and 4 ft. wide, if the box contains 60 cu. ft.?

$$\frac{\overset{3}{\cancel{60}}}{\underset{1}{\cancel{5}} \times \underset{1}{\cancel{4}}} = 3$$

Since the box is 5 ft. long and 4 ft. wide, the area of the base is 20 sq. ft.

If the box were 1 ft. deep, the volume would be 20 cu. ft. Therefore the depth of the box must be as many times 1 ft. as 60 cu. ft. ÷ 20 cu. ft., or 3. Hence the box is 3 ft. deep.

The number of cubic units of volume, divided by the product of the numbers of units of two dimensions, is equal to the number of units of the third dimension.

This is sometimes expressed thus: *The volume divided by the product of two dimensions is equal to the third dimension.*

2. What is the area of the base of a block $2\frac{1}{2}$ in. high, if the block contains 70 cu. in.?

$$70 \div \frac{5}{2} = \frac{\overset{14}{\cancel{2}} \times \cancel{70}}{\underset{1}{\cancel{5}}} = 28$$

If the block were 1 in. high, the volume would be 70 cu. in. ÷ $2\frac{1}{2}$, or 28 cu. in., and the area of the base of such a block is 28 sq. in. Hence the area of the base is 28 sq. in.

The volume divided by the height is equal to the area of the base.

3. What is the thickness of a block containing 100 cu. in., if the area of the base is 30 sq. in.?

$$\frac{10\cancel{0}}{3\cancel{0}} = 3\tfrac{1}{3}$$

If the block were 1 in. thick, it would contain 30 cu. in. Therefore the block is as many times 1 in. thick as 100 cu. in. ÷ 30 cu. in., or $3\frac{1}{3}$. Hence the thickness is $3\frac{1}{3}$ in.

The volume divided by the area of the base is equal to the height.

PROBLEMS ON VOLUMES

1. A man wishes to place a rectangular tank in the attic of his house. He wishes to have it contain 10 bbl., which he takes as 42 cu. ft. He can allow a height of only 3 ft. 6 in. on account of the roof. What area must he give to the base of the tank?

2. A rectangular tank with base 9 ft. by $16\frac{2}{3}$ ft. contains 900 cu. ft. How deep is the tank?

3. A box with base 13 in. by 20 in. contains 3900 cu. in. How deep is the box?

4. A rectangular tank 4 ft. deep contains 450 cu. ft. What is the area of the base of the tank?

5. A hall 22 ft. high and 30 ft. wide contains 28,600 cu. ft. Find the length of the hall.

6. A block containing 700 cu. ft. has a base area of 56 sq. ft. How high is the block?

7. A hall contains 41,341.3 cu. ft. It is 41.3 ft. long and 36.4 ft. wide. How high is the hall?

8. A box contains 42,159 cu. in. The area of the bottom is 1081 sq. in. How deep is the box?

9. A block of granite is 14 in. long, 8.3 in. wide, and contains 836.64 cu. in. How thick is the block?

10. The area of the floor of a hall is 1386 sq. ft. The hall contains 15,246 cu. ft. How high is the hall?

11. A storeroom is 14.3 ft. long, 13 ft. 6 in. wide, and has a capacity of 2316.6 cu. ft. How high is the room?

12. The floor of a bedroom is square and contains 144 sq. ft. If the room contains 1368 cu. ft., find the three dimensions.

Let the pupils first find the two equal factors of 144.

USING WHAT YOU HAVE LEARNED

1. Henry visited his uncle, who owns a large farm. He told his uncle about the arithmetic he was studying. His uncle told him that in the barn there was an oat bin 8 ft. long and 5 ft. wide. He asked Henry to find how many bushels of oats there were in the bin. Henry took a stick and measured the depth of the oats, finding it to be 3 ft. 8 in. He allowed 2150 cu. in. to the bushel. How many bushels of oats were there?

2. Express in bushels the volume of a bin 4 ft. by 6 ft. by 8 ft. $6\frac{1}{2}$ in., allowing 2150 cu. in. to the bushel.

3. A bin 21 ft. 6 in. long and 10 ft. wide is filled with wheat to the depth of 5 ft. Allowing 2150 cu. in. to the bushel, what is the wheat worth at \$2.20 a bushel?

4. How many bars of soap, each 4 in. long, $2\frac{3}{4}$ in. wide, and $1\frac{1}{2}$ in. thick, can be packed in a box 16 in. long, $13\frac{3}{4}$ in. wide, and 1 ft. deep? Will the bars fit in the box?

5. At \$12 a thousand, what is the value of the bricks in a kiln 25 ft. by 60 ft. by 9 ft., each brick being 4 in. by 8 in. by 2 in.?

6. What is a pile of cut stones 27 ft. long, 5 ft. wide, and 6 ft. high worth at \$6.40 a cord?

A cord of stone has the same volume as a cord of wood.

7. If it takes 550 cu. ft. of clover hay to make a ton, how many tons of clover hay will fill a mow 32 ft. long, the width being half the length and the depth being half the width?

8. How many gallons of water are there in a watering trough that is 6 ft. long and 16 in. wide, the depth of the water being three fourths of the width?

9. Express in gallons the capacity of a tank 6 ft. long, 4 ft. wide, and 4 ft. deep.

Capacity and Weight. The laws of the various states usually specify the weight of a bushel of various grains and vegetables, requiring it to weigh about the amount specified below. This table should not be committed to memory, but used for reference.

Corn (in the ear),	70 lb.	Oats,	32 lb.
Barley,	48 lb.	Dried apples,	26 lb.
Corn (shelled), rye,	56 lb.	Timothy seed,	45 lb.
Wheat, clover seed, peas,	60 lb.	Dried peaches,	33 lb.
Beans, white potatoes,	60 lb.	Sweet potatoes,	55 lb.
Buckwheat,	48 lb.	Onions,	48 lb.

The teacher can probably find the exact weights for the state in which the pupils live by writing to the state agricultural college or department.

CAPACITY AND WEIGHT

1. How many tons to 1000 bu. of wheat? to 1000 bu. of oats? to 1000 bu. of timothy seed?

2. How many bushels of white potatoes to the ton?

3. How many bushels to a ton of corn in the ear? How many bushels to a ton of shelled corn?

4. How many tons to 1000 bu. of corn in the ear? How many tons to 1000 bu. of shelled corn?

Find the number of bushels to a ton of the following:

5. Oats.
6. Buckwheat.
7. Clover seed.
8. Onions.
9. Sweet potatoes.
10. Timothy seed.

11. A bushel of rye weighs how many times a bushel of oats?

12. A bushel of timothy seed is what part as heavy as a bushel of buckwheat? as a bushel of peas?

USING WHAT YOU HAVE LEARNED

1. A machinist uses 64 in. of pipe. Express this as feet and inches; as feet and a fraction; as yards and inches.

2. A milk dealer sells 565 qt. of milk. Express this as gallons and quarts; as gallons and a fraction.

3. A milk dealer sells 95 pt. of cream. Express this as quarts and pints; as quarts and a fraction; as gallons and pints; as gallons and a fraction.

4. If you cut off $\frac{7}{8}$ of a piece of tin that contains 5 sq. ft., how many square inches do you cut off?

5. Counting 64 sheets to a pound, what should be the correct weight of 5000 sheets of paper?

6. If evaporation from a tank is estimated at $8\frac{1}{3}\%$ in a week, how many gallons will be lost in a week from an open tank which contains 4800 gal.?

7. What is the reduction in length of a 240-yard roll of cloth that loses $2\frac{1}{2}\%$ of its length by shrinkage?

8. A sign in a loft says that the maximum weight should not exceed 60 lb. per square foot. If the loft floor is $48' \times 62'$, and if there are already $35\frac{1}{4}$ T. of merchandise on the floor, how much more weight may be safely added?

9. While sailing one day, Albert heard the skipper say that the water was 36 fathoms deep in the bay. How many feet is this? How many yards?

On such unusual units the pupil should refer to the note on page 106.

10. How many yards in 42 fathoms? How many feet?

11. How many fathoms in 42 yd.? How many feet?

12. Which of the years 1975, 1988, 2000, and 2004 will be leap years? Write the reason for each of your answers.

FARM-CLUB PROBLEMS

1. In a calf-feeding contest a boy fed a calf during a period of 120 da. The calf gained on an average $1\frac{3}{4}$ lb. per day. If the calf weighed 70 lb. when the contest began, how much did it weigh when the contest ended?

2. A boy bought a calf for \$35 and entered a calf-feeding contest. His expense account showed the following items:

600 lb. whole milk @ \$2.50 per hundredweight;
1200 lb. skim milk @ 50¢ per hundredweight;
2 bu. whole oats @ 65¢ per bushel;
50 lb. oil meal @ \$3 per hundredweight;
120 lb. clover hay @ \$1 per hundredweight.

If he sold the calf for \$72, how much did he receive, above these expenses, for his labor?

3. If a boy feeds some calves 150 lb. of hay at \$20 per ton, how much should he charge to his expense account?

4. A boy feeds a calf for a certain period, and during that time the calf gains 325 lb. in weight. The expenses during the period are as follows:

Whole milk,	\$11.00	Hay,	\$2.00
Skim milk,	5.00	Oil meal,	.50
Oats,	1.65	Labor,	6.50
Bran,	1.20	Medicine,	.75

Find the total cost per hundredweight of gain.

5. How long will $2\frac{1}{2}$ bu. of oats, weighing 32 lb. per bushel, last if a boy feeds 1 lb. twice each day to each of two calves?

6. In a certain year the corn clubs in this country reported that they raised 523,000 bu. of corn at an expense of \$143,000. They sold this corn at 81¢ a bushel. Find the total profit.

SCHOOL-AND-HOME GARDENING

1. Each of two boys in a club raised an acre of potatoes. One boy chemically treated his seed potatoes so that they would grow better, the expense being \$1.10, but the other did not. The potatoes that were chemically treated sold for \$1.48 a bushel, while the others brought only 98¢ a bushel. If each boy harvested 150 bu., how much was gained by the treatment?

2. A bushel of potatoes weighs 60 lb. If a boy carries 30 lb. and his sister carries 20 lb. on a trip from the field, how many bushels will they both carry together in 24 trips?

3. A boy raised 240 bu. of potatoes. He sold 35 bu. for seed potatoes at \$2.25 a bushel, and of the remainder of the crop he sold 120 bu. at \$1.35 a bushel and the rest at 90¢ a bushel. How much money did he receive?

4. A girl raised 10,800 lb. of potatoes, but the yield would have been increased one fifth by spraying. At 60 lb. to the bushel, how many bushels would she have gained by spraying?

5. A member of a club of potato growers turned in the following account of his receipts and expenses:

Receipts		*Expenses*	
Value of seed on hand,	\$4.00	Cost of seed,	\$1.80
Value of table stock,	10.40	Chemical treatment,	.20
Sales, selected,	4.80	Rental of land,	1.50
Sales, common,	2.20	Spraying,	.50
Prize won,	2.00	Labor,	3.80

Find the amount of the member's profit.

6. Allowing 60 lb. to the bushel, find the amount received from the sale of 16,800 lb. of potatoes at \$1.40 a bushel.

CANNING CLUBS

1. A girl puts up 4 qt. of fruit in pint cans each week day during the month of July in a year in which that month had four Sundays. How many pint cans did she put up?

2. When peaches cost 25¢ a pint can, a woman buys peaches to fill such a can for 8¢, the sugar needed for 3¢, and the can itself for 5¢. Allowing 1¢ per can for fuel, how much does she save on 50 cans by putting up the peaches herself?

3. There are eleven girls in a canning club. Six of them put up 35 qt. each and the others put up 29 qt. each. How many quarts did the club put up?

4. A girl was given the use of a tomato bed in a garden. She spent 30¢ for plants, 15¢ for fertilizer and spraying, and 20¢ for other things, and bought 25 cans at 5¢ apiece. She put up 25 quart cans, which she sold at 22¢ each. Find her profit.

5. In a recent year the canning clubs reported that 7900 members canned on an average $25\frac{1}{2}$ qt. of food per member. Find the total number of quarts canned.

6. The clubs in Ex. 5 reported that the total cost of canning the food was 14¢ a quart. If the clubs sold the product at 21¢ a quart, how much did they receive above expenses?

7. Margaret sold 63 lb. of beans at 5¢ a pound, and the purchaser canned all these beans in 35 one-quart cans. How much did the beans cost per quart? How much less would Margaret have received if she had canned the beans herself at a cost of $3\frac{1}{2}$¢ per can, and sold them at 10¢ per can?

8. The canning club in a school near Chicago put up 3304 cans in one season. Rachel Wagner put up 104 of these cans. This was what part of the total number?

WASTE AND SAVING OF FOOD

1. If the average waste of wheat flour in this country is 1 oz. per week for each of the 24,000,000 families, how many pounds does the country waste per year in this way?

2. There are 196 lb. of flour in a barrel, and $4\frac{1}{2}$ bu. of wheat are required to make a barrel of flour. Find to the nearest bushel the amount of wheat that is wasted annually in Ex. 1.

3. Allowing 16 bu. of wheat to the acre, how many acres of land would be required to supply the waste as found in Ex. 2?

4. If a woman throws away on an average a slice of bread every day, and if there are 12 slices to a 10-cent loaf, how much money does she waste in this way in a year?

A year is always to be taken as 365 da. unless the contrary is stated.

5. If 10,000,000 women are as wasteful in this respect as the one referred to in Ex. 4, how much money would they all waste in this way in a year?

6. The average consumption of meat per person in this country is approximately 194 lb. per year, but the government reports that 4 oz. per day is sufficient for the best work. The average consumption for each person is how much more per year than is sufficient for the best work?

7. If meat costs on the average 28¢ a pound, how much might be saved in a year by our population of 105,000,000 people if we brought the average consumption of meat down to 4 oz. per day for each person?

8. If each of 24,000,000 families in this country wastes on an average 1 oz. of meat per day, how many pounds are thus wasted in a year, and how much is this worth at an average price of 28¢ per pound?

II. HOW TO SOLVE PROBLEMS

General Suggestions. You may already have learned that in solving a problem there is one thing that the teacher will ask you to do, and there are two other things that may be required. These three things are (1) to work rapidly and accurately, that is, to take the shortest road to the answer and to be certain that the answer is correct; (2) to put neatly on paper not merely the operation but a brief explanation; (3) to give a brief analysis or oral explanation of the problem, so as to show that you clearly understand it.

For example, if 7 yd. of cloth cost $4.20, how much will 12 yd. cost?

(1) *The number work:*

$$\frac{12 \times \$4.20}{7} \qquad \$0.60 \times 12 = \$7.20$$

(2) *The written explanation:*

7 yd. cost $4.20.

1 yd. costs $\frac{1}{7}$ of $4.20.

12 yd. cost $12 \times \frac{1}{7}$ of $4.20, or $7.20.

(3) *The oral analysis:*

Since 7 yd. cost $4.20, 1 yd. costs $\frac{1}{7}$ of $4.20, or $0.60, and 12 yd. will cost $12 \times \$0.60$, or $7.20.

Teachers will not need to call for all this work with every example. Sometimes it will be necessary to emphasize (2) and sometimes (3), but the important thing is that the work should be quickly and neatly and, above all, *accurately* done. When it comes to computation, the work is right or it is wrong; there is no halfway point.

USING WHAT YOU HAVE LEARNED

In solving the following problems give first the number work and then the written explanation:

1. If Mollie pays 72¢ for 6 lb. of hominy, how much must she pay for 8 lb.?

2. If Mollie buys 5 packages of cornstarch for 90¢, how much must she pay for 8 packages?

3. If 6 lb. of prunes cost $1.26, how much will 7 lb. cost?

4. If 8 cans of salmon cost $2.32, how much will 5 cans cost?

5. If 4 cans of cottonseed oil cost $3.52, how much will 7 cans cost? How much will 8 cans cost?

6. Mr. Ray's rent for 3 mo. was $192. How much would it be for 5 mo.? How much would it be for 9 mo.?

7. If 8 cans of cocoa cost $2.56, how much will 7 cans cost?

8. A dealer pays $63.75 for 5 gross of toothbrushes. How much must he pay for 14 gross?

9. If 42 yd. of linoleum cost $94.50, how much will 34 yd. cost? How much will 84 yd. cost?

10. If 16 yd. of flannel cost $10.88, how much will 28 yd. cost? How much will 48 yd. cost?

11. If 8 yd. of sheeting cost $8.96, how much will 22 yd. cost?

12. If 12 lb. of raisins cost $4.08, how much will 8 lb. cost?

13. If 6 small bottles of French capers cost $1.08, how much will 8 bottles cost?

14. If 100 sheets of blotting paper cost $6.50, how much will 250 sheets cost? How much will 400 sheets cost?

15. If 11 yd. of cotton scrim cost $3.63, how much will 18 yd. cost? How much will 44 yd. cost?

Work Rapidly and Accurately. In considering a problem there are four matters which are especially important. These are suggested on this page, and this is the first one.

The business man demands rapidity and accuracy. Reasonable rapidity usually leads to accuracy. The most important thing is accuracy. To insure accuracy, always check your work. It is a good check to estimate the answer in advance.

Work as the Business Man does. The business man does not put his work in steps, with signs like + and ÷. He writes his work in columns, because it is quicker to do so and he is less likely to make mistakes.

In addition, add upwards as the business man does.

In short division, put the quotient below, as he does, so that it will be where you can do further work with it.

Make your figures neatly. Slovenly work is unbusinesslike. It leads to inaccuracies.

Analyze your Problem. This means that you are to think out the steps in advance and state the reasons to yourself.

Think. That is what makes a business man successful.

Think. That is what makes a pupil a leader.

Use Short Methods. Do not use pencil and paper in such a problem as finding the cost of $12\frac{1}{2}$ yd. at 64¢ a yard. Do the work "in your head."

The greatest defect shown in business arithmetic is the inability to add, subtract, multiply, and divide quickly and accurately.

Use short methods whenever it is reasonable to do so.

The teacher should enliven the work and make it real through correlated problems and projects with some force to them, but it should not be forgotten that the greatest defect is found in the numerical computations with integers and fractions.

USING WHAT YOU HAVE LEARNED

1. How many packages of tea, each containing 25 lb. at 48¢ a pound, can be bought for $3000?

2. If 32 bricks are required to pave 1 sq. yd., how many bricks will be needed to pave a space 42 ft. by 69 ft.?

3. If a certain homing pigeon makes a flight of 1000 mi. in 2 da. $9\frac{3}{4}$ hr., what is the average speed per hour?

4. A yacht in one of the international races made 30 mi. in 3 hr. 12 min. 30 sec. What was the average time per mile?

5. A cubic foot of pure cast gold weighs 1204 lb., and a cubic foot of granite weighs 172 lb. Gold is how many times as heavy as granite?

6. How many tons of pressure will a hurricane exert on the front of a city building 150 ft. high and 100 ft. wide if the average pressure is 45 lb. to the square foot?

7. How many minutes will it take a column of soldiers, the column being $\frac{1}{2}$ mi. long, to pass a fixed point if the soldiers in a minute take 120 steps averaging 2 ft. 6 in.?

8. In a certain arid region in the West 250 mi. of main irrigating ditches were put in at an average cost of $10,000 a mile, and 400 mi. of side ditches at an average cost of $1250 a mile. Find the total cost.

9. A load of coal was sold for $17.76. The wagon and coal together weighed 4160 lb., and the wagon was known to weigh 1200 lb. At what price per ton was the coal sold?

10. A fair breeze blows 880 ft. a minute, and a hurricane 7040 ft. a minute. How many feet does each blow in a second? Answer to two decimal places only. How many miles does each blow in an hour?

SHORT METHODS IN MULTIPLICATION

All work oral

1. Multiply by 10: 27, 2.7, and 270.
2. State the short method of multiplying by 10; by 100.
3. Multiply by $\frac{10}{2}$: 20, 44, 88, and 440.
4. State the short method of multiplying by 5.
5. Multiply by $\frac{100}{4}$: 40, 88, and 160.
6. State the short method of multiplying by 25.
7. Multiply by $\frac{100}{3}$: 3, 9, 18, and 66.
8. State the short method of multiplying by $33\frac{1}{3}$.
9. Multiply by $\frac{1000}{8}$: 8, 16, 24, and 32.
10. State the short method of multiplying by 125.
11. How much is $10 \times 15 - 15$? How much is 9×15?
12. State the short method of multiplying by 9.
13. Multiply 14 by 9, using a short method.

Multiply the following by 5:

14. 18.	16. 28.	18. 36.	20. 54.	22. 86.
15. 24.	17. 32.	19. 46.	21. 68.	23. 84.

Multiply the following by 25:

24. 16.	26. 28.	28. 48.	30. 64.	32. 72.
25. 24.	27. 36.	29. 56.	31. 68.	33. 84.

Multiply the following by $33\frac{1}{3}$:

34. 15.	35. 21.	36. 27.	37. 36.	38. 42.

Multiply the following by 125:

39. 80.	40. 48.	41. 72.	42. 64.	43. 96.

Short Methods in Multiplication Summarized. We shall now state the important short methods of multiplication. *Always save time by using short methods whenever possible.*

To multiply by 10, move the decimal point one place to the right, annexing a zero if necessary.

Thus, $10 \times 7.25 = 72.5$, and $10 \times 856 = 8560$.

The reason was seen when studying pages 26, 144, and 149.

To multiply by 100 or 1000, move the decimal point to the right two or three places respectively, annexing zeros if necessary.

Thus, $100 \times 8.3 = 830$, and $1000 \times 62 = 62{,}000$.

To multiply by 5, multiply by 10 and divide by 2.

This follows from the fact that $5 = \frac{10}{2}$,

and so $$5 \times 48 = \frac{10}{2} \times 48 = \frac{10 \times 48}{2}.$$

To multiply by 25, multiply by 100 and divide by 4.

This follows from the fact that $25 = \frac{100}{4}$.

To multiply by 125, multiply by 1000 and divide by 8.

This follows from the fact that $125 = \frac{1000}{8}$.

To multiply by $33\frac{1}{3}$, multiply by 100 and divide by 3.

This follows from the fact that $33\frac{1}{3} = \frac{100}{3}$.

To multiply by 9, multiply by 10 and subtract the multiplicand.

This is because $9 = 10 - 1$,

and so $$9 \times 78 = 10 \times 78 - 1 \times 78 = 780 - 78 = 702.$$

To multiply by 11, multiply by 10 and add the multiplicand.

This is because $11 = 10 + 1$,

and so $$11 \times 62 = 10 \times 62 + 1 \times 62 = 620 + 62 = 682.$$

Begin at the left to multiply in simple cases.

Thus, 9×63 is $540 + 27$, or 567.

Think simply **"540, 27, 567."**

Short Methods in Per Cents Summarized. We shall now review the important short methods in finding per cents.

To find 50% of a number, divide the number by 2.

This follows from the fact that $50\% = \frac{1}{2}$.

To find 25% of a number, divide the number by 4.

This follows from the fact that $25\% = \frac{1}{4}$.

To find $12\frac{1}{2}\%$ of a number, divide the number by 8.

This follows from the fact that $12\frac{1}{2}\% = \frac{1}{8}$.

To find $33\frac{1}{3}\%$ of a number, divide the number by 3.

This follows from the fact that $33\frac{1}{3}\% = \frac{1}{3}$.

To find $16\frac{2}{3}\%$ of a number, divide the number by 6.

This follows from the fact that $16\frac{2}{3}\% = \frac{1}{6}$.

To find $66\frac{2}{3}\%$ of a number, take $\frac{2}{3}$ of the number.

This follows from the fact that $66\frac{2}{3}\% = \frac{2}{3}$.

Teachers will find it advantageous to review other cases, like $37\frac{1}{2}\%$, $62\frac{1}{2}\%$, $87\frac{1}{2}\%$, $6\frac{1}{4}\%$, and 75%. See also pages 26, 87–89, 166–170.

WRITTEN DRILL WORK

Multiply, in turn, by 10, by 100, by 5, by 25, and by 125:

1. 5656. 2. 8448. 3. 17,184. 4. 15,048. 5. 14,120.

Multiply, in turn, by $33\frac{1}{3}$, by $16\frac{2}{3}$, by 9, and by 11:

6. 16,974. 7. 64,050. 8. 12,210. 9. 14,601. 10. 10,803.

Find, in turn, 50%, 25%, and $12\frac{1}{2}\%$ of the following:

11. 11,360. 12. 16,816. 13. 83,864. 14. 396.56. 15. 77.768.

Oral Analysis. Many problems are so easy that they may be solved without pencil and paper. Merchants solve their problems in this way, and we should solve our own problems by this method whenever it is possible. In analyzing a problem speak briefly and accurately. Use your own language; do not feel that you must use the exact language of the book or of the teacher.

Illustrative Problems. 1. If a dealer pays \$10 for 8 books of the same kind, what is the cost of each book?

Since 8 books cost \$10, 1 book costs $\frac{1}{8}$ of \$10, or \$1.25.

2. If a stationer buys 100 sheets of blotting paper for \$6.50 and retails them at 8¢ a sheet, how much does he gain?

Since he sells 1 sheet for 8¢, he sells 100 sheets for 100 times 8¢, or \$8. He therefore gains \$8 − \$6.50, or \$1.50.

3. A dealer pays \$48 for 120 doz. pencils. How much does he pay for one pencil?

Since he pays \$48 for 120 doz. pencils, for 1 doz. he pays $\frac{1}{120}$ of \$48, or 40¢. Therefore he pays for 1 pencil $\frac{1}{12}$ of 40¢, or $3\frac{1}{3}$¢.

4. If a dealer pays \$7.20 for a dozen knives and sells them at 90¢ each, how much does he gain? His gain is what part of the cost?

Since the dealer sells 12 knives at 90¢ each, he receives 12 times 90¢, or \$10.80. He therefore gains \$10.80 − \$7.20, or \$3.60. Then \$3.60 is $\frac{3.60}{7.20}$, or $\frac{1}{2}$, of the cost.

5. If you can buy baseballs at \$9.50 a dozen and are allowed the dozen rate for $\frac{1}{2}$ doz., how much do you save in buying 6 baseballs by the half dozen instead of buying at 95¢ each?

Since 6 baseballs cost 6 × 95¢, or \$5.70, if bought singly, and $\frac{1}{2}$ of \$9.50, or \$4.75, if bought by the half dozen, the saving is \$5.70 − \$4.75, or 95¢.

PURCHASES AT THE STORE

All work oral

Using short processes when possible, find the cost of the following:

1. 25 yd. of felt @ \$2.40; @ \$2.48; @ \$2.20.
2. 12½ yd. of denim @ 40¢; @ 56¢; @ 64¢.
3. 5 yd. of dimity @ 34¢; @ 38¢; @ 46¢; @ 50¢.
4. 12½ lb. of feathers @ 96¢; @ \$1.20; @ \$1.60.
5. 32 yd. of cambric @ 40¢; @ 50¢; @ 60¢; @ 75¢.
6. 8 yd. of cashmere @ \$1.25; @ \$1.37½; @ \$1.50.
7. 16 yd. of gingham @ 40¢; @ 50¢; @ 75¢; @ 80¢.
8. 20 yd. of buckram @ 40¢; @ 50¢; @ 60¢; @ 75¢.
9. 25 yd. of crinoline @ 30¢; @ 40¢; @ 48¢; @ 56¢.
10. 11 yd. of percaline @ 40¢; @ 50¢; @ 60¢; @ 70¢.
11. 10 pieces of beading @ 33¢; @ 34¢; @ 36¢; @ 40¢.
12. 25 yd. of eider down @ \$1.20; @ \$1.60; @ \$1.80.
13. 12 yd. of chambray @ 70¢; @ 80¢; @ 90¢.
14. 22 yd. of nainsook @ 50¢; @ 60¢; @ 70¢; @ 80¢.
15. 12 yd. of linen lawn @ \$1.50; @ \$1.75; @ \$2.
16. 20 whalebones @ 40¢; @ 42¢; @ 44¢; @ 50¢.
17. 7 gross of bone buttons @ 30¢; @ 31¢; @ 40¢.
18. 10 yd. of jute burlap @ 65¢; @ 78¢; @ 87½¢.
19. 3 doz. spools of cotton @ 80¢; @ 81¢; @ 82¢; @ 83¢.
20. 7 yd. of torchon lace @ 30¢; @ 31¢; @ 32¢.
21. 4 doz. spools of basting cotton @ 30¢; @ 32¢; @ 40¢.
22. 11 yd. of cotton scrim @ 43¢; @ 45¢; @ 55¢; @ 60¢.
23. 30 yd. of cheesecloth @ 15¢; @ 20¢; @ 25¢; @ 30¢.

DRILL WORK IN MULTIPLICATION

Multiply the following:

1. $10 \times \$0.35$.
2. $10 \times \$9257$.
3. $10 \times \$8.25$.
4. $10 \times \$7.90$.
5. $100 \times \$0.47$.
6. $100 \times \$5.87\frac{1}{2}$.
7. $1000 \times \$0.85$.
8. $1000 \times \$7.37\frac{1}{2}$.
9. $50 \times \$32{,}700$.
10. $50 \times \$854.20$.
11. $50 \times \$962.50$.
12. $25 \times \$282$.
13. $25 \times \$720$.
14. $25 \times \$925$.
15. $625 \times \$88$.
16. $25 \times \$38.60$.
17. $25 \times \$85.20$.
18. $12\frac{1}{2} \times \$7200$.
19. $12\frac{1}{2} \times \$4080$.
20. $125 \times \$720$.
21. $125 \times \$7200$.
22. $125 \times \$9600$.
23. $33\frac{1}{3} \times 369$.
24. $33\frac{1}{3} \times 486$.
25. $33\frac{1}{3} \times 1440$.
26. $33\frac{1}{3} \times \$612.30$.
27. $16\frac{2}{3} \times 18{,}630$.
28. $16\frac{2}{3} \times \$696.30$.
29. $5 \times 98{,}664.8$.
30. $5 \times \$310{,}850$.
31. $125 \times \$85{,}280$.
32. $125 \times \$852.80$.
33. $16\frac{2}{3} \times 5{,}400{,}000$.
34. How much will 25 books cost at \$1.60 each?
35. How much will 25 wagons cost at \$120 each?
36. How much will 50 cows cost at \$84 a head?
37. How much will 100 books cost at $87\frac{1}{2}¢$ each?
38. How much will 25 tons of hay cost at \$18.40 a ton?
39. How much will $12\frac{1}{2}$ yards of cloth cost at \$1.68 a yard?
40. How much will 25 doz. pencils cost at \$0.64 a dozen?
41. How much will 48 office desks cost at \$75 each?
42. At \$25 an acre, how much is a tract of 38,364 acres of mountain land worth?
43. How much will 25 automobile tires cost at \$36 each? at $\$33.33\frac{1}{3}$ each? at $\$66.66\frac{2}{3}$ each?
44. How much will a dozen tables weigh at 25 lb. each? at 50 lb. each?

Short Methods in Division. Certain divisors are used so frequently in business that it is desirable to know the shortest methods of dividing by them. These include the numbers 5, 10, 25, 100, 125 (with 12.5 or $12\frac{1}{2}$), and $33\frac{1}{3}$. We have met with these methods already in our work, and shall now review them.

SHORT METHODS IN DIVISION

All work oral

1. Divide by 10: \$960, \$8300, 430, \$725, 675 lb., 9 mi.

2. State the short method of dividing by 10.

3. Since $\frac{1}{5} = \frac{2}{10}$, we may take $\frac{1}{5}$ of a dollar by taking how many tenths of it? State the short method of dividing a number by 5; that is, of taking $\frac{1}{5}$ of a number.

4. Divide by 5: \$2210, \$1240, 5280 ft., 3210 mi., 2220.

5. Divide by 100: \$700, 8000, 6800, \$760, \$475, \$38.

6. State the short method of dividing by 100.

7. State the short method of finding $\frac{1}{25}$ (or $\frac{4}{100}$) of a dollar; of dividing a number by 25.

8. Divide by 25: \$1200, \$2000, \$2100, 3200, \$23, \$11.

9. State the short method of dividing by 125. ($\frac{1}{125} = \frac{8}{1000}$.)

10. Divide by 125: \$11,000, 2000, \$3000, \$60, \$700.

11. State the short method of dividing by $33\frac{1}{3}$. (Take $\frac{3}{100}$.)

12. Divide by $33\frac{1}{3}$: 300, \$200, \$5, \$10, \$60,000, \$21,000.

13. In a certain year Delaware had 346.2 mi. of railroad, and the District of Columbia had $\frac{1}{10}$ as many miles. How many miles had the District of Columbia?

14. In a certain year the United States had 231,179 mi. of railroad, and Ecuador had 0.001 as many. How many had Ecuador?

Short Methods in Division Summarized. From the work on page 245, from similar questions, and because division is the inverse of multiplication, we have the following:

To divide by 10, move the decimal point one place to the left, prefixing a zero if necessary.

Because $4.8 \times 10 = 48$,
we see that $48 \div 10 = 4.8$.

To divide by 100 or 1000, move the decimal point to the left two places or three places respectively, prefixing zeros if necessary.

Because $9.42 \times 1000 = 9420$,
we see that $9420 \div 1000 = 9.42$.
Because $0.084 \times 1000 = 84$,
we see that $84 \div 1000 = 0.084$.

To divide by 5, multiply by 2 and divide by 10.

To multiply by 5 we take $\frac{10}{2}$; so to divide by 5 we take $\frac{2}{10}$.

To divide by 25, multiply by 4 and divide by 100.

To multiply by 25 we take $\frac{100}{4}$; so to divide by 25 we take $\frac{4}{100}$.

To divide by 125, multiply by 8 and divide by 1000.

To multiply by 125 we take $\frac{1000}{8}$; so to divide by 125 we take $\frac{8}{1000}$.

To divide by $12\frac{1}{2}$, multiply by 8 and divide by 100.

To multiply by $12\frac{1}{2}$ we take $\frac{100}{8}$; so to divide by $12\frac{1}{2}$ we take $\frac{8}{100}$.

To divide by $33\frac{1}{3}$, multiply by 3 and divide by 100.

To multiply by $33\frac{1}{3}$ we take $\frac{100}{3}$; so to divide by $33\frac{1}{3}$ we take $\frac{3}{100}$.

To divide by $66\frac{2}{3}$, multiply by 3 and divide by 200.

To multiply by $66\frac{2}{3}$, we take $\frac{200}{3}$; so to divide by $66\frac{2}{3}$ we take $\frac{3}{200}$.

To divide by $16\frac{2}{3}$, multiply by 6 and divide by 100.

To multiply by $16\frac{2}{3}$ we take $\frac{100}{6}$; so to divide by $16\frac{2}{3}$ we take $\frac{6}{100}$.

SHORT METHODS IN DIVISION

Divide by short methods, writing only the answers:

1. $8745 \div 5$.
2. $2675 \div 25$.
3. $8725 \div 12\frac{1}{2}$.
4. $3678 \div 33\frac{1}{3}$.
5. $4375 \div 125$.
6. $7386 \div 66\frac{2}{3}$.
7. $6789 \div 16\frac{2}{3}$.
8. $3445 \div 33\frac{1}{3}$.
9. $8929 \div 12\frac{1}{2}$.
10. $5378 \div 12\frac{1}{2}$.
11. $4986 \div 16\frac{2}{3}$.
12. $4169 \div 66\frac{2}{3}$.
13. $7872 \div 33\frac{1}{3}$.
14. $47{,}625 \div 125$.
15. $89{,}500 \div 125$.

16. Divide by 5: 125; 685; \$475; \$37,295; \$975,265.

17. Divide by 25: 750; 825; \$475; 62,750; \$389,000.

18. Divide by 125: \$250,125; \$375,375; \$276,525; \$452,625.

19. At 25¢ a yard, how many yards of ribbon can be bought for \$34.25? for \$28.25? for \$76.75? for \$92.50?

20. Divide 452,346,125 by 125 and also by 25, using long division and also the short method. Time each method.

21. Divide by $12\frac{1}{2}$: 75; 175; \$650; \$126; \$7250.

22. Divide by $33\frac{1}{3}$: 87; 251; \$725; \$435; \$2175.

23. Divide by $66\frac{2}{3}$: 68; 386; \$374; \$428; \$12,750.

24. Divide by $16\frac{2}{3}$: 49; 984; \$738; \$566; \$13,825.

25. At \$12.50 each, how many chairs can be bought for \$225?

26. At \$125 each, how many mules can be bought for \$2125?

27. Give a short method of dividing by $37\frac{1}{2}$ and illustrate the method by dividing 2625 by $37\frac{1}{2}$.

Remember that $37\frac{1}{2} = \frac{3}{8}$ of 100.

28. Give a short method of dividing by $62\frac{1}{2}$ and illustrate the method by dividing 11,875 by $62\frac{1}{2}$.

29. Give a short method of dividing by $87\frac{1}{2}$ and illustrate the method by dividing 20,125 by $87\frac{1}{2}$.

Comparing Numbers. Comparing the numbers in a problem will often lead to a simple solution. For example:

1. If 6 T. of hay cost \$96, how much will 18 T. cost?

Since 18 is 3 times 6, we see that the cost of 18 T. of hay will be $3 \times \$96$, or \$288.

2. At \$18.50 a ton, how much will 17,500 lb. of hay cost?

Since 2000 lb. cost \$18.50, we see that 1 lb. costs $\frac{1}{2000}$ of \$18.50, and 17,500 lb. cost $\frac{17500 \times \$18.50}{2000}$, or \$161.88.

$$\frac{8.75}{\cancel{17500} \times \$18.50}{\cancel{2000}} = \$161.875$$

From solving Ex. 2 we have the following convenient rule:

Given the price per ton, to find the cost of a given number of pounds; point off three figures from the number of pounds, divide by 2, and multiply the price per ton by the quotient.

COMPARING NUMBERS

1. If 8 T. of coal cost \$84, how much will 16 T. cost? 32 T.?
2. If 3 T. of hay cost \$45, how much will 27 T. cost? 48 T.?

Find the cost of the following at the given prices per ton:

3. 22,500 lb. of coal @ \$13.20.
4. 8400 lb. of coal @ \$9; @ \$9.50; @ \$11.20.
5. 38,100 lb. of coal @ \$9.50; @ \$9.75; @ \$11.45.
6. 94,300 lb. of coal @ \$9.25; @ \$9.90; @ \$12.20.
7. 127,500 lb. of coal @ \$8.75; @ \$9.30; @ \$11.75.
8. 242,600 lb. of coal @ \$9.15; @ \$8.75; @ \$10.25.

Coal is usually billed by the pound in this way.

Calculating by Hundreds, Thousands, and Dozens. In buying some kinds of goods it is the custom to calculate by the hundred (C), by the thousand (M), or by the dozen. If you were going to the grocery for eggs, would you ask how much ten, thirteen, or seventeen eggs cost? How would you ask the price? Here are the ways in which some things are sold at retail:

Oranges are sold by the dozen or by the box.

Bananas are sold by the dozen, or by the bunch, or frequently "3 for 10," that is, 3 bananas for 10¢.

Envelopes are sold by the package of 25, by the hundred, or by the box of 500.

Coal is sold by the ton of 2000 lb. or by the sack.

Lumber is sold by the thousand feet.

The method of measuring lumber need not be considered at this time.

USING WHAT YOU HAVE LEARNED

All work oral

1. At 60¢ a dozen, how much will $1\frac{1}{2}$ doz. oranges cost?
2. At 30¢ a dozen, how much will $2\frac{1}{2}$ doz. bananas cost?
3. At 50¢ a dozen, how much will 6 doz. eggs cost?
4. At \$12 per M, how much will 12 M bricks cost?
5. At 60¢ per C, how much will 12 C envelopes cost?
6. If 7 M bricks cost \$70, how much will 14 M cost?
7. At 55¢ per 500, how much will 2500 sheets of paper cost?
8. If 6 C sheets of blotting paper cost \$20, what is the cost of 36 C sheets? of 42 C sheets? of 72 C sheets?
9. If 2 M feet of finest oak cost \$400, what is the cost of 8 M feet? of 14 M feet? of 52 M feet?

CALCULATING BY LARGE UNITS

1. At 75¢ a dozen, find the cost of $4\frac{1}{4}$ doz. oranges.

The grocer will often consider as a whole cent any fraction of a cent that is due him, but in this book the rule given on page 90 is followed. The custom varies in different places.

2. At \$9.75 per M, how much will $5\frac{1}{2}$ M bricks cost?

3. If 7 M ft. of lumber cost \$630, how much will a dealer have to pay for 28 M ft.?

Have the pupils solve this problem by one multiplication.

4. If the freight on 9 M lb. of goods is \$18.70, how much will it be on 27 M lb. at the same rate?

5. Find the cost of 4500 bricks at \$9.80 per M.

In this problem have the pupil notice that 4500 = 4.5 M.

6. A dealer buys 7500 envelopes at \$6.80 per M and sells them at 90¢ per C. How much did he gain?

7. At \$11 per ton, find the cost of 5000 lb. of coal.

8. At \$12.20 per ton, find the cost of 8400 lb. of coal.

9. If blotting paper is sold at \$34.50 per M sheets, how much must a dealer pay for 500 sheets?

10. At \$1.20 per C lb., how much is the express charge on 1250 lb. of goods?

11. A man paid \$29.40 for $3\frac{1}{2}$ M bricks. At the same rate, how much must he pay for $10\frac{1}{2}$ M bricks?

12. A dealer can buy pasteboard boxes for his goods at \$32.50 per M. How much is this per C? per dozen?

13. At \$158.40 per gross, how much must a dealer pay for 6 doz. knives?

14. A shipper paid \$97.68 freight on some goods at 66¢ per 1000 lb. How many tons were there?

Problems in Profit and Loss. In business one of the merchant's chief problems relates to the profit on his goods.

For example, a merchant bought 960 yd. of linen lawn at a total cost of $\$1.12\frac{1}{2}$ a yard, including freight and all expenses of sale. He sold 738 yd. at $\$1.35$ a yard, and the rest at a bargain sale at $95¢$ a yard. Find his profit.

960	738	960	222	996.30
$1\frac{1}{8}$	1.35	738	0.95	210.90
120	3690	222	1110	1207.20
960	2214		1998	1080.
1080, cost	738		210.90	127.20
	996.30			

The merchant will probably not use the dollar signs, and he will not multiply $\$1.12\frac{1}{2}$ by 960, but will multiply 960 by $1\frac{1}{8}$. The full written analysis is as follows:

$960 \times \$1.12\frac{1}{2} = \$1080$, the cost.
$738 \times \$1.35 = \$996.30$, received at regular sale.
960 yd. $-$ 738 yd. $=$ 222 yd., sold at bargain sale.
$222 \times \$0.95 = \$210.90$, received at bargain sale.
$\$996.30 + \$210.90 = \$1207.20$, the total receipts.
$\$1207.20 - \$1080 = \$127.20$, the profit.

Such charges as rent, insurance, light, bookkeeping, and interest on the money invested are called *overhead charges* or simply *overhead*. The merchant must also consider other charges such as the cost of selling the goods, but for the present we shall speak of overhead as covering all expenses besides the cost.

In Book Three these technicalities will be considered in detail.

PROFIT AND LOSS

1. A dealer buys 75 sewing machines at \$68 apiece and sells them at \$82 apiece. He pays \$44.75 freight, and his other overhead charges are \$2.40 per machine. Find his profit.

2. A merchant buys 840 yd. of muslin at 28¢ a yard and sells it at the rate of a dozen yards for \$4. His overhead charges are $1\frac{3}{4}$¢ per yard. How much is his profit or loss?

3. A merchant buys 12 gross of spools of silk at \$9.76 a gross. He sells the spools at \$1.20 a dozen. The overhead charges being computed at 0.1¢ per spool, how much is his profit?

4. If a dealer buys 740 yd. of scrim for \$400 and sells three fourths of it at 45¢ a yard and the rest at 30¢ a yard, how much is his profit or loss, not counting overhead charges?

5. A dealer buys 380 reams of paper at \$2.22$\frac{1}{2}$. He sells 90 reams, which were damaged, at 85¢, and the rest at \$2.65. Find his profit or loss, not counting overhead charges.

6. A dealer buys 720 yd. of cloth at \$1.36 a yard and thinks he should make a profit of about \$90 on the lot. Find to the next higher cent the price per yard which he must ask for the cloth, not counting overhead charges.

7. A dealer buys 1600 yd. of Victoria lawn for \$600. If the overhead charges are \$40, for how much per yard must he sell the lawn to make a profit of $3\frac{5}{8}$¢ a yard? He sees that this is an awkward price and so sells the cloth at 45¢ a yard. Find his total profit and the profit per yard.

8. A dry-goods dealer buys 840 yd. of percaline at 38¢ a yard and estimates the overhead charges at $1\frac{5}{8}$¢ a yard. He sells 500 yd. at 45¢ a yard, 120 yd. at 46¢ a yard, and the rest at 18¢ a yard. How much is his profit or loss?

USING WHAT YOU HAVE LEARNED

1. This girl bought for her mother 9 yd. of silk at $2.35 a yard and 8 yd. of muslin at $1.60. Find the cost.

2. This girl buys 5 yd. of serge at $2.75 a yard. How much does it cost? How much change should she receive from $15?

3. If the girl buys at another time toweling for $1.20, muslin for $1.50, needles for 85¢, thread for 18¢, beads for 35¢, and braid for 34¢, how much change should she receive from $5?

4. The girl saw a woman buy 13 yd. of broadcloth and pay $68.25 for it. How much did it cost a yard?

5. If the merchant charges $69.30 for 21 yd. of velvet, how much does he charge a yard?

6. If the merchant sells tweeds at $4.25 a yard, how much will he receive for 19 yd.? how much for 46 yd.?

7. If a woman buys 27 yd. of silk at $2.45, 3 yd. of velvet at $2.75, and 22 yd. of serge at $2.60, how much is her bill?

8. A man pays $4.48 for 14 lb. of meat. How much does it cost him a pound?

9. A merchant receives $30.60 for some velvet at $3.60 a yard. How many yards does he sell?

Stating a Problem in One Step. We have often seen that it is a good plan to indicate the solution of a problem in a single step, so as to shorten the work by canceling. For example:

Soldiers marching quickstep take 120 steps a minute, averaging 30 in. each. At this rate how many miles will a company of soldiers march in an hour?

In an hour the company will march $60 \times 120 \times 30$ in. To express this result in miles we must divide by the number of inches in a mile, which is 5280×12 in.

We therefore have

$$\frac{60 \times 120 \times 30}{5280 \times 12}.$$

We may now cancel and thus complete the solution more easily than by multiplying step by step.

STATING A PROBLEM IN ONE STEP

1. Finish the solution of the above problem.

2. When sound travels 1125 ft. per second, how many miles away is a lightning flash which is seen 13.2 sec. before the thunder is heard?

3. If 1 cu. ft. of water weighs $62\frac{1}{2}$ lb., and gold is 19.5 times as heavy as water, how much is 1 cu. ft. of gold worth at \$18.60 an ounce avoirdupois?

4. If the circumference of a wheel is $3\frac{1}{7}$ times the diameter, and the drive wheel of a locomotive is 6 ft. in diameter, how many revolutions will it make in going 143 mi.?

5. What is the expense of covering a floor with plain matting 1 yd. wide, the room being 18 ft. by $21\frac{1}{2}$ ft., and the matting being worth 30¢ a yard, allowing 6 in. extra on each strip?

USING WHAT YOU HAVE LEARNED

1. If 475 horse power from several engines costs $13,490 a year, what will 60 horse power cost at the same rate?

2. A dealer bought 25 bu. of grass seed at $2.90 a bushel. He paid $1.20 for paper bags, and sold the seed in bags at 10¢ a quart. How much did he gain?

3. Two steamers started for Europe on the same route at the same time. After 4 da. one had averaged 448.7 mi. a day, and the other 502.1 mi. a day. How far apart were they?

4. A box of arithmetics cost a dealer $53.94, including $1.48 freight. He sold them for 50¢ each, thus gaining 7¢ on the price the publisher charged him. How many did he buy?

5. A man bought two adjacent farms containing the same number of acres. For one he paid $95 per acre, and for the other $130 per acre. The two farms cost him $31,950. How many acres did he buy?

6. If it costs $702 to excavate a cellar $27' \times 39' \times 9'$, how much will it cost to excavate another cellar $30' \times 36' \times 12'$ at the same rate?

7. If 100 lb. of bituminous (soft) coal contain 2.32 lb. of ash and 76.05 lb. of carbon, how many pounds of each in 15 T.?

8. From Ex. 7 find the number of pounds of ash and the number of pounds of carbon in $8\frac{1}{2}$ T. of coal.

9. If 2000 lb. of anthracite (hard coal) contain 1828 lb. of carbon, how much carbon will 100 lb. contain? What is the per cent of carbon?

10. A gallon contains 231 cu. in. Alcohol is 0.84 as heavy as water. Find the number of cubic inches of alcohol that will weigh as much as a gallon of water.

USING WHAT YOU HAVE LEARNED

1. Mr. Wheeler works six full days each week and earns $31.50 per week. Last month he was ill and lost 9 da. How much did he lose in wages?

2. A grocer charges 14¢ a pound for sugar and 60¢ a dozen for eggs. At a special sale one of his customers bought 6 lb. of sugar for 75¢ and 2½ doz. eggs for $1.25. How much did the customer save in buying at the special sale?

3. At the rate of three for $1, find the cost of 5 collars.

4. A family requires 3½ qt. of milk and 1 pt. of cream daily. If they pay 14¢ a quart for the milk and 20¢ a half pint for the cream, how much do they pay for milk and cream in July?

5. A family of four go to the movies on an average of two nights a week. If the tickets are 20¢ each, how much do they cost in a year? If the house rent of this family is $38 a month, the money spent for movies would pay the rent for how many months?

6. Mr. Howard lives 140 mi. from Chicago, and he decides to make a trip there and back in his automobile. The price of gasoline is 27¢ a gallon and his car will make 10 mi. on a gallon. On the trip he also pays 38¢ for oil. Find the cost of gasoline and oil for the trip to Chicago and back.

7. A farmer who planted 40 acres of corn found that by carefully selecting the seed corn he could have raised 45 bu. instead of 30 bu. per acre. If he had done this and had sold the excess corn raised at $1.10 a bushel, he could have bought with the money some more stock that he wanted. If the stock costs $70 a head, how many head could he have bought, and how much money would he have left over?

USING WHAT YOU HAVE LEARNED

1. It is estimated that each family in this country wastes about $7\frac{1}{2}\%$ of the total amount that it spends for food. Allowing an average of \$470 per year for the food of each of the 24,000,000 families in the country, find the approximate value of the food wasted per year in the United States.

2. From Ex. 1 find the average amount of money wasted per family on the expenditure for food and make out a list of articles that a family might profitably buy with this money.

3. Allowing $4\frac{1}{2}$ bu. of wheat for making a barrel of flour, how much does the wheat cost for 75 barrels of flour, the wheat being worth \$2.10 a bushel?

4. If 15 bu. of wheat are grown to an acre, and $4\frac{1}{2}$ bu. are needed for making a barrel of flour, how many acres are needed to produce enough wheat for 100 barrels of flour?

5. Allowing 12 oz. of wheat flour to a loaf of bread, how many loaves can be made from a barrel of flour, the weight of a barrel of flour being 196 lb.?

6. If 1 bu. of wheat weighs 60 lb., and $4\frac{1}{2}$ bu. are needed for a barrel of flour weighing 196 lb., what fractional part of wheat actually goes into flour?

The result may be stated as a common fraction reduced to its lowest terms, as on page 54, or it may better be expressed as a decimal fraction to the nearest hundredth.

7. During the World War our people made every effort to increase the production of our farms so as to help the starving inhabitants of the countries abroad. In one year, by attending carefully to the potato crop, the crop increased 62% over that of the previous year. If it was 285,000,000 bu. the previous year, how much was it after the increase?

III. LITTLE EXAMINATIONS

i.

1. $47\frac{1}{2}$ ft. = (?) in.
2. $7\frac{1}{4}$ pk. = (?) qt.
3. $42\frac{1}{2}$ gal. = (?) qt.
4. $3\frac{1}{3}$ hr. = (?) min.
5. 5275 × \$5276.45.
6. \$329.75 ÷ 25.
7. 4876 × \$328.72.
8. 875 × \$8642.75.

ii.

1. Find the area of a rectangle 38 ft. 6 in. by 42 ft. 6 in.
2. Find the area of a triangle with base 18 ft. and height 9 ft.
3. Find the area of a parallelogram with base 9 ft. 6 in. and height 5 ft. 2 in.
4. Find the area of a trapezoid with parallel sides 5 ft. 6 in. and 4 ft. 4 in., and height 3 ft. 6 in.
5. Find the volume of a rectangular solid 3 ft. 3 in. long, 2 ft. 8 in. wide, and 18 in. deep.

iii.

1. Find the area of a triangle with base 224 ft. 8 in. and height 111 ft. 10 in.
2. The triangular gable of a house has a base 42 ft. and a height 14 ft. 8 in. Find the area of the gable.
3. How many cubic yards must be removed in digging a tunnel 468 ft. long, 21 ft. wide, and 18 ft. 6 in. high?
4. If 40 mules cost \$5000, how much will 60 mules cost?
5. At \$120 for $\frac{3}{4}$ A., how much will 1 A. of land cost? How much will 3 A. of land cost?

iv.

1. $61\frac{3}{8}$ in. + $3\frac{1}{2}$ in.
2. $7\frac{3}{8}$ in. − $5\frac{3}{4}$ in.
3. $8\frac{3}{4} \times 9\frac{1}{2}$ in.
4. $7\frac{7}{8}$ ft. ÷ $4\frac{7}{16}$ ft.
5. $5.75 \div 2\frac{1}{2}$.
6. 38% of \$9.60.
7. \$5.25 ÷ $1\frac{1}{4}$.
8. $\frac{2}{5}$ of $13\frac{1}{2}$ ft.
9. $\frac{7}{8}$ of 17 ft. 4 in.
10. 75% of 14 lb. 4 oz.

IV. REVIEW AND DRILL

MINIMUM ESSENTIALS

1. Write in our common numerals the number eighty million, three hundred seventy-seven thousand, two hundred nine.

2. Write in our common numerals the number ninety-two and forty-four thousandths.

Add the following:

3.	4.	5.	6.	7.
\$7.98	\$4.62	\$8.43	\$3.89	\$9.48
6.85	3.46	4.84	4.94	8.21
4.79	1.34	7.06	7.43	6.32
5.68	3.62	8.06	8.52	5.87
7.52	9.75	9.54	9.48	6.78
9.48	8.25	8.46	4.87	4.22
8.72	3.38	9.50	5.96	8.95
6.55	4.42	8.72	8.96	2.05

8. From the sum of \$98.86 and \$74.69 subtract \$109.57.

9. Multiply 48.756 by 0.078.

10. From the sum of \$95.42 and \$87.69 subtract \$129.78.

11. Divide 37.84 by 6.93, carrying the result to two decimal places; that is, to the nearest hundredth.

Perform the following operations:

12. $\frac{2}{3}+\frac{1}{2}$.	17. $\frac{3}{16}+\frac{1}{2}$.	22. $1\frac{1}{2}+2\frac{7}{8}$.	27. 0.7×1.2.
13. $\frac{3}{4}-\frac{3}{8}$.	18. $\frac{9}{16}-\frac{1}{2}$.	23. $3\frac{1}{3}+2\frac{1}{2}$.	28. 1.7% of 380.
14. $\frac{7}{8}+\frac{3}{4}$.	19. $\frac{1}{2}$ of $\frac{9}{16}$.	24. $6\frac{1}{4}-3\frac{7}{8}$.	29. 2.3% of 9.25.
15. $\frac{7}{8}-\frac{1}{4}$.	20. $\frac{7}{16}+\frac{1}{2}$.	25. $7\frac{2}{3}-2\frac{1}{2}$.	30. 0.6% of 750.
16. $\frac{1}{2}-\frac{1}{3}$.	21. $\frac{9}{16}+\frac{3}{4}$.	26. $2\frac{1}{2} \times 2\frac{1}{2}$.	31. $1.44 \div 1.2$.

PROBLEMS WITHOUT NUMBERS

All work oral

1. I have measured in feet the length, width, and height of this room. How do I find the number of square feet in the walls and ceiling, including all openings?

2. I have measured in feet the length and width of a room. How do I find the number of square yards of linoleum needed to cover the floor?

3. I know the dimensions of this room in feet and the dimensions of all the doors and windows. How do I find the number of square yards of plastering needed for the walls and ceiling, allowing for the doors and windows?

4. How do you find the number of square feet of flooring needed to cover the floor of your schoolroom?

5. How do you find the cost of a sidewalk to be laid in front of the schoolhouse?

6. If you know what an automobile dealer pays for a car and the per cent above cost at which he sells it, how do you find the selling price?

7. If you know the three dimensions of this room in feet and inches, how do you find the number of cubic feet in the room?

8. If you were asked to find how many yards of carpet were needed for your sitting room or stairs, how would you find it?

9. If you knew or were told the number of cubic inches in a bushel, how would you find, by first measuring the dimensions, the number of bushels of wheat that could be put into a bin?

10. If you know the cost of a certain number of articles and wish to find the cost of a certain other number, how do you do this in one step?

PROBLEMS FOR COMPLETION

1. In a recent year the average consumption of meat for each person in this country was as follows: beef, 82.7 lb.; mutton, 7 lb.; pork, 103.5 lb.; other kinds of meat, 0.3 lb. Complete the problem and solve it.

As stated on page 70, the problem may be completed in more than one way. Any reasonable problem, if correctly solved, should be accepted.

Complete and solve the following problems:

2. Butter costs 64¢ a pound and a family uses 4 oz. a day.

3. A woman who used gas for cooking bought a fireless cooker and found that her gas bills, which had been \$3.85, \$3.80, \$3.75, \$3.20, and \$2.90 for the first five months of last year, were now \$3.10, \$3, \$2.90, \$2.40, and \$2.10 for the first five months of this year.

4. Fred ran a thrift garden. He raised tomatoes and sold 150 of them at $2\frac{1}{2}$¢ each, and also sold twice as many at 2¢ each.

5. John bought a pig for \$5.75. He fattened the pig, spending \$16.75 for feed. When the pig weighed 240 lb., John sold it at 19¢ a pound.

6. A man works in a machine shop and earns \$36 a week. He pays \$10 a week for his board and room. His other expenses amount to 20% of his earnings.

7. Mary's recipe for making molasses candy called for 2 cups of molasses, 1 tablespoon of vinegar, 2 tablespoons of butter, $\frac{1}{2}$ teaspoon of baking powder, 1 teaspoon of ginger extract. This was enough to make 1 lb. of candy. She wished to make 3 lb. of candy for a church sale.

Teachers should read the note after Ex. 1 on page 70 and also the one at the foot of that page.

ALERTNESS TEST

In the three lines *a*, *b*, *c*, given below, a certain relation exists between the first two words, and a line is drawn under one of the words in the parentheses that has the nearest similar relation to the third word. If the teacher writes on the blackboard ten such sets of words, copy the words, notice that relation, and draw a line under the one word in the parentheses that has the nearest similar relation to the third word. You will be allowed 5 min. in which to copy the words and draw the lines.

a. Addition, sum; subtraction, (product, difference, quotient).

b. Principal, interest; selling price, (commission, time, loss).

c. Cost, profit; selling price, (loss, brokerage, gain).

This is a specimen of the general-intelligence or alertness tests commonly given in such examinations. It may be used or not at the discretion of the teacher. Such general alertness tests are now given in many schools. It is, of course, undesirable to print the sets of words that are to be given to the class, for then the pupils would be studying them in advance. The sets should be written by the teacher on the blackboard and covered with a map. They should be uncovered when the class is ready to begin, and no more than 5 min. should be allowed for ten sets. Mark each paper by the number of rights, and grade each pupil by this number. Keep the record and compare it with the records secured in later tests.

The following are typical sets:

1. Addition, addend; multiplication, (product, subtrahend, multiplicand).
2. Fraction, numerator; quotient, (divisor, dividend, answer).
3. Foot, inch; twenty-four, (three, seven, two).
4. Dividend, divisor; numerator, (fraction, quotient, denominator).
5. Subtrahend, difference; multiplier, (multiplicand, product, number).
6. Product, multiplier; difference, (subtrahend, subtraction, remainder).
7. Hundred, ten; thousand, (unit, ten thousand, hundred).
8. Half, six; three, (ten, thirty-six, forty-eight).
9. Zero, ten; one, (twelve, eleven, ten).
10. Hundredth, one; fiftieth, (half, two, fourth).

Industrial Problems

SCHOOL POULTRY CLUB

1. Willard bought 3 geese at \$2 each and paid \$3.50 for feed. At the end of the season he had raised 13 young geese which he sold to the butcher for \$1.50 apiece. Find his profit.

2. Albert belonged to a poultry club, and at the end of a year he was able to report that he had received from eggs and poultry sold a total of \$161.30, while he had paid for feed and other items \$37.24. Find his profit.

3. In Ex. 2, Albert set 260 eggs and hatched 85% of them. How many chickens hatched?

4. Albert sold 134 of his chickens for a total of \$57.62. Find the average price received per chicken.

5. Albert's hens laid 699 eggs in April, 544 in May, 522 in June, 425 in July, 290 in August, and 112 in September. How many dozen eggs did his hens lay in the 6 mo.?

6. In Ex. 5, Albert sold the eggs at an average price of 48¢ a dozen. How much did he receive from the sale of his eggs?

7. In School District No. 12 there was a poultry club which reported that its members had sold 1345 eggs in March, 1209 in April, and 1142 in May, and that the average price received was $42\frac{1}{2}$¢ a dozen. How much was received from the sale of eggs in these three months?

8. In Ex. 7, if there were 77 hens laying eggs, what was the average number of eggs laid per hen during this period?

9. In School District No. 9 the poultry club's records showed 1276 eggs sold in March, 1140 in April, and 1070 in May, and that the average price received was $44\frac{1}{2}$¢ a dozen. How much was received from the sale of eggs in these three months?

FEEDING PIGS

1. Twenty pigs averaging 55 lb. each were fed in a dry lot until they weighed on the average 155 lb. each. For each 100 lb. increase in weight the average consumption of feed was 354 lb. of shelled corn worth $1.12 per bushel of 56 lb., and 51 lb. of tankage worth 5¢ per pound. Find the cost of the feed for all the pigs.

2. The 20 pigs in Ex. 1 were sold at $10 per hundredweight. If the cost of labor and the original cost of the pigs were not considered, find the total profit.

3. Twenty pigs averaging 55 lb. each, running on 3 A. of alfalfa, were fed in a self-feeder until they averaged 205 lb. each. For each 100 lb. increase in weight the average consumption of feed was 343 lb. of shelled corn worth 2¢ a pound, and 32 lb. of tankage worth 5¢ per pound. Not counting the alfalfa, what was the cost per pig of the feed consumed?

4. If the pigs in Ex. 3 were sold at $12 per hundredweight, the receipts from each pig were how much more than the cost of the corn and tankage required to fatten it?

5. From the 3 A. of alfalfa upon which the pigs in Ex. 3 grazed, there were taken 3 T. of alfalfa per acre, worth $15 per ton; while from 3 A. where no pigs had grazed 4 T. per acre were taken. How much more should be charged to the cost of feeding each pig in Ex. 3 for the alfalfa?

6. Considering now the cost of the alfalfa, how much more was gained on the 20 pigs grazed on alfalfa as in Exs. 3 and 4 than on those fed in a dry lot as in Exs. 1 and 2?

As is always the case with problems relating to a particular industry, the teacher should omit all that do not relate to the needs of the pupil. In a city school, most or all of pages 263–272 should be omitted.

7. Twenty pigs averaging 55 lb. each were pastured on 2 A. of blue grass at a total cost of $17.50 per acre. After a certain period their average weight was 225 lb. For each 100 lb. increase in weight the average consumption of feed was 350 lb. of shelled corn worth $1.12 a bushel of 56 lb., and 50 lb. of tankage worth 4¢ a pound. The pigs were then sold at $10 per hundredweight. The receipts were how much more than the cost of feeding?

8. A farmer fattened 36 hogs by turning them into a field of 9 A. of corn in which he had also planted soy beans, rape, and rye. He also fed 40 lb. of tankage, worth 5¢ a pound, to every 100 lb. gained in weight. The hogs gained on an average 150 lb. each, and were sold for $10 per hundredweight. How much did the farmer realize per acre for the corn?

9. From a field of 10 A. of alfalfa three cuttings were made, averaging $1\frac{1}{6}$ T. per acre for each cutting. At $15 per ton, how much was the value of the crop for the year?

10. From the field mentioned in Ex. 9 two crops of alfalfa were cut the next year, averaging $1\frac{1}{6}$ T. per acre for each cutting, and a third crop was cut, averaging only $\frac{1}{2}$ T. per acre. The reason for the small size of the third crop was that some hogs were turned in after the second crop was cut. If the year's crop of alfalfa sold at $15 per ton, how much should be charged for hog pasture in order to make the value of the crop equal to that of the first year, as described in Ex. 9?

11. In a field of 10 A. of corn and 10 A. of alfalfa 60 hogs were pastured for 2 mo., each hog gaining on the average 120 lb. The hogs were also fed on tankage worth 5¢ a pound at the rate of 20 lb. to every 100 lb. of weight gained. If the farmer allowed $15 an acre for the alfalfa, and sold the hogs at $10 per hundredweight, how much did he realize per acre for the corn?

HERBERT'S FARM ACCOUNT

1. Below is given the story of Herbert Koelper, a real boy, told substantially in his own way except that some items in the accounts are omitted, their places being marked by stars. Fill the places which are marked by the stars.

Last year I joined the garden club. I bought 25 lb. of onion-set seed, and then I rented 69 sq. rd. of land from my grandfather. My uncle sowed my onion seed for me. My mother and grandfather instructed me in onion culture and advised me as to some of the latest and best methods of raising sets. I followed their advice, and my crop turned out to be very nice. The profit which I have made on my crop I am going to give to my mother, and she says that she is going to use the money for music lessons for me and my sister.

Following is the itemized account of my receipts and expenditures. The items are all of this year except the first two expenditures, which were made in November of last year.

The itemized account appears on page 267.

The problem is given for the purpose of serving as a good model for a project to be carried out by a pupil during a year. Such a project is real to the one who carries it out or to those who know the actual circumstances. It is of little value to a class in a city school, but the local conditions in a city frequently allow of the use of projects that represent just as real conditions as are here set forth. It should be observed that the actual arithmetic involved takes but a short time, and the teacher should continually be on guard not to use more time than is reasonable in such work. In order to increase the amount of computation required in a project of this sort, Ex. 2 on page 267 is included as a suggestion of the type of problem which may be made up and solved from the data given. The chief danger of such projects is that they consume a great deal of time in teaching facts that relate to other things than arithmetic, leaving the pupils unable to perform ordinary computations with accuracy and reasonable speed.

Receipts

Sept. 24.	6466 lb. onion sets at 5½¢	$***
Oct. 1.	587 lb. over runs at 2¼¢	13.21
	Total	$***

Expenditures

Nov. 2.	Manure and hauling same	$9.00
Nov. 4.	Plowing	1.80
Feb. 17.	25 lb. onion seed at 75¢	***
April 10.	Preparing land for seeding	1.00
April 10.	Sowing 25 lb. onion seed	.65
May 14.	Wheel-hoeing, 4½ hr. at 15¢	***
May 18.	Weeding, 20 hr. at 15¢	***
May 28.	Wheel-hoeing, 4½ hr. at 15¢	***
June 7.	Weeding, 20 hr. at 15¢	***
Aug. 6.	Loosening sets with harvester	.75
Aug. 6.	Harvesting 485 baskets at 5¢	***
Aug. 6.	5 men assisting at harvesting, stacking, and covering onion crates, 25 hr. at 40¢	***
Aug. 6.	Tar paper to cover stacks	.59
Aug. 6.	26 lunches at 25¢	***
Aug. 6.	Rent of 114 onion crates at 7¢	***
Sept. 19.	Grading 192 bu. onion sets at 3¢	***
Sept. 19.	Marketing	7.00
Oct. 1.	Rent of land	7.50
	Total	***
	Balance	***

2. If Herbert had reckoned his time at twice as much on May 14, May 18, May 28, and June 7, how much in all would he have charged to expenses for these four days?

SCHOOL-AND-HOME GARDENING

1. In his garden plot Raymond planted 5 lb. of beans which cost him 12¢ a pound and 4 ears of popcorn which cost him 5¢ an ear. From the garden he sold 55 lb. of beans at 17¢ a pound and 2 bu. of popcorn at $2.35 a bushel. Find his total gain.

2. Ralph planted a piece of land to corn at an expense of $29. An early frost damaged the crop so that it sold for only $49.50, which was half of what Ralph expected it would bring. Ralph wanted to put $100 in the bank, and if he had secured a full crop how much would he have needed, in addition to his profit on the corn, to make up the $100?

3. Richard planted potatoes on 35½ sq. rd. of land which he had rented from his father for 20¢ a square rod. His father gave him the seed potatoes and charged him $1.25 for plowing the ground, Richard doing the rest of the work. If Richard sold 37 bu. of potatoes at $1.30 a bushel, how much did he make?

4. In January Arnold paid $5 for some hens, and by August he had raised 41 chickens that his hens had hatched from their own eggs. He sold the chickens at $1.50 each, and before he set the hens he had sold 10 doz. eggs at 60¢ a dozen. He found that total expenses for care and feed were $25.50. How much was his gain?

5. Oscar planted onions in his home garden, paid out $4.15 for expenses, and sold the entire yield for $52.25. With the profits he bought a bicycle for $20 and 2 pigs at $8 each. How much money had he left?

6. Harry had a plot of ground 22 ft. wide and 275 ft. long. On this plot he raised 20 bu. of potatoes at a total expense of $3.85, and he sold the potatoes at $1.75 a bushel. Find his profit, the yield per acre, and the profit per acre at this rate.

7. Mr. Miller gave his son John the use of a piece of land and told John that he could have what he could make on it. John raised 44 bu. of potatoes which he sold at \$1.45 a bushel and 90 bags of onions which he sold at \$1.40 a bag. If his profits were \$156.20, how much were his total expenses?

8. Clement rented a piece of land from his father, planting it to potatoes and sweet corn. In July he sold 5 bu. of early potatoes at \$2 a bushel, and in September he sold 28 bu. of later potatoes at \$1.75 a bushel. At one time he sold 120 ears of sweet corn at 20¢ a dozen, and at another time, 180 ears at 15¢ a dozen. The rest of his sales amounted to \$7.30. His expenses being \$7.45, how much was his profit?

9. Fred has made up these problems about his school garden: (1) My garden is 200 ft. long and 40 ft. wide. Find the perimeter in rods and the area in square rods. (2) I had 39 rows of beets and they yielded 4 sacks to a row, each sack containing 5 pk. I received 45¢ a bushel for the beets, and my expenses were as follows: sacks, $9\frac{1}{2}$¢ each; rent, \$1.75; fertilizer, \$1.50; plowing, \$1; seed, \$1.50; marketing, \$8.40. How much was my profit? (3) Last year I raised 12 bu. of potatoes and sold them at \$1.45 a bushel. My expenses in raising the potatoes being \$6.35, how much was my profit? Solve the three problems that Fred made up.

Problems such as those given on pages 263–272 are suggestive of certain project work which may occasionally be undertaken with profit. Such real projects as raising garden produce, fattening pigs, or running a poultry yard will not be within the means of most pupils, but imaginary projects also have their value. For example, a class might find the cost of furnishing a room, of keeping an automobile for a year, of maintaining a family of five persons for a year, of clothing themselves, and the like. While such work is often of little relative value because of the waste of the time involved, it may prove to be of much service if used with discretion.

A SCHOOL-GARDEN PROJECT

1. The teacher of a country school rented for a school-garden project a piece of land 20 rd. long and 16 rd. wide, paying a rental of $12.50 an acre. The length of the garden ran east and west. She divided the entire piece into 10 plots, beginning at the northwest corner, and assigned these plots as follows: to John, a strip 20 rd. long and 3 rd. wide; to William, Clarence, and Roger, each a strip 20 rd. long and 2 rd. wide; to Mary, Susan, Ella, and Ann, each a piece 5 rd. square; to Henry and Frank, each a strip 10 rd. long and 2 rd. wide. Draw a plan of the land with its divisions, using the scale of 1 in. to 4 rd.

2. The teacher paid $4.10 for plowing the whole piece and also paid for 5 loads of stable fertilizer at $2.50 a load. How much did the rent, plowing, and fertilizing cost?

3. From the result of Ex. 2, compute the share that each of the pupils should pay, considering the fractional part of the land that each uses.

That is, if John has $\frac{3}{16}$ of the land, he should pay $\frac{3}{16}$ of the expenses. If he does not make $\frac{3}{16}$ of the profits, that is his concern.

4. John planted his strip to early potatoes. He paid \$1.80 for seed potatoes and \$5.60 for marketing. He sold 18 sacks of potatoes at \$4.60 a sack. Deducting his share of the expenses as found in Ex. 3, find his profit.

Since his compensation is in the profit, no account is taken of the value of his labor, and so in all similar cases. In the examples following, deduct the share of the expenses as in Ex. 4.

5. William planted his strip to tomatoes. He paid \$16.55 for seed and for a hotbed and raised 108 boxes of tomatoes, which he sold at the garden at an average price of 90¢ a box. Find his profit.

6. Clarence raised onion sets on his strip, paying \$10.72 for seed and \$29.68 for marketing. He sold 2986 lb. of onion sets at $3\frac{1}{2}$¢ per pound. Find his profit.

7. Roger raised onions on his strip, paying \$1.50 for seed. He sold the onions at the garden, selling 98 bags at 50¢ a bag. Find his profit.

8. Henry raised celery on his strip, paying \$1.88 for seed and planting and \$24.25 for marketing. He sold 262 bunches of celery at 25¢ a bunch. Find his profit.

9. Frank raised lettuce and cabbage on his strip, paying \$4.80 for seeds and plants and \$13.20 for marketing. He sold 68 boxes of lettuce at 60¢ a box and 270 cabbages at 5¢ a head. Find his profit.

10. From the results found in Exs. 4–9, find the total profit made by the six boys.

11. Considering the results in Exs. 4 and 5, would John probably have done better if he had planted tomatoes as William did? Give the results on which you base your conclusion.

12. Mary raised onions, lettuce, and celery on her piece of land, paying \$1.60 for seed and \$11.58 for marketing. She sold 17 boxes of lettuce at 45¢ a box, 12 bags of onions at 50¢ a bag, and 246 bunches of celery at 20¢ a bunch. Find her profit.

13. Susan raised radishes, beans, and spinach on her piece of land, paying \$1 for seed and \$7.50 for marketing. She sold 18 boxes of beans at 60¢ a box, 2200 bunches of radishes at 1¢ a bunch, and 41 boxes of spinach at 55¢ a box. Find her profit.

14. Ella raised lettuce and cabbage on her piece of land, paying \$6 for seed and plants and \$14 for marketing. She sold 86 boxes of lettuce at 60¢ a box and 330 cabbages at 5¢ a head. Find her profit.

15. Ann raised celery on her piece of land, paying \$2.35 for seed and planting and \$17.80 for marketing. She sold 286 bunches of celery at 25¢ a bunch. Find her profit.

16. From the results found in Exs. 12–15, find the total profit made by the four girls.

17. When the year's project had been carried out, the question arose as to which had made the most money per square rod of land cultivated. Find this profit in each case to the nearest cent and determine who stood highest, who next, and so on.

18. The question also arose as to the total profit of the group of the ten children, and what this profit amounted to per acre. Find these two results.

It should be repeated that this project is merely a suggestion. A project is real when it is carried out by a school; a textbook can merely suggest. The cases given in this book are prepared from actual records, but of course the prices, the crops, and numerous other conditions vary, as do also the industries of the people, in various parts of the country.

PACKING ORANGES

1. In shipping oranges north, Florida growers put 144 oranges of a certain size in a standard box, but they put half as many more smaller oranges in the same size standard box. How many oranges of the smaller size do they put in the box?

2. In packing oranges for shipping, the packers put in each box 4 layers of a certain size of orange, there being 24 oranges in a layer. How many oranges are there in each box?

3. A shipping box is divided in the middle by a partition. In each half of the box, for a certain size of orange, the packers put in five layers, the first, third, and fifth layers having 18 oranges each, and the second and fourth layers having 17 oranges each. How many oranges do they put in the whole box?

4. For a small orange the packers can put in 6 layers, 21 in a layer, in each half of the box. How many oranges can they ship in such a box?

5. They can pack 28 large grapefruits, or pomelos, in a box, using 3 layers and having 5 pieces in the top layer and 5 in the bottom layer of each half of the box. How many pieces of fruit are there in the middle layer of each half of the box?

6. In shipping small China King oranges the packers put 144 oranges in each box, 18 in each layer in each half of the box. How many layers of oranges are there?

7. In shipping small limes the packers put 420 limes in each box, putting them in 6 layers. How many limes are there in each layer of each half of the box?

8. If a box 33 in. long, 12 in. wide, and 12 in. deep is divided crosswise by a partition $\frac{1}{2}$ in. thick, how many cubic inches are there in each half?

RAILROAD PROBLEMS

1. If railroad ties are 8″ wide and are placed 18″ apart, how many ties are there to a mile?

2. In a recent year our railroads used 98,000,000 new ties, averaging 8″ × 8′ 6″ × 6″. Find to the nearest cubic foot the amount of timber which they used for ties.

3. The modern coal car usually has a capacity of 100,000 lb. At 35 cu. ft. to the ton of 2000 lb., what is the volume?

4. If a rail is 30′ long and weighs 120 lb. to the yard, and if a man can carry 160 lb., how many men will it take to carry a rail?

5. The standard gauge of railroads in America and England is 4′ 8½″. Express this in meters, the meter being 39.37 in.

6. The standard American and English gauge (see Ex. 5) is $\frac{1}{5}$ wider than the gauge of a certain mountain road. What is the gauge of the latter to the nearest hundredth of a foot?

7. If the annual cost of maintaining the single track of a certain railroad averages $2870 a mile, what is the annual cost of maintaining its 378 mi. of double track?

8. A company ships oil in tank cars holding 1200 cu. ft. Allowing 7½ gal. to 1 cu. ft., how many gallons will such a car carry? How many gallons will 32 cars carry?

9. Allowing 1¼ cu. ft. to a bushel, how many bushels of wheat can be placed in a freight car 38 ft. long, 8 ft. 6 in. wide, if the depth of the wheat cannot exceed 5 ft.?

10. A hardware dealer shipped 16 T. of merchandise on which the freight rate charged by the railroad was 47½¢ per hundredweight. How much were the transportation charges?

11. Allowing 60 lb. to the bushel, find the railroad's charge for transporting 12,000 bu. of wheat at 33½¢ per hundredweight.

V. REVIEW EXAMINATIONS

ORAL EXAMINATION

1. State rapidly the following sums:

87	46	89	67	49	85	74
95	28	98	85	86	49	38

2. Make change from \$10, the purchases amounting to:

\$6.75 \$4.80 \$6.92 \$3.78 \$5.69 \$8.17

3. State the results of the following:

3×34	6×33	$294 \div 14$	$198 \div 18$
7×61	8×32	$330 \div 15$	$264 \div 12$

4. Express the following as common fractions:

20% 50% 90% $12\frac{1}{2}\%$ $33\frac{1}{3}\%$ $16\frac{2}{3}\%$

5. State the results of the following:

$12\frac{1}{2}\%$ of 88 $37\frac{1}{2}\%$ of 88 $16\frac{2}{3}\%$ of 24 $66\frac{2}{3}\%$ of 24

6. How many rods in a mile? square rods in an acre?

7. How many days from March 7 to April 7?

8. If $16\frac{2}{3}\%$ of the pupils in a school of 180 pupils were not in school last year, how many new pupils are there this year?

9. A rectangle 6 in. by 8 in. was increased 50% in length and $33\frac{1}{3}\%$ in width. What is the area after the increase?

10. In a school of 220 pupils, if there should be an increase of 5% next year, how many pupils would there be then?

Arranged as they are on the cumulative plan and with exactly ten questions in each, these examinations furnish a series of excellent reviews and at the same time they allow for easy rating on the scale of ten or a hundred.

WRITTEN EXAMINATION

1. Add the following:

$4\frac{7}{8}+1\frac{3}{8}$ $\quad 6\frac{5}{8}+7\frac{3}{4}$ $\quad 9\frac{3}{4}+5\frac{2}{3}$ $\quad 16\frac{7}{12}+15\frac{3}{4}$

2. Subtract the following:

$4\frac{1}{24}-1\frac{9}{16}$ $\quad 9\frac{1}{48}-3\frac{7}{8}$ $\quad 8\frac{5}{24}-7\frac{1}{4}$ $\quad 18\frac{1}{12}-12\frac{7}{8}$

3. Perform the operations indicated:

$5\frac{3}{4}\times 2\frac{3}{8}$ $\quad 6\frac{3}{4}\times 7\frac{1}{2}$ $\quad 2\frac{1}{2}+1\frac{1}{4}$ $\quad \frac{9}{16}\div\frac{3}{8}$

$7\frac{5}{8}\times 8\frac{3}{4}$ $\quad 28\times 75\frac{3}{4}$ $\quad 13\frac{3}{4}\div 2\frac{3}{4}$ $\quad 15\frac{1}{4}\div 18\frac{3}{8}$

4. Copy and complete the following:

$2\frac{1}{2}$ mi. = (?) rd. $\qquad$ 49 ft. = (?) in.

4800 lb. = (?) T. $\qquad$ 384 cu. ft. = (?) cd.

5. Find the values of the following:

25% of 844	0.75 × 368	90% of 480
75% of 724	$\frac{3}{4}$ of 5040	69% of 539

6. Find the net amounts of bills with discounts as follows:

\$850, 10%	\$240, $12\frac{1}{2}$%	\$450, $66\frac{2}{3}$%
\$900, 25%	\$360, $16\frac{2}{3}$%	\$270, $33\frac{1}{3}$%

7. Increase each of the following by the per cent indicated:

\$120, 40%	\$640, $37\frac{1}{2}$%	\$1296, 25%
\$696, $16\frac{2}{3}$%	\$1250, 10%	\$18.75, $33\frac{1}{3}$%

8. Decrease each of the following by the per cent given:

\$250, 10% $\qquad$ \$6750, 20% $\qquad$ \$4800, $33\frac{1}{3}$%

9. Compare 25% of 50% of \$1000 with 50% of 25% of \$1000, and each with $12\frac{1}{2}$% of 50% of \$2000.

10. The number 85 is 15% less than what number?

WRITTEN EXAMINATION

1. Write the number 1925 in three different ways.

2. A man has two pieces of land: one of 240 A., which he values at $125 per acre; and one of 65 A., which he values at $80 per acre. What is the average value of all the land per acre?

3. A state road 471 mi. long cost $2,325,798. What was the average cost per mile?

4. A certain farmer has 160 A. of land worth $150 per acre, 10 cows worth $85 each, 30 T. of hay worth $14.50 per ton, 800 bu. of corn in the crib worth 90¢ per bushel, $125 worth of poultry, farm machinery worth $450, an automobile worth $750, 20 hogs worth $13.50 each, and other property worth $1125. Make an inventory of his property.

5. A careless farmer failed to test his seed corn, and only four grains grew out of every five that were planted. The farmer's yield on 40 A. was 1280 bu. If the corn was worth 96¢ per bushel, how much did his neglect cost him?

6. If corn shrinks 10% in weight when carried over to the next summer, what is the loss on 80 A. of corn yielding 45 bu. per acre, the price remaining 90¢ a bushel?

7. Draw a plan of a village lot 200 ft. long and 100 ft. wide, using the scale $\frac{1}{200}$.

8. Draw a plan of a living room 18′ × 24′, with a bedroom 10′ × 12′ adjoining, using the scale $\frac{1}{48}$.

9. In a school of 492 pupils $16\frac{2}{3}\%$ were not there last year. How many of the present number were in the school last year?

10. Using short methods, find the amount of the following bill of goods: 14 pr. gloves @ $2.25, $12\frac{1}{2}$ yd. cloth @ 64¢, 16 rugs @ $12.50, and 4 doz. spools thread @ $62\frac{1}{2}$¢ per dozen.

WRITTEN EXAMINATION

1. A man bought a hat which was marked \$5.50 for 20% below the marked price. Find the cost.

2. A farmer had $5\frac{1}{2}$ A. that yielded 12 T. of ensilage to the acre. He put this into a silo and fed 20 cows an average of 30 lb. each a day from November 1 to June 1. How many tons did he have left in the silo on June 1?

3. By selection of the seed the yield from 80 A. of corn was increased 20% over 45 bu. per acre. Find the total yield.

4. A new road was built along one side of the farm in Ex. 3 at a cost of \$4320 per mile. If the corn is worth 90¢ a bushel, the yield will pay for what part of a mile of this road?

5. A poultry raiser has a lot 10 rd. square which he wishes to fence into four equal square lots, placing the fence posts 1 rd. apart. How many posts will he need? Draw a diagram.

6. A man paid \$1200 for an automobile. He sold it after 2 yr. for a price which showed a discount of 25% at the end of 1 yr. and a further discount the next year of 25% on the remainder. What price did he get for his car?

7. A salesman is offered a salary of \$2250 a year, or half that amount and 3% commission on his sales. If the sales amount to \$35,000 a year, which is the better plan for him?

8. Find the interest on \$1250 for 2 yr. 4 mo. at 6%.

9. A secretary who received a salary of \$125 a month lent her brother, who was in college, 10% of her total salary for 9 mo. He returned the loan 4 yr. 6 mo. afterwards with 6% interest. How much did he pay her?

10. If an automobile travels at the rate of 24 mi. an hour, how long does it take to go 0.8 mi.?

WRITTEN EXAMINATION

1. Write the following as decimal fractions:

$\frac{1}{10}$ $\frac{4}{10}$ $\frac{232}{1000}$ $\frac{723}{1000}$ $\frac{8}{10}$

2. Write as whole numbers or mixed decimals:

$\frac{13}{10}$ $\frac{150}{10}$ $\frac{225}{100}$ $\frac{3007}{1000}$ $\frac{2800}{1000}$

3. Add, and also subtract, the following:

127.42	403.61	512.71	623.42	803.40
109.64	362.83	329.86	435.64	637.68

4. Divide the following as indicated:

$41.6 \div 0.4$	$2.22 \div 0.02$	$36.08 \div 3.28$
$0.25 \div 0.5$	$3.33 \div 0.03$	$1.036 \div 0.56$

5. A dealer pays \$113 for some chairs at \$5.65 each. How many chairs does he buy?

6. The product of two numbers is 156.25, and one of the numbers is 5000. What is the other number?

7. If wind blows in a hurricane at the rate of 8796.6 ft. per minute, what is the rate per second?

8. A contractor bought a shipment of bricks for \$249.90, paying \$10.20 a thousand. How many bricks did he buy?

9. In a recent year there were 614 coöperative creameries in Minnesota, and they made 74,934,940 lb. of butter in that year. Find to the nearest pound the average amount per creamery.

10. A farmer found that he could average 2.76 T. of alfalfa per acre on his 40-acre field. At \$15 per ton, what was this crop worth? How much greater was the value of the alfalfa than that of the clover in another 40-acre field averaging 1.85 T. per acre and worth \$16.40 per ton?

WRITTEN EXAMINATION

1. Multiply the following:

2×3.73	1.3×48	4.25×64	3.42×28.6
6×0.95	2.9×62	6.75×84	4.25×66.4

2. Multiply the following:

$4\frac{1}{2} \times 6.8$	8.2×6.8	8.36×4.8	5.33×2.96
$3\frac{3}{4} \times 7.2$	7.7×8.8	9.82×6.7	4.56×8.21

3. Divide, carrying the quotients to two decimal places:

$9.5 \div 0.5$	$6.25 \div 25$	$3.49 \div 26$	$8.34 \div 26$
$9.5 \div 0.05$	$6.75 \div 75$	$47.2 \div 34$	$342.9 \div 0.16$

4. Divide, carrying the quotients to three decimal places:

$2 \div 3.7$	$8.48 \div 1.7$	$39.4 \div 2.8$	$426.9 \div 0.31$

5. If a locomotive costs \$42,000, a steel day coach \$9000, and a steel sleeping car \$32,000, how much will 2 locomotives, 36 day coaches, and 5 sleeping cars cost?

6. In a year when Montana produced 285,719,918 lb. of copper and Michigan produced 155,715,286 lb., Montana produced how much more than Michigan?

7. A contractor bought a lot of shingles for \$1239.75, paying \$7.12$\frac{1}{2}$ a thousand. How many thousand shingles did he buy?

8. In a year when Iowa produced 378,766,000 bu. of corn worth 50¢ a bushel, what was the value of the crop?

9. In a year when this country produced 304,043,400 lb. of wool, the total value being \$75,819,251, what was the average value per pound, to the nearest cent?

10. How many times will a roller 4.25 ft. in circumference turn in rolling the length of a lawn 136 ft. long?

WRITTEN EXAMINATION

1. A man bought 16 Government Savings Stamps at $4.27 each. How much did they cost?

The price of government securities of this kind varies according to the time at which they are bought and according to market conditions.

2. How many Treasury Savings Certificates at $84.80 each can be bought for $1275, and how much money will be left over?

3. A man buys 12 Treasury Savings Certificates for $1022.40. How much does he pay for each?

4. A workman in a factory had his pay raised from $52\frac{1}{2}¢$ an hour to $59\frac{1}{4}¢$ an hour. What was his increase in pay for a week of 44 working hours?

5. A workman had his pay raised from $21.12 to $25.08 for a week of 44 working hours. Find the increase in pay per hour.

6. From our food we obtain heat, and the heat values of our foods are measured in units known as calories. From 60¢ worth of beefsteak we can secure about 600 calories, and from 15¢ worth of mashed potatoes about 250 calories. Find to the nearest hundredth of a cent the cost of one of the calories in each of these foods. Which food gives us the larger number of calories for our money?

7. A woman needs $31\frac{1}{2}$ in. of lace for a waist. What part of a yard does she need?

8. Kate needs $\frac{5}{8}$ yd. of ribbon for a hat. How many inches of ribbon does she need?

9. How many cubic yards of earth are removed in excavating a cellar 24 ft. 6 in. long, 18 ft. wide, and $5\frac{1}{2}$ ft. deep?

10. Multiply 7656 by 375 and multiply the result by 25, using the shortest methods you know.

WRITTEN EXAMINATION ON THRIFT

1. Fred's mother saves on an average $\$1.37\frac{1}{2}$ a month by doing her marketing personally instead of by telephone. How much does she save a year in this way?

2. Fred and his mother make on an average $\$8.37$ a month for 5 mo. by raising vegetables and canning what they do not eat. How much do they make in the 5 mo.?

3. By purchasing at a special sale, Fred saved $16\frac{2}{3}\%$ on a suit of clothes that had been selling for $\$33$. Find the amount that Fred saved.

4. Fred earned $\$1.25$ a week by taking care of a furnace for a neighbor during 7 mo. Allowing $4\frac{3}{7}$ wk. to a month, how much did he earn in this way?

5. By raising their own chickens, Fred and his mother saved on an average $\$2.12\frac{1}{2}$ a month over all expenses for the poultry yard. Find the saving in a year.

6. Fred mowed 18 lawns for his neighbors at an average price of $37\frac{1}{2}¢$ each. How much did he earn?

7. Fred earned $8\frac{1}{2}¢$ a quart by picking 240 qt. of berries. How much did he earn in this way?

8. By making half a dozen shirts for Fred instead of buying them, Fred's mother saved $87\frac{1}{2}¢$ on each shirt. How much did she save in this way?

9. Fred and his mother saved on an average $62\frac{1}{2}¢$ a month on gas by being careful to turn it out when not in use. How much did this saving amount to in a year?

10. Taking their earnings and savings as found above, how many Government Savings Stamps could Fred and his mother buy at $\$4.23$ each, and how much would they have left?

WRITTEN EXAMINATION ON THRIFT

1. In December a man bought 150 Government Savings Stamps at $4.23 each. Four years later he received from the government $5 for each stamp. How much more did he receive for them than he paid?

2. A man's bill for electric lights averages $3.80 a month. If he pays cash, he is entitled to a discount of 5%. How much does the discount amount to in a year?

3. A girl in a canning club spent $22.75 in raising and canning tomatoes. She sold the goods for 200% more than her expenses. How much did she receive for her labor?

4. If 1 oz. of edible material is wasted every day in each of our 24,000,000 homes, how many pounds are wasted in a year?

5. By saving $3 every month and investing it in Government Savings Stamps a man receives from the government $199.16 in 5 yr. At the same rate, how much, to the nearest cent, will he receive if he invests $10 a month?

6. If a man receiving wages of $6 each week day saves $\frac{1}{8}$ of it, how much will he save in a year of 365 da. of which 60 da. are either Sundays or holidays?

7. If you could carry out a project costing you $16.42 and paying you $78.50, how many Government Savings Stamps could you buy with the profit, allowing $4.23 each, and how much would you have left over? When, 4 yr. later, the government pays you $5 per stamp, how much will you have gained from your project and your thrift?

Find the amount saved in 305 working days of a year if the amount saved each day is:

8. $0.75; $1.25. 9. $0.65; $0.85. 10. $1.75; $2.50.

WRITTEN EXAMINATION ON FRACTIONS

1. What number must be added to $7\frac{3}{8}$ to make $11\frac{1}{4}$? What number must be multiplied by $7\frac{3}{8}$ to make $11\frac{1}{4}$? What number must be divided by $7\frac{3}{8}$ to make $11\frac{1}{4}$?

2. A man spent $\frac{1}{5}$ of his income for rent, $\frac{3}{16}$ for food, $\frac{1}{8}$ for Government Savings Stamps, and $\frac{1}{16}$ for sundries. What part of his income was left?

3. A boss reported to the contractor who had undertaken to construct a building that $\frac{3}{16}$ of the job was completed in September, $\frac{5}{24}$ in October, and $\frac{1}{4}$ in November. What part of the work was there still to do?

4. A district school rented $2\frac{5}{8}$ A. of land for a school garden. For purposes of cultivation, the land was then divided equally among 21 pupils of the school. How many square rods were assigned to each pupil to cultivate?

5. A passenger on a train finds, by watching the mile posts, that it takes the train $1\frac{1}{4}$ min. to go a mile. If the train continues at this rate, how long will it be before the train will reach a station that is $3\frac{1}{2}$ mi. away?

6. In a motor race one of the cars averaged $1\frac{5}{16}$ mi. a minute. How many miles did it travel in $8\frac{1}{2}$ min.?

7. At the rate of 3 lb. 10 oz. of sugar a week, how much sugar will a family use in the $4\frac{3}{7}$ wk. of March?

8. If a sheet of blotting paper is $\frac{3}{64}$ in. thick, how many sheets are there in a pile that is $4\frac{1}{16}$ in. thick?

9. How many curtains, each requiring $3\frac{1}{3}$ yd., can be cut from a piece of cloth of the desired width and $28\frac{1}{2}$ yd. long?

10. If a bottle holds $\frac{4}{5}$ qt., how many gallons will it take to fill 75 such bottles?

WRITTEN EXAMINATION ON DECIMALS

1. A man having 127.3 A. of land buys 36.8 A. from one neighbor and 57.6 A. from another. He then sells half of his total holdings and keeps the rest. How much does he keep?

2. The distance from Chicago to Jackson is 209.31 mi., and from Jackson to Detroit, 75.43 mi. On the road from Detroit to Chicago by way of Jackson, how far is it from Chicago to the halfway point?

3. The distance from New Orleans to Baton Rouge is 89.4 mi., and from New Orleans to Natchez by way of Baton Rouge it is 214.1 mi. Find the distance from Natchez to a point halfway between New Orleans and Baton Rouge.

4. At the rate of 48.6 mi. an hour, how far will an express train go in 2 hr. 50 min.?

5. A dealer in real estate cuts a tract of land into building lots, each lot containing 0.68 A. He sells all the lots with the exception of 13 lots. How many acres has he remaining?

6. In a year when the average yield of oats in this country was 32.3 bu. per acre, and the average price on the farm was 56.4¢ per bushel, what was the average value to the nearest cent of the oat crop per acre?

7. If 1 cu. ft. of a certain kind of granite weighs 173.75 lb., and 1 cu. ft. of water weighs 62.5 lb., this kind of granite is how many times as heavy as water?

8. Taking the circumference of a wheel as 3.14 times the diameter, find the diameter of a wheel that is 4.71 ft. in circumference; of a wheel that is $7\frac{3}{4}$ ft. in circumference.

9. Express as decimals $\frac{3}{16}$; $\frac{5}{32}$; $\frac{7}{64}$; $\frac{8}{40}$.

10. Express as common fractions 0.425; 0.875; $0.333\frac{1}{3}$.

WRITTEN EXAMINATION ON MEASURES

1. Express 16 ft. 6 in. as inches; as yards.

2. Express $31\frac{1}{2}$ sq. ft. as square yards; as square inches.

3. Express $94\frac{1}{2}$ cu. ft. as cubic yards, and express $94\frac{1}{2}$ cu. yd. as cubic feet.

4. Taking a pile of wood 8 ft. by 4 ft. by 4 ft. as a cord, how many cords of wood are there in a pile that is 76 ft. long, 6 ft. high, and 4 ft. wide? How many cords of wood are there in a pile 64 ft. long, 7 ft. high, and 4 ft. wide?

5. Allowing $1\frac{1}{4}$ cu. ft. to a bushel, and taking 1 bu. of wheat as weighing 60 lb., find the weight of the wheat that will fill a bin 10 ft. long and $7\frac{1}{2}$ ft. wide to a depth of $5\frac{1}{2}$ ft.

6. There are 231 cu. in. in a gallon and 537.6 cu. in. in a peck. From these measures find the number of cubic inches in a liquid quart and the number in a dry quart.

From this the pupil will see that the dry quart is not the same as the liquid quart, although the difference is not great.

7. If a carpenter can build 30 ft. 8 in. of a certain kind of fence in 1 da., how many days will it take him to build a fence of this kind that is 245 ft. 4 in. long?

8. How many square tiles $5\frac{3}{4}$ in. on a side will it take to pave a courtyard 28 ft. 9 in. wide by 69 ft. long? How many tiles will it take if the courtyard is half as long and half as wide?

9. How many cans, each containing $\frac{7}{8}$ qt., can be filled from 2 bbl. of sirup, each containing $31\frac{1}{2}$ gal.?

10. If 1 cu. ft. of water weighs $62\frac{1}{2}$ lb., and ice is 0.92 times as heavy as water, find to the nearest ton the weight of a mass of ice 8 in. thick on a pond the surface of which covers 7500 sq. yd.; a pond the surface of which covers 2 A.

WRITTEN EXAMINATION ON MEASURES

1. If during a rainstorm $\frac{7}{8}$ in. of rain fell on a field containing 40 A., how many cubic feet of water fell on the field?

2. In Ex. 1, how many tons did the water weigh, at $62\frac{1}{2}$ lb. per cubic foot?

3. If the rainfall in a certain place is 28.5 in. a year, what is the weight of water that falls on 40 A. in that place in 1 yr.?

4. If there are 20 oz. of nitrogen in every bushel of wheat, what is the weight of nitrogen taken from a field of 40 A. that produces 24 bu. of wheat to the acre?

Teachers should call attention to the fact that nitrogen is a plant food that plants extract from the soil. This is the reason why land needs to be fertilized with plant foods from time to time.

5. If a wheel of circumference 12 ft. 1 in. makes 1716 revolutions in going a certain distance, how many revolutions will a wheel of circumference 11 ft. 11 in. make?

6. If a bushel contains 2150.42 cu. in., how much of an error, expressed in cubic inches, do we make in taking 1 bu. as $1\frac{1}{4}$ cu. ft.? Is the approximation $1\frac{1}{4}$ cu. ft. too large or too small?

7. Express 348 oz. as pounds, 348 lb. as ounces, 348 T. as pounds, 348 ft. as inches, and 348 in. as feet.

8. Express $25\frac{3}{8}$ lb. as ounces, $25\frac{3}{8}$ oz. as pounds, and $25\frac{3}{8}$ T. as pounds.

9. A nautical mile is 1.152 common miles. How many feet are there in a nautical mile?

10. The troy pound which is used in weighing gold contains 5760 grains, and the avoirdupois (or common) pound contains 7000 grains. Therefore 1 lb. of gold weighs what part as much as 1 lb. of feathers?

WRITTEN EXAMINATION

1. A Florida gardener sends to New York 475 crates of strawberries, each containing 32 quart boxes. The berries retail at 48¢ a box. Find the total amount received at the retail price.

2. A man's electric-light bill is $13.80 for the three summer months. If the shorter days in the fall require the use of 25% more light, what will be the average cost of electricity per month for the three fall months?

3. A dealer sends an express package weighing 225 lb. and agrees to pay $\frac{2}{3}$ of the express. If the rate is $2.40 per 100 lb., how much express should the dealer pay?

4. If a man pays $9.40 per M for shingles and sells them for 15% more than he pays for them, how much does he gain on 35 M? on 62 M? on 78 M?

5. If a man and a team can seed 8 A. a day, how long will it take them to seed a rectangular field that is 180 rd. long and 120 rd. wide, and how much will it cost at $9.75 a day?

6. If 1 cu. yd. of earth weighs 2825 lb., how many tons do 37 cu. yd. weigh?

7. Multiply 17 ft. 8 in. by $24\frac{1}{2}$ and check the result by dividing it by $24\frac{1}{2}$ and showing that the quotient is 17 ft. 8 in.

8. In an exercise in division the divisor is 49.84 and the quotient is $\frac{3}{4}$ as much. What is the dividend?

9. The product of two numbers is $4612\frac{1}{2}$, and one of the numbers is $73\frac{4}{5}$. Find the other number and check the work.

10. The difference between two numbers is $22\frac{1}{2}$, and one of the numbers is $49\frac{4}{5}$. Find the other number. If it is possible to find more than one number that answers the conditions, find all the numbers that you can.

WRITTEN EXAMINATION ON PER CENTS

1. Goods costing \$1725 are sold at a loss of 8%. Find the loss and the amount received for the goods.

2. Goods costing \$3400 are sold at a profit of $16\frac{1}{2}\%$. Find the profit and the amount received for the goods.

3. If the price of an article which sells for \$5 is marked up 20% and then reduced 25%, what is the final selling price?

4. A dealer sold a watch at a loss of 5%. The loss was \$15. How much did the watch cost the dealer?

If \$15 is 5%, or $\frac{1}{20}$, of the cost, how much was the cost?

5. A man can buy an automobile for 15% less than the price which the dealer first asked. If the dealer first asked \$1600 for the car, for how much can the man buy it now?

6. A grocer bought two chests of tea for \$18 a chest. On the first he gained $17\frac{1}{2}\%$ and on the second he lost 6%. Find his total gain.

7. A man borrowed \$2750 and agreed to pay 6% interest. If he repaid the loan 1 yr. 4 mo. afterward with interest, how much interest did he pay?

8. If a manufacturer sells shoes at a profit of 21% on the cost, and the cost of manufacture averages $\$4.66\frac{2}{3}$ a pair, how much is his profit on 3500 pairs of shoes?

9. A farmer can have his land poorly plowed for \$6.90 an acre, but it will cost him $66\frac{2}{3}\%$ more to have a good job done. How much will it cost to have 75 acres well done?

10. How much butter fat is there in 685 lb. of milk brought to a creamery, the creamery test showing that 4.1% of the weight of the milk is butter fat?

WRITTEN EXAMINATION ON PER CENTS

1. A bicycle was marked $48, but was sold at 16% discount. What was the net price?

2. A piano was listed at $720, but was sold at 25% discount. What was the selling price?

3. A jobber bought goods amounting to $4850. He was allowed a discount of 24%. Find the net amount of the bill.

4. A merchant bought 1680 yd. of muslin at 15¢ a yard, less 22%. How much did it cost him?

5. A merchant bought 2450 yd. of lawn for $1225, less 24%. How much did he pay per yard?

6. A grocer bought 36 doz. cans of soup listed at $3.50 a dozen. He was allowed a discount of 22½%. What was the net amount of the bill?

7. One cow averages 28 lb. of milk a day, testing 3.8% of butter fat; another averages 24 lb., testing 4.2%. Butter fat being worth 36¢ a pound, which cow is the more profitable per month of 30 da., and how much more?

8. A farmer has 110 trees on an acre of woodland, of which he decides to cut 50%. If wood is worth $9.75 a cord, and he can cut 3 cords from 5 trees, how much will he receive for the wood that he cuts?

9. A dressmaker bought 37½ yd. of chiffon velvet at $6.50 a yard, and received a reduction of 12% for paying cash. She sold the velvet at $6.50 a yard. How much profit did she make on the transaction?

10. A grocer bought 24 doz. pound packages of macaroni for $62.40, less 15%. He sold it for 30¢ per package. The overhead being 96¢ in all, how much did he gain?

TABLES FOR REFERENCE

Length

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
$31\frac{1}{2}$ gallons = 1 barrel (bbl.)
2 barrels = 1 hogshead (hhd.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
About 4 weeks = 1 month (mo.)
12 months = 1 year (yr.)
365 days = 1 common year
366 days = 1 leap year

Money

10 mills = 1 cent (¢ or ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)

Angles and Arcs

60 seconds (60″) = 1 minute (1′)
60 minutes = 1 degree (1°)

Counting

12 units = 1 dozen (doz.)
12 dozen, or 144 units = 1 gross (gr.)
12 gross, or 1728 units = 1 great gross

Paper

24 sheets, or 25 sheets = 1 quire
20 quires = 1 ream

For stationery and folded paper a quire is usually 24 sheets; for unfolded paper a quire is usually 25 sheets. Paper is also sold by the pound.

SUGGESTIONS TO TEACHERS

It frequently happens, in solving problems, that pupils are taught to calculate properly in oral work but are allowed to use long, primitive, laborious methods in written work. Economical methods should be used in both written and oral calculations. Here, as elsewhere, good judgment should control, in order that the calculations may be in reality economical. Pupils should be encouraged to use the pencil as little as possible in the solution of all problems, and the short methods given in this series of arithmetics should be used whenever possible.

Closely allied to training for skill in calculation is training in estimating reasonable answers to problems, as a preliminary to their solution. This training may begin early in school life in estimating lengths, areas, capacities, and weights. Much oral and written work of this character with all kinds of problems may profitably be given until pupils have established the habit of doing mathematical work with an expectation of arriving at a reasonable result. Good textbooks give frequent suggestions and opportunities for this training.

There are two kinds of exercises in arithmetic, namely, abstract and concrete. The former lead to skill in manipulation of figures, and the latter lead to interpretation and also to calculation. The concrete problem should be typical of life; its conditions should be within the comprehension of the pupils and within actual domestic, school, or business experiences. Local projects are of great value.

A pupil should not be expected to solve a problem if he does not understand the social or business relations involved or the language used. It matters not how practical to the teacher a problem may appear, it is not practical to the pupil unless it is within his comprehension and comes or has come within his experience, or is within easy range of his imagination.

The distinction between abstract and concrete numbers (page 22) is not observed so much as it formerly was, for all *number* is essentially abstract. We may specify the units (feet, yards, apples, etc.), but the *number* is still abstract. The custom of specifying the units in solving a problem is also tending to disappear, the pupil keeping the units in mind just as the business man does.

For example, consider the following cases:

If one book costs 20¢ and another book costs 30¢, both books together cost *as many cents* as $20 + 30$, or 50¢.

If one book costs 20¢ and another book costs 30¢, the second book costs *as many cents* more than the first as $30 - 20$, or 10¢.

If 1 orange costs 5¢, 4 oranges will cost *as many cents* as 4×5, or 20¢.

If 1 orange costs 5¢, one can buy *as many oranges* for 20¢ as $20 \div 5$, or 4 oranges.

If 4 oranges cost 20¢, each orange costs *as many cents* as $20 \div 4$, or 5¢.

In the study of problems pupils should be led to consider (1) what is given, (2) what is required, and (3) what arithmetic processes are necessary for solving them. Much trouble arises because pupils are not able to discriminate between (1) and (2).

In dealing with a problem there are three methods of procedure: (1) let the pupil tell what is to be done, without performing the operations, or let him indicate the solution, or diagram the work; (2) let him give an approximate answer; (3) let the pupil do all the work but in the most economical way. In order to give more practice in reasoning than is possible under the last method, (1) and (2) should be employed more frequently than is usually the case.

The habit of checking work as it progresses should be fixed in the early grades. This checking consists of repeating one process before the next process is begun. The repetition should be made after each part of the problem has been solved rather than after the entire problem has been solved. This is the business custom.

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Page 3. 17. Thirty-five thousand, eight hundred fifty. 18. Sixty-four thousand, nine hundred seven. 19. Three hundred thirty thousand, three hundred thirty. 20. Six hundred thousand, six. 21. Seven million, six hundred forty-two thousand, eight hundred fifty-eight. 22. Forty-two million, four hundred twenty-nine thousand, eight hundred forty-five. 23. 7,025,059. 24. 87,687. 25. 604,820. 26. 1,002,000,160.

Page 5. 1. 14. 2. 61. 3. 70. 4. 25. 5. 41. 6. 610. 7. 71. 8. 97. 9. 1610. 10. 87. 11. 234. 12. 1926. 13. XXVII. 14. XXXII. 15. XLVI. 16. LIV. 17. LXXVIII. 18. LXXXVII. 19. MDCCLXXV. 20. MDCCCCXX, or MCMXX. 21. MDCCCCXXVIII, or MCMXXVIII. 22. MDCCCCXXX, or MCMXXX.

Page 7. 1. 48. 2. 41. 3. 56. 4. 60. 5. 39. 6. 48. 7. 56. 8. 53. 9. 51. 10. 38. 11. 42. 12. 37. 13. 52. 14. 50. 15. 41. 16. 42. 17. 48. 18. 45. 19. 62. 20. 56. 21. 46. 22. 64. 23. 47. 24. 50. 25. 62. 26. 48. 27. 47. 28. 47. 29. 42. 30. 56. 31. 44. 32. 48. 33. 47. 34. 53.

Page 8. 1. 78. 2. 87. 3. 82. 4. 79. 5. 87. 6. 89. 7. 83. 8. 91. 9. 79. 10. 109.

Page 11. 1. 12,250 cu. ft. 2. $19.98. 3. 643 mi. 4. 3401. 5. 3334 mi. 6. $529.34. 7. $15,340. 8. 107,462 lb.

Page 12. 1. Lost: 54; 62; 66; 75; 79; 82; 97; 98; total, 613. Won: 100; 90; 88; 78; 74; 71; 57; 55; total, 613. 2. $35,684.18. 3. $24,494.25. 4. $21,564.75. 5. $32,955.77.

Page 14. 1. $137.40. 2. 1196. 3. 2352. 4. 2545. 5. 896. 6. 1315. 7. 4554. 8. 1699. 9. 5384. 10. 478. 11. 2642. 12. 4425. 13. 2214. 14. 4427. 15. 4072. 16. 2319.

Page 16. 1. 222,075. 2. 350,565. 3. $493.64. 4. $190.07. 5. $332.87. 6. $483.48. 7. $479.88. 8. $256.30. 9. $14.69. 10. $378.38. 11. 355,571. 12. 217,893. 13. $661.82. 14. $678.79. 15. $406.24. 16. $188.86. 17. $78.86. 18. $43.62. 19. $269.47. 20. $172.35. 21. 153,643. 22. 571,035. 23. $409.29. 24. $321.48. 25. $272.84. 26. $312.50. 27. $388.26. 28. $424.59. 29. $232.37. 30. $196.98. 31. 41,124. 32. 323,993. 33. $330.24. 34. $141.79. 35. $373.95. 36. $391.44. 37. $377.15. 38. $297.10. 39. $184.07. 40. $187.88.

82 322.3

Page 17. 1. $4.75. 2. $5.48. 3. 738. 4. $78.35. 5. $1091. 6. 150 rd., loss; 386 rd., gain. 7. $725.10. 8. $3.95.

Page 18. 1. $132.25. 2. $18.35. 3. $11,935.35.

Page 20. 1. 17,017. 2. 14,387. 3. 20,834. 4. 10,655. 5. 36,791. 6. 22,601. 7. 14,754. 8. 42,636. 9. 28,917. 10. 40,123. 11. 32,877. 12. 64,274. 13. $9560.73; $2997.82. 14. $14,618.55; $3464.05.

Page 21. 1. 39; 12. 2. 15. 3. $3095.26. 4. $44,500. 5. 246. 6. 51. 7. 146 mi.

Page 23. 1. 18,112 ft.; 101,880 ft.; 135,840 ft. 2. $53,725. 3. $33,150. 4. $284,375. 5. $16,500. 6. $25,218.50. 7. $94,873.50.

Page 24. 1. 34,300. 2. 324,562. 3. $243; $351; $729. 4. $189.75. 5. $900. 6. $118,212.50. 7. $13,260. 8. $19,385. 9. $14,790.72. 10. $38,556. 11. $1027.20. 12. $4836.72.

Page 25. 1. 494,868. 2. 114,048. 3. 495,132. 4. 809,009. 5. 209,235. 6. 257,856. 7. 270,803. 8. 270,096. 9. 423,852. 10. 726,612. 11. 451,136. 12. 186,354. 13. 658,440. 14. 499,284. 15. 679,936. 16. 419,148. 17. 291,132. 18. 430,837. 19. 439,956. 20. 316,043. 21. 78,982. 22. 274,164. 23. 2,427,732. 24. 6,038,591. 25. 1,758,736. 26. 4,397,226. 27. 5,131,323. 28. $12,702.69. 29. $25,735.50. 30. $45,749.76. 31. $45,936. 32. $138,587.94. 33. $720,149.

34. I. 91,368; 98,415; 53,703; 95,256; 114,210; 167,427; 139,239; 84,807; 185,409. 152,280; 83,096; 147,392; 176,720; 259,064; 215,448; 131,224; 286,888. 89,505; 158,760; 190,350; 279,045; 232,065; 141,345; 309,015. 86,632; 103,870; 152,269; 126,633; 77,129; 168,623. 184,240; 270,088; 224,616; 136,808; 299,096. 323,830; 269,310; 164,030; 358,610. 394,797; 240,461; 525,707. 199,977; 437,199. 266,287.

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III. 91,936; 79,118; 284,206; 365,534; 400,452; 190,944; 224,094; 252,824; 167,076. 87,232; 133,744; 172,016; 188,448; 89,856; 105,456; 118,976; 78,624. 115,097; 148,033; 162,174; 77,328; 90,753; 102,388; 67,662. 531,761; 582,558; 277,776; 326,001; 367,796; 243,054. 749,262; 357,264; 419,289; 473,044; 312,606. 391,392; 459,342; 518,232; 342,468. 219,024; 247,104; 163,296. 290,004; 191,646. 216,216.

IV. 361,382; 226,009; 248,087; 268,422; 191,730; 260,288; 133,049; 549,626; 161,518. 241,958; 265,594; 287,364; 205,260; 278,656; 142,438; 588,412;

172,916. 166,103; 179,718; 128,370; 174,272; 89,081; 367,994; 108,142. 197,274; 140,910; 191,296; 97,783; 403,942; 118,706. 152,460; 206,976; 105,798; 437,052; 128,436. 147,840; 75,570; 312,180; 91,740. 102,592; 423,808; 124,544. 216,634; 63,662. 262,988.

V. 145,092; 85,654; 112,096; 130,402; 144,640; 182,382; 110,288; 90,174; 94,920. 243,318; 318,432; 370,434; 410,880; 518,094; 313,296; 256,158; 269,640. 187,984; 218,683; 242,560; 305,853; 184,952; 151,221; 159,180. 286,192; 317,440; 400,272; 242,048; 197,904; 208,320. 369,280; 465,639; 281,576; 230,223; 242,340. 516,480; 312,320; 255,360; 268,800. 393,816; 321,993; 338,940. 194,712; 204,960. 167,580.

VI. 282,768; 243,312; 388,806; 733,224; 400,314; 612,390; 252,354; 575,400; 797,340. 101,824; 162,712; 306,848; 167,528; 256,280; 105,608; 240,800; 333,680. 140,008; 264,032; 144,152; 220,520; 90,872; 207,200; 287,120. 421,916; 230,351; 352,385; 145,211; 331,100; 458,810. 434,404; 664,540; 273,844; 624,400; 865,240. 362,815; 149,509; 340,900; 472,390. 228,715; 521,500; 722,650. 214,900; 297,790. 679,000.

a. 74,601; 107,406; 141,183; 54,918; 199,746. 135,694; 178,367; 69,382; 252,354. 256,802; 99,892; 363,324. 131,306; 477,582. 185,772.

b. 178,976; 78,208; 233,872; 241,392; 129,344. 99,008; 296,072; 305,592; 163,744. 129,376; 133,536; 71,552. 399,324; 213,968. 220,848.

c. 373,005; 72,495; 157,545; 153,495; 119,880. 164,859; 358,269; 349,059; 272,616. 69,631; 67,841; 52,984. 147,431; 115,144. 112,184.

d. 72,930; 142,103; 94,367; 109,616; 104,533. 212,190; 140,910; 163,680; 156,090. 274,561; 318,928; 304,139. 211,792; 201,971. 234,608.

e. 159,544; 324,184; 181,104; 226,184; 349,664. 336,589; 188,034; 234,839; 363,044. 382,074; 477,179; 737,684. 266,574; 412,104. 514,684.

f. 432,400; 425,820; 155,100; 300,800; 228,890. 833,520; 303,600; 588,800; 448,040. 298,980; 579,840; 441,222. 211,200; 160,710. 311,680.

g. 565,669; 297,648; 308,672; 556,023; 513,305. 354,672; 367,808; 662,547; 611,645. 193,536; 348,624; 321,840. 361,536; 333,760. 601,215.

h. 79,647; 290,511; 131,217; 279,624; 175,911. 70,473; 31,831; 67,832; 42,673. 116,103; 247,416; 155,649. 111,752; 70,303. 149,816.

i. 149,023; 199,628; 330,154; 139,251; 244,300. 244,244; 403,942; 170,373; 298,900. 541,112; 228,228; 400,400. 377,454; 662,200. 279,300.

j. 302,148; 288,414; 212,114; 320,460; 740,110. 149,688; 110,088; 166,320; 384,120. 105,084; 158,760; 366,660. 116,760; 269,660. 407,400.

Page 27. 16. $78,750. 17. $646,500. 18. $658. 19. $1974.

Page 28. 1. $2,218,900. 2. $38,610. 3. $2898. 4. $25.75. 5. $186.80; $46.80. 6. $259.50. 7. $12.

Page 29. 1. 490; 4900; 49,000. 2. \$4.90; \$49; \$490. 3. 574; 5740; 57,400. 4. \$5.74; \$57.40; \$574. 5. 1018; 10,180; 101,800. 6. \$101.80; \$1018; \$10,180. 7. 1654; 16,540; 165,400. 8. \$165.40; \$1654; \$16,540. 9. 2046; 20,460; 204,600; 10. \$20.46; \$204.60; \$2046. 11. 2175; 21,750; 217,500. 12. \$217.50; \$2175; \$21,750. 13. 2799; 27,990; 279,900. 14. \$279.90; \$2799; \$27,990. 15. 1920; 19,200; 192,000. 16. \$19.20; \$192; \$1920. 17. 21,440; 214,400; 26,800; 268,000. 18. \$254; \$2540; \$317.50; \$3175. 19. 28,840; 288,400; 36,050; 360,500. 20. \$237.20; \$2372; \$296.50; \$2965. 21. 26,520; 265,200; 33,150; 331,500. 22. \$173.60; \$1736; \$217; \$2170. 23. 33,680; 336,800; 42,100; 421,000. 24. \$152.40; \$1524; \$190.50; \$1905.

25. \$6611.85; \$66,118.50; \$661,185; \$712,610.50. 26. \$7003.20; \$70,032; \$700,320; \$765,975. 27. \$4371.15; \$21,855.75; \$190,457.25; \$218,557.50. 28. \$5679.60; \$56,796; \$2839.80; \$283,980; \$567,960. 29. 11,850; 41,250 ft.; \$1425; \$44,020. 30. 3350; 22,550 ft.; \$990; \$19,610. 31. 8000; 32,000; 363,000; \$16,400; \$312,200. 32. \$20; \$10; \$8; \$4. 33. \$100; \$50; \$40; \$20. 34. \$80; \$40; \$32; \$16. 35. \$120; \$60; \$48; \$24. 36. \$180; \$90; \$72; \$36. 37. \$2400; \$1200; \$960; \$480. 38. \$72,828. 39. 2,345,600 ft. 40. \$51,840,000. 41. \$1,507,940. 42. \$2,689,500. 43. \$1,989,760. 44. 2,704,800 yd. 45. \$39,775,000. 46. \$3,795,930. 47. \$635,470.

Page 34. 1. 28. 2. 29. 3. 31. 4. 59. 5. 86. 6. 84; 60. 7. 393; 400 lb. 8. 6; \$250. 9. 42; \$1000. 10. 189. 11. 181. 12. 231. 13. 146. 14. 57; 1000 *remainder*. 15. 149; 500 *remainder*. 16. 91.

Page 36. 1. 42. 2. 24. 3. 38. 4. 42. 5. 66. 6. 37. 7. 48. 8. 56. 9. 82. 10. 75. 11. 43. 12. 66. 13. 77. 14. 37. 15. 99. 16. 33. 17. 42. 18. 53. 19. 72. 20. 37. 21. 68. 22. 98. 23. 42. 24. 77. 25. 79. 26. 88. 27. 89. 28. 82. 29. 66. 30. 38. 31. 46. 32. 90. 33. 62. 34. 360. 35. 93. 36. 35. 37. 124. 38. 79. 39. 89. 40. 87. 41. 94. 42. $57\frac{3}{8}$. 43. $106\frac{3}{7}$. 44. $316\frac{3}{19}$. 45. 273. 46. $78\frac{20}{99}$. 47. $108\frac{9}{88}$. 48. 110. 49. $110\frac{8}{75}$. 50. $62\frac{7}{86}$. 51. $46\frac{11}{95}$. 52. $39\frac{8}{79}$. 53. $71\frac{18}{95}$. 54. $96\frac{37}{71}$.

Page 37. 1. \$139. 2. \$412. 3. 13. 4. 4411. 5. 527. 6. 26 da. 7. 2009. 8. 1250. 9. 4620. 10. \$31.25. 11. 5025. 12. 2,788,518 lb.

Page 38. 1. \$8.78. 2. \$3.88. 3. \$154.70. 4. \$7.40. 5. \$5.12. 6. \$3.08. 7. \$3234.40. 8. \$1527.30. 9. \$493.76.

Page 39. 1. 185. 2. 84. 3. 78. 4. 1727. 5. 1950. 6. 8761. 7. 504. 8. 1060. 9. 626. 10. 1050. 11. 365. 12. 467. 13. 675. 14. 4268. 15. \$4.68. 16. \$2.07. 17. \$7.83. 18. \$8.58. 19. \$3.67. 20. \$7.80. 21. \$13.32. 22. \$18.76. 23. \$33.97. 24. \$20.87. 25. \$22.67. 26. $1846\frac{7}{14}$. 27. $466\frac{3}{8}$. 28. $3298\frac{1}{3}\frac{3}{8}$. 29. $743\frac{4}{4}\frac{5}{8}$. 30. $1584\frac{5}{5}\frac{3}{6}$. 31. $746\frac{12}{5}\frac{9}{6}$. 32. $1615\frac{10}{4}\frac{9}{5}$. 33. $760\frac{4}{4}\frac{3}{6}$. 34. $1145\frac{3}{5}\frac{9}{5}$. 35. $1319\frac{12}{5}\frac{9}{6}$. 36. $1258\frac{2}{7}\frac{5}{2}$. 37. $1208\frac{65}{333}$. 38. $939\frac{43}{234}$. 39. $2705\frac{11}{31}\frac{8}{5}$. 40. $845\frac{45}{655}$. 41. $844\frac{1}{4}\frac{1}{27}$.

42. $2935\frac{163}{238}$. 43. $281\frac{270}{578}$. 44. $112\frac{308}{432}$. 45. $263\frac{857}{346}$. 46. $125\frac{950}{248}$. 47. $119\frac{172}{800}$. 48. $57\frac{121}{985}$. 49. $1473\frac{197}{583}$. 50. $1062\frac{137}{325}$.

Page 40. 1. $420; $1260. 2. $1020; $4080; $12,750. 3. 325. 4. 1351. 5. 3919. 6. $25. 7. $4938. 8. $18,246. 9. $1750. 10. $137.50. 11. $5.50. 12. $342.81.

Page 41. 1. $6.25. 2. $68.75. 3. $32.15. 4. $5.75. 5. $34.88. 6. 85¢. 7. $4.23. 8. $84.60. 9. $5475.

Page 42. 1. $82.50; $330. 2. $29.53; $206.71; $826.84; $620.13. 3. $650; $910; $390; $1040. 4. $133. 5. 33¢. 6. $455. 7. 479; 10. 8. 38.

Page 45. 13. $\frac{1}{2}$. 14. $\frac{1}{4}$. 15. $\frac{2}{3}$. 16. $\frac{3}{3}$. 17. $\frac{5}{8}$. 18. $\frac{5}{8}$. 19. Five eighths. 20. Seven eighths. 21. Five sixteenths. 22. Nine sixteenths. 23. Seven twelfths. 24. Five twenty-fourths. 25. Two and two thirds. 26. Three and seven eighths. 27. Nine and fifteen thirty-seconds. 28. Seven and twenty-one sixty-fourths. 29. Twenty-five dollars, twelve and one half cents; $25.13. 30. Six dollars, sixty-six and two thirds cents; $6.67. 31. Thirty-three dollars, thirty-seven and one half cents; $33.38. 32. Seventy-five dollars, sixteen and seven eighths cents; $75.17.

Page 48. 1. $\frac{1}{2}$. 2. $\frac{2}{3}$. 3. $\frac{1}{3}$. 4. $\frac{3}{5}$. 5. $\frac{1}{3}$. 6. $\frac{2}{3}$. 7. $\frac{9}{13}$. 8. $\frac{4}{7}$. 9. $\frac{1}{2}$. 10. $\frac{37}{41}$. 11. $\frac{13}{14}$. 12. $\frac{11}{20}$.

Page 49. 13. $\frac{1}{2}$. 14. $\frac{1}{2}$. 15. $\frac{2}{3}$. 16. $\frac{4}{3}$. 17. $\frac{6}{3}$. 18. $\frac{3}{2}$. 19. $\frac{2}{4}$. 20. $\frac{12}{4}$. 21. $\frac{2}{4}$. 22. $\frac{12}{4}$. 23. $\frac{14}{4}$. 24. $\frac{16}{4}$. 25. $\frac{6}{8}$. 26. $\frac{8}{8}$. 27. $\frac{20}{8}$. 28. $\frac{28}{8}$. 29. $\frac{2}{8}$. 30. $\frac{4}{8}$. 31. $\frac{4}{12}$. 32. $\frac{9}{12}$. 33. $\frac{8}{12}$. 34. $\frac{14}{12}$. 35. $\frac{18}{12}$. 36. $\frac{2}{12}$. 37. $\frac{3}{4}$. 38. $\frac{3}{3}$. 39. $\frac{1}{3}$. 40. $\frac{1}{2}$. 41. $\frac{1}{2}$. 42. $\frac{1}{3}$. 43. $\frac{4}{8}$ in.; $\frac{3}{8}$ in.; $\frac{2}{3}$ in. is greater. 44. $\frac{8}{12}$ in.; $\frac{9}{12}$ in.; $\frac{3}{4}$ in. is greater.

Page 50. 1. $\frac{2}{3}$. 2. $\frac{7}{8}$ in.; no; no. 3. $\frac{1}{4}$; $\frac{2}{8}$ in.; $\frac{4}{16}$ in. 4. $\frac{4}{32}$; $\frac{5}{32}$ in. 5. $\frac{3}{8}$ in. 6. $\frac{3}{8}$; $\frac{14}{32}$; $\frac{8}{16}$; $\frac{5}{8}$; $\frac{3}{4}$. 7. $\frac{28}{64}$. 8. $\frac{36}{64}$. 9. $\frac{18}{64}$. 10. $\frac{30}{64}$. 11. $\frac{32}{64}$. 12. $\frac{6}{64}$. 13. $\frac{24}{36}$. 14. $\frac{27}{36}$. 15. $\frac{4}{36}$. 16. $\frac{6}{36}$. 17. $\frac{20}{36}$. 18. $\frac{18}{36}$. 19. $\frac{14}{24}$. 20. $\frac{8}{24}$. 21. $\frac{1}{24}$. 22. $\frac{3}{24}$. 23. $\frac{10}{24}$. 24. $\frac{22}{24}$.

Page 51. 4. $\frac{4}{4}$; $\frac{5}{4}$; $\frac{7}{4}$; $\frac{8}{4}$. 5. $\frac{5}{5}$; $\frac{6}{5}$; $\frac{7}{5}$; $\frac{8}{5}$. 6. $\frac{6}{8}$; $\frac{7}{8}$; $\frac{11}{8}$; $\frac{12}{8}$; $\frac{13}{8}$; $\frac{17}{8}$; $\frac{18}{8}$. 7. $\frac{8}{7}$; $\frac{16}{7}$; $\frac{22}{7}$. 8. $\frac{8}{8}$; $\frac{16}{8}$; $\frac{17}{8}$; $\frac{19}{8}$. 9. $\frac{10}{9}$; $\frac{20}{9}$; $\frac{27}{9}$. 10. $\frac{33}{10}$; $\frac{37}{10}$; $\frac{40}{10}$. 11. $\frac{12}{8}$; $\frac{24}{12}$; $\frac{36}{12}$; $\frac{48}{12}$; $\frac{25}{12}$; $\frac{41}{12}$; $\frac{55}{12}$; $\frac{59}{12}$. 12. $\frac{16}{16}$; $\frac{32}{16}$; $\frac{48}{16}$; $\frac{64}{16}$; $\frac{17}{16}$; $\frac{18}{16}$; $\frac{33}{16}$; $\frac{37}{16}$; $\frac{55}{16}$; $\frac{73}{16}$.

Page 52. 1. 31; 33; 39; 22. 2. 27; 39. 3. $\frac{24}{8}$. 4. $\frac{18}{3}$. 5. $\frac{6}{2}$. 6. $\frac{12}{6}$. 7. $\frac{11}{3}$. 8. $\frac{28}{6}$. 9. $\frac{28}{9}$. 10. $\frac{13}{3}$. 11. $\frac{10}{3}$. 12. $\frac{22}{5}$. 13. $\frac{42}{9}$. 14. $\frac{33}{5}$. 15. $\frac{23}{3}$. 16. $\frac{77}{9}$. 17. $\frac{32}{9}$. 18. $\frac{320}{10}$. 19. $\frac{13}{4}$. 20. $\frac{53}{8}$. 21. $\frac{12}{4}$. 22. $\frac{41}{8}$. 23. $\frac{55}{8}$. 24. 102.

Page 53. 3. 11. 4. $7\frac{5}{8}$. 5. 7. 6. $15\frac{3}{8}$. 7. $5\frac{5}{24}$. 8. $18\frac{5}{8}$. 9. $7\frac{3}{8}$. 10. $102\frac{2}{32}$. 11. $19\frac{3}{4}$. 12. $15\frac{5}{8}$. 13. $13\frac{9}{32}$. 14. $17\frac{1}{4}$. 15. $31\frac{3}{4}$. 16. $7\frac{1}{4}$. 17. $2\frac{29}{32}$. 18. $12\frac{3}{8}$. 19. $10\frac{1}{2}$. 20. $7\frac{3}{8}$. 21. $2\frac{3}{8}$. 22. $22\frac{31}{32}$. 23. $7.25. 24. $12; $12.50.

Page 54. 7. $\frac{1}{3}$. 8. $\frac{1}{8}$; $\frac{5}{8}$; $\frac{3}{3}$. 9. $\frac{1}{4}$. 10. $\frac{1}{3}$. 11. $\frac{1}{4}$. 12. $\frac{1}{2}$. 13. $\frac{1}{4}$. 14. $\frac{1}{7}$. 15. $\frac{3}{8}$.

Page 56. 10. $\frac{4}{8}$, $\frac{6}{8}$. 11. $\frac{9}{12}$, $\frac{8}{12}$. 12. $\frac{9}{12}$, $\frac{10}{12}$. 13. $\frac{3}{24}$, $\frac{20}{24}$. 14. $\frac{6}{8}$, $\frac{6}{8}$. 15. $\frac{21}{24}$, $\frac{8}{24}$. 16. $\frac{6}{8}$, $\frac{4}{8}$. 17. $\frac{8}{24}$, $\frac{16}{24}$. 18. $\frac{8}{15}$, $\frac{10}{15}$. 19. $\frac{24}{40}$, $\frac{24}{40}$. 20. $\frac{14}{24}$, $\frac{11}{24}$. 21. $\frac{9}{48}$, $\frac{10}{48}$. 22. $\frac{21}{24}$, $\frac{10}{24}$. 23. $\frac{15}{24}$, $\frac{11}{24}$. 24. $\frac{21}{24}$, $\frac{12}{24}$. 25. $\frac{28}{36}$, $\frac{8}{36}$. 26. $\frac{20}{32}$, $\frac{11}{32}$. 27. $\frac{14}{32}$, $\frac{5}{32}$. 28. $\frac{30}{36}$, $\frac{31}{36}$. 29. $\frac{28}{32}$, $\frac{27}{32}$. 30. $\frac{9}{24}$, $\frac{7}{24}$. 31. $\frac{21}{24}$, $\frac{17}{24}$. 32. $\frac{20}{36}$, $\frac{9}{36}$. 33. $\frac{68}{96}$, $\frac{15}{96}$. 34. $\frac{68}{96}$, $\frac{48}{96}$. 35. $\frac{4}{96}$, $\frac{3}{96}$. 36. $\frac{20}{96}$, $\frac{9}{96}$. 37. $\frac{28}{96}$, $\frac{27}{96}$. 38. $\frac{14}{48}$, $\frac{9}{48}$. 39. $\frac{24}{48}$, $\frac{11}{48}$. 40. $\frac{42}{64}$, $\frac{7}{64}$. 41. $\frac{32}{64}$, $\frac{11}{64}$. 42. $\frac{28}{64}$, $\frac{15}{64}$. 43. $\frac{16}{192}$, $\frac{9}{192}$. 44. $\frac{80}{192}$, $\frac{21}{192}$. 45. $\frac{8}{192}$, $\frac{3}{192}$. 46. $\frac{56}{192}$, $\frac{75}{192}$. 47. $\frac{18}{48}$, $\frac{15}{48}$, $\frac{14}{48}$. 48. $\frac{21}{24}$, $\frac{10}{24}$, $\frac{11}{24}$. 49. $\frac{36}{96}$, $\frac{48}{96}$, $\frac{51}{96}$. 50. $3\frac{20}{48}$, $5\frac{24}{48}$. 51. $2\frac{28}{48}$, $3\frac{15}{48}$, $8\frac{14}{48}$. 52. $1\frac{36}{96}$, $2\frac{48}{96}$, $3\frac{33}{96}$.

Page 58. 7. $\frac{9}{16}$ in. 8. $\frac{7}{32}$ in. 9. 1 in. 10. $\frac{41}{64}$ in. 11. $1\frac{11}{24}$. 12. $1\frac{17}{24}$. 13. $\frac{29}{48}$. 14. $\frac{21}{48}$. 15. $\frac{37}{48}$. 16. $\frac{43}{48}$. 17. $\frac{31}{48}$. 18. $1\frac{17}{48}$. 19. 2 in. 20. $\frac{7}{8}$. 21. $1\frac{3}{8}$. 22. $\frac{2}{3}$. 23. $1\frac{1}{2}$. 24. $\frac{13}{24}$. 25. $1\frac{3}{8}$.

Page 59. 1. $2\frac{1}{2}$ lb. 2. $4\frac{3}{4}$ yd. 3. 73. 4. $52\frac{1}{2}$ ft. 5. 11. 6. 9. 7. $11\frac{2}{3}$. 8. $10\frac{1}{2}$. 9. $58\frac{2}{3}$. 10. $63\frac{1}{2}$. 11. $76\frac{2}{3}$. 12. 50. 13. 54. 14. $61\frac{1}{3}$. 15. 60. 16. $110\frac{2}{3}$.

Page 60. 1. $16\frac{1}{8}$ ft. 2. $15\frac{17}{48}$. 3. $84\frac{3}{4}$ lb. 4. $94\frac{11}{24}$ yd. 5. $3\frac{1}{8}$. 6. $4\frac{2}{3}$. 7. $4\frac{7}{24}$. 8. $4\frac{11}{16}$. 9. $9\frac{1}{8}$. 10. 6. 11. $6\frac{7}{16}$. 12. 7. 13. $11\frac{9}{16}$. 14. $12\frac{3}{16}$. 15. $9\frac{1}{16}$. 16. $8\frac{14}{16}$.

Page 61. 1. $\frac{1}{8}$ yd. 2. $\frac{3}{8}$ mi. 3. $\frac{5}{16}$. 4. $\frac{9}{16}$. 5. $\frac{5}{8}$. 6. $\frac{1}{4}$. 7. $\frac{9}{16}$. 8. $\frac{3}{8}$. 9. $\frac{3}{8}$. 10. $\frac{1}{8}$. 11. $\frac{5}{8}$. 12. $\frac{1}{3}$. 13. $\frac{1}{8}$. 14. $\frac{1}{8}$. 15. $\frac{5}{16}$. 16. $\frac{1}{12}$. 17. $\frac{3}{16}$. 18. $\frac{7}{16}$. 19. $\frac{1}{8}$. 20. $\frac{1}{30}$. 21. $\frac{1}{10}$. 22. $\frac{1}{24}$. 23. $\frac{7}{10}$. 24. $\frac{11}{24}$. 25. $\frac{5}{16}$. 26. $\frac{1}{12}$. 27. $\frac{1}{8}$. 28. $\frac{5}{24}$. 29. $\frac{1}{8}$. 30. $\frac{1}{8}$. 31. $\frac{17}{32}$. 32. $\frac{3}{10}$. 33. $\frac{1}{16}$. 34. $\frac{3}{16}$.

Page 62. 1. $11\frac{1}{8}$ in. 2. $15\frac{1}{2}$ in. 3. $44\frac{1}{4}$ mi. 4. $3\frac{3}{8}$. 5. $1\frac{5}{8}$. 6. $2\frac{3}{8}$. 7. $3\frac{3}{8}$. 8. $10\frac{3}{8}$. 9. $15\frac{3}{8}$. 10. $12\frac{3}{8}$. 11. $28\frac{9}{16}$. 12. $31\frac{7}{16}$. 13. $21\frac{9}{16}$. 14. $5\frac{1}{4}$ in. 15. $4\frac{27}{32}$ in. 16. $4\frac{1}{4}$. 17. $14\frac{1}{3}$. 18. $1\frac{5}{8}$.

Page 63. 1. $22\frac{1}{2}$ in. 2. $2\frac{7}{16}$ in. 3. $2\frac{3}{8}$ in. 4. $24\frac{5}{8}$. 5. $5\frac{5}{8}$. 6. $5\frac{1}{8}$. 7. $23\frac{7}{8}$. 8. $7\frac{5}{8}$. 9. $8\frac{5}{8}$. 10. $7\frac{3}{4}$. 11. $7\frac{13}{24}$. 12. $14\frac{1}{10}$. 13. $5\frac{5}{8}$. 14. $4\frac{7}{12}$. 15. $15\frac{7}{12}$. 16. $15\frac{1}{12}$. 17. $6\frac{13}{16}$. 18. $6\frac{23}{40}$. 19. $3\frac{1}{8}$ in. 20. $17\frac{3}{8}$ in.

Page 64. 1. 12. 2. 21. 3. 9. 4. 18. 5. 24. 6. 56. 7. 16. 8. 6. 9. 12. 10. 30. 11. 24. 12. 24. 13. 140. 14. $14\frac{1}{2}$. 15. $20\frac{1}{3}$. 16. $20\frac{1}{3}$. 17. $16\frac{7}{10}$. 18. $22\frac{8}{9}$. 19. $90\frac{1}{16}$. 20. $21\frac{7}{16}$. 21. $33\frac{7}{12}$. 22. $41\frac{7}{8}$. 23. $32\frac{1}{8}$. 24. $25\frac{1}{2}$. 25. $32\frac{1}{16}$. 26. $114\frac{27}{40}$. 27. $91\frac{3}{4}$ in. 28. $33\frac{7}{8}$.

Page 65. 1. 315. 2. 140. 3. $\frac{3}{40}$. 4. $\frac{5}{12}$ yd. 5. $6\frac{1}{4}$. 6. $6\frac{1}{8}$. 7. $4\frac{1}{8}$. 8. $5\frac{3}{8}$. 9. $3\frac{5}{8}$. 10. $5\frac{1}{2}$. 11. $3\frac{1}{12}$. 12. $3\frac{7}{12}$. 13. $4\frac{41}{48}$. 14. $14\frac{7}{40}$. 15. $7\frac{5}{24}$ in. 16. $2\frac{1}{8}$ yd. 17. $2\frac{3}{4}$. 18. $15\frac{1}{8}$. 19. $13\frac{3}{16}$. 20. $34\frac{5}{12}$. 21. $2\frac{1}{4}$. 22. $3\frac{7}{8}$. 23. $5\frac{5}{16}$. 24. $31\frac{9}{16}$. 25. $\frac{1}{24}$. 26. $\frac{17}{30}$. 27. $3\frac{13}{16}$. 28. $4\frac{15}{16}$. 29. $1\frac{33}{40}$. 30. $4\frac{29}{32}$. 31. $6\frac{1}{8}$. 32. $8\frac{1}{2}$. 33. $9\frac{7}{8}$. 34. $11\frac{5}{8}$. 35. $21\frac{2}{16}$. 36. $28\frac{13}{16}$. 37. $8\frac{1}{4}$. 38. $17\frac{1}{12}$. 39. $28\frac{5}{32}$. 40. $32\frac{11}{12}$.

Page 66. i. 1. 1621. 2. 259,091. 3. $363\frac{3}{8}$. 4. $552\frac{7}{8}$. 5. \$4,902,375. 6. \$3,605,250. 7. \$3717.53. 8. \$3.19. ii. 1. 1591. 2. 1,146,442. 3. 313,429. 4. 293,501,180. 5. 47,128,875. 6. \$17.98. 7. \$6049.45$\frac{5}{7}$. 8. 71. iii. 1. 1296. 2. 299,465. 3. 365,241. 4. 579,106,080. 5. 27,749,020. 6. 44,624,400. 7. \$1.98.

8. 186. iv. 1. 184. 2. 992,311. 3. 872,986. 4. $324,084.75. 5. 27,052,830. 6. 54,711,695. 7. $6.66. 8. $418\frac{1}{2}\frac{4}{7}$. v. 1. $\frac{26}{3}$. 2. $14\frac{1}{8}$. 3. $16\frac{1}{8}$. 4. $463\frac{7}{8}$. 5. $3\frac{1}{4}$. 6. $4\frac{1}{8}$. 7. $12\frac{1}{4}$. 8. $15\frac{1}{40}$. vi. 1. $\frac{32}{5}$. 2. 5. 3. $19\frac{3}{8}$. 4. $4\frac{1}{12}$. 5. $353\frac{1}{8}$. 6. $8.12. 7. $76\frac{8}{11}$. 8. $14\frac{3}{16}$ ft. 9. $6\frac{7}{8}$ in. 10. $5072.59.

Page 68. 1. Three million, six hundred twenty-eight thousand, three hundred nine; fifteen hundred ninety-three. 2. 19,000,006. 3. 121,823. 4. $473.82. 5. 614,272. 6. $843.87. 7. 174,045,410. 8. $461,466.80. 9. 545. 10. 2566; 1565. 11. $\frac{3}{4}$; $\frac{3}{4}$; $\frac{1}{2}$; $\frac{9}{16}$; $\frac{3}{4}$; 1. 12. $\frac{28}{4}$; $\frac{29}{4}$; $\frac{39}{4}$. 13. $\frac{104}{3}$; $\frac{206}{8}$. 14. 17; $11\frac{3}{4}$; $7\frac{5}{8}$. 15. $24\frac{3}{4}$. 16. $\frac{7}{8}$ in. 17. $57\frac{5}{8}$. 18. 7; 4 in. 19. 265,625. 20. 768,912. 21. 8,781,696. 22. 13,048,035. 23. 10,596,960. 24. 477,391. 25. 16,461,296. 26. 144. 27. 216. 28. 129. 29. 144. 30. $169\frac{48}{51}$. 31. 408. 32. 912. 33. $965\frac{1}{2}\frac{7}{2}$.

Page 71. I. $35.71; $37.66. II. $35.77; $19.98.

Page 72. 1. 36. 2. $12. 3. $2.58. 4. $6.80. 5. $20.

Page 73. 1. $56\frac{4}{7}$ sec. 2. $1\frac{3}{8}$ yd. 3. $82\frac{1}{2}$. 4. $16\frac{7}{8}$ in. 5. $\frac{11}{30}$; $\frac{9}{15}$; $\frac{1}{2}$.

Page 76. 28. $\frac{1}{13}$. 29. $\frac{1}{5}$. 30. $\frac{38}{13}$. 31. $\frac{45}{64}$. 32. $\frac{7}{51}$.

Page 77. 6. $2\frac{2}{3}$. 7. $6\frac{5}{12}$. 8. 27. 9. $7\frac{1}{2}$. 10. 36.

Page 78. 5. 70; 56. 6. 80; 48. 7. 196; 28. 8. 549. 9. 2863. 10. 6470. 11. 6105. 12. 1290. 13. 6480. 14. 2110. 15. 1465. 16. 1866.

Page 81. 1. $\frac{1}{8}$ hr.; $\frac{1}{12}$ hr. 2. $1\frac{5}{12}$ min. 3. $\frac{1}{2}$. 4. $\frac{5}{8}$. 5. $\frac{1}{2}$ mi. 6. $\frac{5}{16}$ yd. 7. $\frac{48}{88}$. 8. $\frac{1}{10}$. 9. $\frac{1}{2}$. 10. $\frac{1}{3}$. 11. $\frac{2}{5}$. 12. $\frac{1}{3}$. 13. $\frac{24}{64}$. 14. $\frac{3}{5}$. 15. $\frac{5}{8}$. 16. $\frac{5}{32}$. 17. $\frac{5}{16}$. 18. $\frac{1}{6}$. 19. $\frac{3}{8}$. 20. $\frac{7}{24}$ mi.; $\frac{7}{48}$ mi. 21. $1\frac{9}{40}$ yd. 22. $\frac{1}{4}$; $\frac{1}{8}$.

Page 82. 1. $6\frac{3}{8}$. 2. $2\frac{13}{8}$. 3. $4\frac{1}{2}$ in. 4. $2\frac{11}{32}$ ft. 5. $4\frac{13}{32}$ in. 6. $1\frac{25}{32}$ yd. 7. $8\frac{7}{8}$ mi. 8. 17 mi. 9. $1\frac{13}{16}$ in. 10. $8\frac{1}{16}$ lb. 11. $5\frac{13}{16}$ in. 12. $11\frac{5}{8}$ yd. 13. $7\frac{19}{32}$ yd. 14. $9\frac{27}{32}$ mi. 15. $15\frac{5}{16}$ mi. 16. $6\frac{1}{4}$ ft. 17. $12\frac{1}{8}$ ft.

Page 85. 1. 20. 2. 30. 3. 21. 4. $34\frac{7}{8}$. 5. $43\frac{1}{2}$. 6. 9. 7. $17\frac{1}{3}$. 8. 51. 9. $141\frac{3}{4}$. 10. $336\frac{7}{8}$. 11. $6\frac{1}{4}$. 12. 15. 13. $55\frac{25}{32}$. 14. $26\frac{2}{15}$. 15. $65\frac{27}{64}$. 16. $115\frac{1}{2}$. 17. $13\frac{3}{4}$. 18. 30. 19. 70. 20. 168. 21. 42. 22. $153\frac{1}{3}$. 23. $24\frac{1}{2}$. 24. $42\frac{2}{3}$. 25. $233\frac{1}{3}$. 26. 645. 27. 207. 28. $4\frac{5}{16}$. 29. $15\frac{3}{32}$. 30. $4\frac{17}{64}$. 31. $9\frac{83}{128}$. 32. $25\frac{37}{128}$. 33. $83\frac{1}{4}$ mi. 34. $63\frac{7}{8}$ mi. 35. 84. 36. 132. 37. 324. 38. 235. 39. 198. 40. 98. 41. 268. 42. $26\frac{1}{4}$. 43. 116. 44. $53\frac{3}{4}$. 45. $19\frac{9}{64}$. 46. $194\frac{29}{64}$. 47. $34\frac{1}{8}$. 48. $17\frac{3}{16}$. 49. $5\frac{1}{32}$.

Page 86. 1. $7\frac{1}{2}$. 2. $6\frac{3}{4}$. 3. 27 in. 4. $24\frac{1}{2}$. 5. 5. 6. $\frac{567}{160}$. 7. $\frac{2}{5}$; $\frac{6}{25}$. 8. $31\frac{1}{2}$. 9. $\frac{5}{8}$. 10. $\frac{3}{4}$; $\frac{3}{8}$; $\frac{9}{16}$. 11. $57\frac{1}{2}$. 12. 568. 13. 2696. 14. 1912. 15. $112\frac{1}{8}$. 16. $199\frac{3}{4}$.

Page 88. 1. $8.67; $76. 2. $26.67; $18.67. 3. $25. 4. $165.33. 5. $294. 6. $144. 7. $14; $28. 8. $22; $44. 9. $26; $52. 10. $32; $64. 11. $180; $360. 12. $8; $6. 13. $16; $12. 14. $24; $18. 15. $32; $24. 16. $40; $30.

Page 89. 16. $14. 17. $30. 18. $6. 19. $2. 20. $4. 21. $6. 22. $8. 23. $15. 24. $24. 25. $24. 26. $37.50. 27. $45. 28. $62. 29. $48. 30. $65. 31. $63. 32. $5. 33. $9. 34. $33. 35. $33. 36. $66. 37. $105. 38. $46.87½. 39. $83.33⅓. 40. $23.33⅓. 41. $115.50. 42. 6¼. 43. 41⅔. 44. 31¼. 45. 43¾. 46. 3⅛. 47. 9⅜.

Page 90. 1. $57.36; $2.64. 2. $45.01; $4.99. 3. $21.99; $3.01. 4. $26.55; $3.45. 5. $14.03; $5.97. 6. $33.54; $16.46.

Page 94. 1. ⅛. 2. 2⅝ yd. 3. 1⅛ yd. 4. $\frac{5}{16}$; 2½ hr. 5. $\frac{5}{16}$; ⅝. 6. 22½ mi.; 22½ mi. 7. ⅛. 8. ⅛. 9. $\frac{2}{35}$. 10. ⅛. 11. ⅔. 12. ⅜. 13. $\frac{5}{56}$. 14. ⅜. 15. ⅓. 16. ⅓. 17. ⅜. 18. $\frac{5}{24}$. 19. $\frac{3}{14}$. 20. $\frac{3}{32}$. 21. $\frac{1}{16}$. 22. ⅜. 23. $\frac{1}{12}$. 24. $\frac{1}{16}$. 25. $\frac{1}{24}$. 26. $\frac{5}{32}$. 27. $\frac{1}{7}$. 28. ⅛. 29. $\frac{5}{64}$. 30. $\frac{1}{32}$. 31. 8⅛. 32. 6⅜. 33. 13⅛. 34. 3⅛. 35. 16⅓. 36. 15¾. 37. 17$\frac{3}{16}$. 38. 126$\frac{9}{10}$.

Page 95. 1. 3. 2. 36. 3. 412. 4. 1728. 5. 256. 6. 384. 7. 180. 8. 288. 9. 576. 10. 10. 11. 7. 12. 19. 13. 47. 14. 19. 15. 12. 16. 29. 17. 57. 18. 118. 19. 83. 20. 66. 21. 88. 22. 188. 23. 168. 24. 248.

Page 98. 1. $1\frac{5}{16}$; $\frac{16}{21}$. 2. $1\frac{1}{9}$; $\frac{8}{10}$. 3. $\frac{8}{20}$; $2\frac{2}{3}$. 4. $3\frac{1}{8}$; $\frac{8}{25}$. 5. $1\frac{1}{24}$; $\frac{24}{25}$. 6. ½; 2. 7. $1\frac{1}{3}$; ⅝. 8. $1\frac{1}{3}$; ¾. 9. $1\frac{1}{3}$; ⅝. 10. $\frac{10}{21}$; $2\frac{1}{10}$. 11. $\frac{8}{10}$; $1\frac{1}{3}$. 12. $4\frac{2}{3}$; $\frac{3}{14}$. 13. $\frac{5}{14}$; $2\frac{2}{3}$. 14. $1\frac{1}{3}$; ⅝. 15. $\frac{5}{16}$; $3\frac{1}{3}$. 16. ¼; 4. 17. 5. 18. $3\frac{3}{7}$. 19. $7\frac{1}{3}$. 20. $42\frac{2}{3}$. 21. 96. 22. $\frac{8}{13}$. 23. $1\frac{1}{3}$. 24. $1\frac{7}{8}$. 25. ⅜. 26. $1\frac{15}{24}$. 27. $\frac{24}{40}$. 28. $1\frac{13}{24}$. 29. $\frac{54}{84}$. 30. $1\frac{2}{3}$. 31. ⅔. 32. $1\frac{1}{3}$. 33. ⅓.

Page 99. 1. 18; 30. 2. 15. 3. 16. 4. 19. 5. 9. 6. 5. 7. 12; 25½; 54. 8. 27. 9. 35. 10. 100. 11. 16. 12. $4\frac{1}{2}$. 13. $12\frac{12}{13}$. 14. 30. 15. 5. 16. 4. 17. 4. 18. 7. 19. $3\frac{33}{40}$. 20. $1\frac{5}{16}$. 21. 2. 22. 3. 23. 3.

Page 102. 1. ⅓. 2. ¼. 3. ⅝. 4. ⅚. 5. $\frac{8}{15}$. 6. ⅛. 7. $\frac{3}{10}$. 8. $\frac{5}{14}$. 9. ⅓. 10. ⅔. 11. ¾. 12. ¼. 13. ⅓. 14. $\frac{20}{77}$. 15. $\frac{7}{10}$. 16. ½; ¾. 17. 20. 18. 35. 19. 36. 20. 49. 21. 1. 22. 2. 23. ⅘. 24. $4\frac{2}{3}$. 25. 10. 26. 6. 27. 16. 28. 10½. 29. 19⅛. 30. 2 mi. 31. ⅝.

Page 103. 1. 1. 2. $4\frac{4}{9}$. 3. $\frac{33}{50}$. 4. 1. 5. 7245. 6. 24,990. 7. 10,680. 8. 211,200. 9. 1824. 10. 6021¾.

Page 104. 1. ⅝. 2. $\frac{8}{15}$. 3. $10\frac{4}{5}$. 4. $7\frac{8}{15}$. 5. $5\frac{5}{9}$. 6. $12\frac{7}{15}$. 7. $16\frac{18}{20}$. 8. $17\frac{1}{2}$. 9. $\frac{3}{10}$. 10. $\frac{7}{20}$. 11. $7\frac{3}{10}$. 12. $2\frac{7}{20}$. 13. $1\frac{1}{6}$. 14. $1\frac{19}{24}$. 15. $4\frac{7}{8}$. 16. $1\frac{5}{8}$. 17. 12. 18. 79. 19. $15\frac{3}{4}$. 20. $45\frac{3}{8}$. 21. $28\frac{11}{16}$. 22. $18\frac{3}{5}$. 23. $15\frac{13}{16}$. 24. $30\frac{3}{5}$. 25. $\frac{25}{88}$. 26. ⅓. 27. $5\frac{7}{16}$. 28. 11. 29. $1\frac{3}{4}$. 30. $\frac{8}{15}$. 31. $4\frac{3}{4}$. 32. 5. 33. $\frac{8}{25}$ cu. ft. 34. 215 sq. ft. 35. 574 sq. in. 36. ⅓ cu. in. 37. 4365 sq. ft. 38. 819 sq. in. 39. 32. 40. $1\frac{3}{33}$ in. 41. 16,464. 42. $3.88.

Page 106. 1. 1320. 2. 440. 3. 99. 4. 198. 5. 18. 6. 126. 7. 24¾. 8. 54. 9. 8¼. 10. 2640. 11. 135. 12. 660 ft.; 220 yd.; ⅛ mi. 13. 9 mi. 3960 ft. 14. 64 in.; 5 ft. 4 in.

Page 107. 1. 504. 2. 31½ sq. ft. 3. 60½ sq. yd.; 544½ sq. ft. 4. 63; 272¼.

Page 108. 7. ¼. 8. $\frac{1}{16}$. 9. 24 sq. in. 10. ⅛.

Page 109. 1. 7200. 2. 64. 3. 1890 cu. in. 4. 378 cu. in. 5. 15,232 cu. in. 6. 42,624 cu. in.

Page 110. 4. 5000. 5. 15,500. 6. 31,500. 7. 34,500. 8. 53,500. 9. 1¼. 10. 1½. 11. 1⅞. 12. 2⅛. 13. 40. 14. 44. 15. 52. 16. 120. 17. 156.

Page 111. 1. 6. 2. 12. 3. 24. 4. 16. 5. 48. 6. 2. 7. 4. 8. 7. 9. 4. 10. 60. 11. 1. 12. 8½. 13. 8. 14. 2. 15. 38.

Page 112. 1. 1. 2. 15. 3. 16. 4. 56. 5. 64. 6. 4½. 7. 6. 8. 60. 9. 80. 10. 240. 11. 1. 12. 2. 13. 14. 14. 38. 15. 35. 16. 1. 17. 8. 18. ½. 19. 16. 20. 24.

Page 113. 1. 150. 2. 5. 3. 180. 4. 90. 5. 2160. 6. 36. 7. 5. 8. 60. 9. 54. 10. 168. 11. 126. 12. 11. 13. 133. 14. 161. 15. 266.

Page 115. 1. 67½. 2. 33¾. 3. 128¼. 4. 360,000; 6000. 6. 80¢; $6. 7. 5832. 8. First; $6. 9. $3.50. 10. $2.50; $1.25; $6.

Page 117. 1. 117. 2. 117. 3. 154. 4. 88. 5. 20. 6. 624. 7. 1428. 8. 120. 9. 116. 10. 14,580. 11. 5,971,840. 12. 188. 13. 380.

Page 118. 1. 10⅛. 2. 2 sq. ft. 82 sq. in. 3. 9⅝. 4. 19$\frac{7}{8}$. 5. 4 lb. 6. 80 hr. 7. 16 bu. 8. 10½ ft. 9. 4 yd. 10. 23 rd. 5½ ft.

Page 119. 1. 77 ft. 9 in. 2. 132 ft. 6 in. 3. 169 sq. ft. 81 sq. in. 4. 171 sq. ft. 106 sq. in. 5. 94 yd. 6. 147 ft. 2 in. 7. 942 lb. 2 oz. 8. 1168 lb. 2 oz.

Page 120. 1. 5 lb. 13 oz. 2. 8 yd. 16 in. 3. 26 lb. 6 oz. 4. 11 bu. 3 pk. 5. 6 gal. 3 qt. 6. 16 bu. 26 qt. 7. 52 qt. 1 pt. 8. 11 ft. 8¼ in. 9. 33 ft. 8 in. 10. 13 lb. 12 oz. 11. 11 yd. 30 in. 12. 37 A. 147 sq. rd. 13. 48 sq. ft. 118 sq. in. 14. 27 cu. ft. 1436 cu. in.

Page 121. 1. 23,437 lb. 8 oz. 2. 94 ft. 3. 58 gal. 2 qt. 4. 19 lb. 8 oz.

Page 122. 1. 5 ft. 11⅛ in. 2. 1 ft. 11 in. 3. 9° 32′. 4. 2 hr. 52 min. 30 sec. 5. 15 mi.; 37½ mi.; 63¾ mi. 6. 12½ mi. 7. 12,571⅞.

Page 123. 1. 184 ft. 2 in. 2. 243 yd. 34 in. 3. 166 ft. 4. 204 gal. 1 qt. 5. 189 ft. 6. 178 bu. 7. 211 ft. 9 in. 8. 271 bu. 9. 8 lb. 1 oz. 10. 4 ft. 2 in. 11. 75 ft. 12. 1 bu. 30 qt. 13. 191 ft. 10 in. 14. 193 lb. 5 oz. 15. 112 yd. 13 in. 16. 5 bu. 15 qt. 17. 308 lb. 8 oz. 18. 185 lb. 12 oz. 19. 243 ft. 9 in. 20. 178 bu. 21. 299 lb. 12 oz.; 599 lb. 8 oz. 22. 591 lb.; 394 lb. 23. 2514 ft.; 3142 ft. 6 in. 24. 1275 bu. 12 qt.; 1700 bu. 16 qt. 25. 22 lb. 14 oz. 26. 83. 27. 61 lb. 12 oz. 28. 131 lb. 11 oz. 29. 108; 1944. 30. 82 lb. 5 oz.; 41 lb. 2½ oz.

Page 124. **i. 1.** 1026 in. **2.** $7\frac{1}{12}$ ft. **3.** 124 oz. **4.** $5\frac{1}{2}$ pk. **5.** 9 ft. 1 in. **6.** $26\frac{1}{4}$ ft. **7.** 48 ft. 4 in. **8.** $27\frac{1}{2}$ ft. **ii. 1.** 561 in. **2.** 3 yd. **3.** $25\frac{1}{2}$. **4.** $6\frac{7}{9}$ sq. yd. **5.** 9 yd. 2 in. **6.** $42\frac{3}{4}\frac{1}{2}$ yd. **7.** 45 bu. 18 qt. **8.** 31. **iii. 1.** $15\frac{1}{2}$ yd. **2.** 4 sq. mi. **3.** $7\frac{3}{8}$. **4.** 1296 sq. in. **5.** 9 lb. 6 oz. **6.** $73\frac{5}{8}$ lb. **7.** 67 lb. 8 oz. **8.** 2 yd. **iv. 1.** $20\frac{1}{2}$ yd. **2.** 3840 sq. rd. **3.** 4500 lb. **4.** 5 sq. yd. **5.** $7\frac{5}{8}$ ft. **6.** $25\frac{5}{8}$ ft. **7.** 42 ft. 8 in. **8.** $5\frac{1}{4}$ ft. **v. 1.** 191 yd. **2.** 2880 A. **3.** 290 qt. **4.** 152 pk. **5.** $8\frac{1}{4}$ hr. **6.** 9 bu. **7.** $70\frac{1}{16}$ yd. **8.** 98 yd. **vi. 1.** 1530 in. **2.** 2 cd. **3.** 68 pt. **4.** 225 min. **5.** $12\frac{2}{3}$ yd. **6.** 16 ft. 2 in. **7.** 78 gal. **8.** 3 yd. 1 ft.

Page 125. **1.** \$4314.16. **2.** \$3402.45. **3.** \$2893.59. **4.** \$3691.37. **5.** \$8776.29. **6.** \$3177.65. **7.** \$774.03. **8.** \$3322.80. **9.** \$775. **10.** \$1550. **11.** \$1.45. **12.** \$1217.05. **13.** \$93. **14.** \$176. **15.** $6\frac{3}{4}$. **16.** \$53.05. **17.** 242 ft. 8 in. **18.** 18 ft. $11\frac{1}{2}$ in. **19.** 47 ft. $1\frac{1}{2}$ in. **20.** 35 ft. 3 in. **21.** 46 ft. $9\frac{1}{8}$ in. **22.** 105 yd. 34 in. **23.** $\frac{5}{12}$. **24.** $1\frac{7}{20}$. **25.** $1\frac{5}{8}$. **26.** $\frac{1}{8}$. **27.** $62\frac{1}{2}$. **28.** $\frac{3}{4}\frac{3}{8}$. **29.** $4\frac{3}{8}$. **30.** $3\frac{7}{8}$. **31.** $6\frac{9}{16}$. **32.** $1\frac{1}{16}$. **33.** $\frac{3}{8}$. **34.** $1\frac{5}{7}$. **35.** $38\frac{1}{4}$ in. **36.** $4\frac{1}{4}$ in. **37.** $4\frac{3}{16}$ ft. **38.** $3\frac{3}{8}$ yd.

Page 129. **1.** 68. **2.** $30\frac{5}{16}$ bu. **3.** \$3.78. **4.** 12 mi. **5.** \$27.28; $5\frac{1}{2}$. **6.** \$6420. **7.** 400.

Page 130. **1.** $\frac{5}{8}$. **2.** $\frac{1}{8}$; $\frac{1}{5}$. **3.** $\frac{9}{10}$. **4.** \$2400. **5.** \$6. **6.** \$1.50. **7.** \$3201.75. **8.** \$27,125. **9.** \$38.25.

Page 131. **1.** 50. **2.** \$121. **3.** $42\frac{3}{4}$¢. **4.** \$26. **5.** 100. **6.** 20. **7.** \$36.40.

Page 132. **1.** \$20. **2.** \$19.20. **3.** \$1.12. **4.** 1600. **5.** 157. **6.** 46. **7.** 18. **8.** \$27. **9.** \$9.50.

Page 136. **15.** 0.12; 0.55; 0.09; 0.01; 0.20. **16.** 0.125; 0.047; 0.010; 0.005.

Page 137. **1.** $\frac{1}{5}$. **2.** $\frac{4}{5}$. **3.** $\frac{7}{10}$. **4.** $\frac{3}{8}$. **5.** $3\frac{3}{4}$. **6.** $\frac{5}{8}$. **7.** $\frac{1}{8}$; \$400. **8.** $\frac{1}{3}$; \$2370.25. **9.** $\frac{2}{3}$. **10.** $\frac{4}{3}$. **11.** $\frac{5}{3}$. **12.** $\frac{8}{3}$. **13.** $\frac{10}{3}$.

Page 138. **1.** 1.75; 2.60. **2.** 0.40; 0.27. **3.** 0.30; 0.03. **4.** 0.700; 0.009. **5.** 0.600; 0.548. **6.** 0.8; 0.9. **7.** 0.70; 0.08. **8.** 0.130; 0.406. **9.** 0.350; 0.008. **10.** 0.69; 0.07. **11.** 24.30; 14.25. **12.** 15.060; 28.003.

Page 139. **1.** 11.35. **2.** 0.75. **3.** 0.625. **4.** 0.875. **5.** 0.8. **6.** 0.063. **7.** 0.188. **8.** 0.813. **9.** 0.031. **10.** 0.156. **11.** 0.833. **12.** 0.889. **13.** 0.36. **14.** 7.256. **15.** 3.36. **16.** 6.275.

Page 140. **1.** 11.6 in. **2.** 453.6 mi. **3.** 2160.5 sq. ft. **4.** 5.01 in. **5.** 72.6. **6.** 2171.7 mi. **7.** 3.5°. **8.** 293 mi. **9.** 5.55 in. **10.** 0.45 in. **11.** 159.8 ft. **12.** 48.8 ft. **13.** 31.04 mi.

Page 142. **1.** 6.6. **2.** 22.4. **3.** 26.75. **4.** 78.26. **5.** 10.81. **6.** 1.22. **7.** 15.9. **8.** 16. **9.** 42.6. **10.** 56.4. **11.** 60.3. **12.** 30.73. **13.** 43.6. **14.** 109.77. **15.** 14.021. **16.** 13.907. **17.** 231.81. **18.** 271.85. **19.** 87.42. **20.** 164.23. **21.** 143.17. **22.** 184.29. **23.** 129.7. **24.** 184.35. **25.** 18.776. **26.** 30.522.

Page 143. 1. $15.99. 2. $77.71. 3. 92.93. 4. 47.85. 5. 92.606. 6. 60.16. 7. 28.36. 8. 46.64. 9. 36.34. 10. 79.43. 11. 328.43. 12. 460.09. 13. 35.082. 14. 4966.2. 15. 27.876. 16. 2.73. 17. 11.91. 18. 22.873. 19. 46,609.822. 20. 193,071.7725. 21. 12,376.5144. 22. 2.3 in.; 4.5 in. 23. 22.8 ft. 24. 24.6 in.

Page 145. 1. 96.75 ft. 2. 2952.75. 3. 266.4 ft. 4. 21.12 ft. 5. 51.04 in. 6. 21.28 mi. 7. 18.48 in. 8. 78.75 ft. 9. 6.72 oz. 10. 5.84 ft. 11. 18.85 yd. 12. 56.56 ft. 13. 8.52 in. 14. 15.24 ft. 15. 44.45 lb. 16. 42.93 oz. 17. 31.92 mi. 18. 51.28 yd. 19. 22.15 in. 20. 89.64 ft. 21. 17 mi. 22. 39.12. 23. 460.32. 24. 308.32. 25. 362.88. 26. 699.55. 27. 305.44. 28. 197.54. 29. 641.16. 30. 324.72. 31. 773.76. 32. 562.52. 33. 6281.08. 34. 5958.26. 35. 3852.2. 36. 7650.39. 37. 13,428.366. 38. 10,115.3. 39. 37,468.34. 40. 430.98. 41. 164.79. 42. 102.85. 43. 332.15. 44. 199.35. 45. 332.88. 46. 236.61. 47. 669.61. 48. 49.657. 49. 70.73. 50. 32.819. 51. 13.547. 52. 14.706. 53. 139.68. 54. 11.808. 55. 17.856. 56. 27.072. 57. 35.87.

Page 147. 1. 4224. 2. 2112. 3. 313.5. 4. $1215.45. 5. 6.15 in. 6. 14.25 ft. 7. 2636.38. 8. 42.10 ft. 9. 57.5 lb. 10. 73.13 lb.

Page 148. 18. 1.47. 19. 2.94. 20. 11. 21. 2.91. 22. 6.96. 23. 16.94. 24. 2.34. 25. 4.98. 26. 57.84. 27. 8.37. 28. 2.43. 29. 55.38. 30. 66.6. 31. 7.92. 32. 654.5. 33. 25.11. 34. 46.4. 35. 52.26. 36. 57.6. 37. 5.46. 38. 120.6. 39. 9.86. 40. 44.4. 41. 3.92. 42. 49. 43. 3.64. 44. 28.2. 45. 2.82. 46. 30.618. 47. 926.4. 48. 19.062. 49. 7217.52. 50. 1666.62. 51. 315.3327. 52. 912.2337. 53. 3108.63. 54. 2832.361. 55. 3703.648. 56. 0.42.

Page 149. 17. 437. 18. 275.6. 19. 546.7. 20. 2592. 21. 1464. 22. 3639.2. 23. 43,548. 24. 46,963. 25. 30,800. 26. 2250. 27. 7450. 28. 413,504. 29. 487,780. 30. 4,667,488. 31. 5,015,920. 32. 2,711,310. 33. 2,286,315. 34. 31,250.

Page 151. 1. 14.4 in. 2. 2.54 in. 3. $5.37. 4. 10.85. 5. 31.83 ft. 6. 712.8. 7. 416.5. 8. 657.8. 9. 11.6. 10. 5.8. 11. 4.8. 12. 0.3. 13. 0.2. 14. 0.2. 15. 10.45. 16. 8.58. 17. 4.02. 18. 28.31. 19. 23.61. 20. 12.39. 21. 12.08. 22. 1.08. 23. 1.68. 24. 1.71. 25. 2.25. 26. 1.85. 27. 2.06. 28. 1.68. 29. 6.83. 30. 14.34. 31. 6.88. 32. 14.08. 33. 6.57. 34. 20.01. 35. 18.61. 36. 4.592. 37. 1.359. 38. 33.882.

Page 152. 1. 2.9. 2. 1.18. 3. 0.57. 4. 13.49. 5. 16.52. 6. 4.69. 7. 15.06. 8. 17.9. 9. 0.63. 10. 2.28. 11. 0.746. 12. 0.683. 13. 2.812. 14. 0.732. 15. 0.826. 16. 0.025. 17. 0.031. 18. 0.042. 19. 0.024. 20. 0.023. 21. 0.041.

Page 154. 1. 23 min.; 31.11 min. 2. 52 min.; 1 hr. 10 min. 3. 13.5. 4. 16 min. 5. 0.11. 6. 0.025. 7. 0.0345. 8. 0.01624. 9. 1.168. 10. 0.01031. 11. 0.0001052. 12. 0.00826. 13. 11.853. 14. 270.565. 15. 14.271. 16. 6.213. 17. 5.792. 18. 99.034. 19. 4.916. 20. 1568.889. 21. 1.050. 22. 3.8 hr. 23. 0.7692.

Page 155. 1. 2. 3. 0.21. 3. 1.33. 4. 0.17. 5. 3.5. 6. 3.33. 7. 56. 8. 0.1. 9. 0.3. 10. 0.1.

Page 156. 1. \$10.20. 2. \$15.66. 3. \$23.95.

Page 157. 1. \$16.05. 2. \$57.15. 3. \$9.10. 4. \$23.50. 5. \$15.10. 6. \$37.50.

Page 158. 1. \$2.19. 2. \$12.67. 3. \$2.24. 4. \$5.33. 5. \$15.08. 6. \$8.27. 7. \$3.04.

Page 159. 1. \$1310. 2. \$2755. 3. \$84. 4. \$216. 5. \$120. 6. \$120. 7. \$21. 8. \$1184. 9. \$52; \$55; \$65. 10. 400. 11. 440. 12. \$8.60. 13. \$10.40; \$15.60. 14. \$468. 15. \$37.

Page 160. 1. 316.7. 2. \$16,050. 3. 51 mi. 4. 44.5 mi. 5. 19.64 mi. 6. 18.35 mi. 7. 36.44 mi.; 39.84 mi. 8. 0.39. 9. Frank.

Page 161. 1. 480 mi. 2. \$10. 3. 168. 4. 193; 172. 5. \$8.26; \$2.36. 6. \$37.84. 7. 201.25. 8. 4632.5. 9. 1000. 10. 0.58 oz.; 0.6 oz. 11. 4.23 oz. 12. 1203.75 lb. 13. 3.22. 14. 1.56. 15. 1.17.

Page 166. 1. 142. 2. 17; 51. 3. 54. 4. 427. 5. 121; 847. 6. 1039. 7. 893. 8. 8195. 9. \$1104.18. 10. 638 bu. 11. 21 ft. 1 in. 12. \$12.41. 13. \$12.22. 14. 16; \$2000; 112.

Page 167. 5. 298. 6. \$2476.20. 7. \$205.76. 8. 698 yd. 9. 1231 bu. 10. 2176 mi.

Page 168. 10. 129. 11. \$1700. 12. 93 mi. 13. 225 ft. 14. 371 lb. 15. $12\frac{1}{2}\%$. 16. 35%. 17. 25%.

Page 169. 1. $\frac{9}{25}$. 2. $\frac{16}{25}$. 3. $\frac{1}{4}$. 4. $\frac{1}{16}$. 5. $\frac{3}{10}$. 6. $\frac{5}{4}$. 7. $\frac{9}{50}$. 8. $\frac{14}{25}$. 9. $\frac{23}{40}$. 10. $\frac{1}{3}$. 11. $87\frac{1}{2}\%$. 12. $62\frac{1}{2}\%$. 13. $87\frac{1}{2}\%$. 14. 20%. 15. 40%. 16. 60%. 17. 80%. 18. $6\frac{1}{4}\%$. 19. $18\frac{3}{4}\%$. 20. $106\frac{1}{4}\%$. 21. 100%. 22. 350%.

Page 171. 8. \$81,000. 9. 1203. 10. 28. 11. 16¢.

Page 172. 1. 20; 30. 2. 17. 3. 372. 4. 32,000. 5. 260. 6. 14. 7. 36. 8. \$1.06. 9. $\frac{3}{5}$; 20; 30. 10. $\frac{1}{4}$ yd.; $49\frac{3}{4}$ yd.

Page 173. 1. 1,125,000. 2. 2,339,020. 3. 100; 225. 4. $86\frac{1}{4}$. 5. 35,658. 6. 270,000; 122,400. 7. 114,400,000. 8. 2801 ft. 9. 13,146 ft.

Page 174. 1. 3400. 2. 12,400. 3. 13 lb. 3.2 oz. 4. \$4634. 5. 11.9 in.; 9.6 in. 6. \$99.20. 7. \$81.60. 8. 244 lb. 9. 1220 lb.

Page 175. 1. \$1080. 2. \$11,520. 3. \$760. 4. \$162.88. 5. 5¢.

Page 177. 9. \$1320. 10. \$22. 11. \$11.90. 12. \$4.20. 13. \$11.20. 14. \$9. 15. \$7.82.

Page 179. 1. \$1084.17. 2. \$2077.25. 3. \$2649.60. 4. \$5717.07. 5. \$1416.39. 6. \$79.50. 7. \$76. 8. \$57.50. 9. \$94.20. 10. \$402.30. 11. \$502.50. 12. \$2112.

13. $2748. 14. $18.16. 15. $4.96. 16. $15.76. 17. $5.60. 18. $300. 19. $451.25. 20. $643.11. 21. $743.75. 22. $744.24. 23. $832.50. 24. $583.20. 25. $26.20. 26. $3618.72. 27. $4963. 28. $49.03. 29. $38.16. 30. $2523.30. 31. $29.82. 32. $42.05.

Page 181. 1. $31.25. 2. $132. 3. $2114.94. 4. $38.50; $346.50. 5. $259.50; $13,234.50. 6. $350. 7. $6.38. 8. $77.74. 9. $58.50. 10. $420.

Page 182. 1. $\frac{1}{10}$; 10%. 2. $\frac{1}{5}$; 20%. 3. $\frac{3}{10}$; 30%. 4. $\frac{1}{50}$; 2%. 5. $\frac{18}{50}$; 38%. 6. $\frac{3}{8}$; 37½%. 7. $\frac{3}{50}$; 6%. 8. $\frac{33}{100}$; 33%. 9. $\frac{1}{3}$; 33⅓%. 10. $\frac{37}{100}$; 37%. 11. $\frac{3}{5}$; 60%. 12. $\frac{2}{3}$; 66⅔%. 13. $\frac{87}{100}$; 87%. 14. $\frac{7}{8}$; 87½%. 15. $\frac{1}{6}$; 16⅔%. 16. 10,000%. 17. 20,000%. 18. 27,500%. 19. 32,500%. 20. 17,800%. 21. 105. 22. 29.4. 23. 46. 24. 123. 25. 43.5. 26. 76. 27. 19. 28. 22.5. 29. 250. 30. 640. 31. 221. 32. 234. 33. 423. 34. 720. 35. 183¾ lb. 36. 36. 37. 140. 38. 27.9. 39. 24.

Page 183. 1. $2625; $6125. 2. $2775. 3. $3024. 4. $1392. 5. $3426.50. 6. $4488. 7. $1.05. 8. $1.50. 9. $3. 10. $2.55. 11. $3.60. 12. $5.25. 13. $21. 14. $42.75. 15. $39.40. 16. $33.75. 17. $1050. 18. $120. 19. $160. 20. $420. 21. 296. 22. 333. 23. 385. 24. 60. 25. 64. 26. 360. 27. 420. 28. $12. 29. $30. 30. $22.50. 31. $30. 32. $48. 33. $1875. 34. $6. 35. $600. 36. $600. 37. $6. 38. 16.422. 39. $225. 40. $120. 41. $800. 42. 0.36. 43. 63.75. 44. 806.25. 45. 1226.25.

Page 184. 1. 137.5. 2. 283.5. 3. 352. 4. $18.84. 5. $35.84. 6. $78.12. 7. 927. 8. 927. 9. 927. 10. $600. 11. $403.75. 12. $660. 13. $897. 14. $1435. 15. $1349. 16. $1300. 17. $8547. 18. $3052. 19. $76.50. 20. $1122. 21. $4050. 22. Second; $23.20. 23. $59. 24. $123.75. 25. $278.

Page 185. 1. 19.08 ft. 2. 39.37. 3. 6082.56. 4. 112. 5. 190. 6. 270. 7. 644. 8. 16. 9. 58.4. 10. 83.25. 11. 33.075. 12. 1655. 13. 59. 14. 723. 15. 723. 16. $400. 17. $374. 18. $216. 19. $162. 20. $422. 21. $21.66. 22. $62.45. 23. $44.17. 24. $46.20. 25. $648.40. 26. $70.60. 27. $145.69. 28. $374.50. 29. $750. 30. $7. 31. $375. 32. $1420. 33. 1100; 1100; 1100. 34. 451; 451; 451.

Page 187. 25. $7.50. 26. $10.50. 27. $11.25, 28. $13.75. 29. $15. 30. $25.50. 31. $40.50. 32. $37.50. 33. $42.50. 34. $14. 35. $3.50. 36. $4. 37. $4.40. 38. $3.60. 39. $7.80. 40. $7.50. 41. $8.75. 42. $9. 43. $19.20. 44. $20.50. 45. $5.50. 46. $8.10. 47. $14. 48. $24.75. 49. $21. 50. $96.25. 51. $75. 52. $85. 53. $168. 54. $97.50.

Page 189. 19. $1.75. 20. $2.50. 21. $5.70. 22. $8.40. 23. $14.25. 24. $23.80. 25. $38. 26. $35.10. 27. $30. 28. $46.20. 29. $30. 30. $65.63. 31. $105.20. 32. $191.25. 33. $375. 34. $14.58; $20.42. 35. $46.92; $78.20; $109.48. 36. $13.75. 37. $540; $135.

Page 190. 1. \$11.25. 2. \$16. 3. \$315. 4. \$103.13. 5. \$47.25. 6. \$43.75. 7. \$7.27. 8. \$402.19. 9. \$198. 10. \$346.67. 11. \$840. 12. \$1300.

Page 191. 1. \$33.75. 2. \$50.63. 3. \$37.50. 4. \$52.50. 5. \$69.38. 6. \$71.25. 7. \$40.63. 8. \$32.50. 9. \$48.75. 10. \$44.69. 11. \$36.56. 12. \$21.68. 13. \$40.38. 14. \$49.30. 15. \$65.88. 16. \$102. 17. \$635. 18. \$3363.75. 19. \$246. 20. \$3535. 21. \$3312. 22. \$7.88.

Page 192. i. 1. $\frac{7}{8}$. 2. $1\frac{1}{12}$. 3. $18\frac{5}{8}$. 4. \$45. 5. \$123.36. 6. 615. 7. $2\frac{1}{2}$. 8. \$6.75. ii. 1. 13.328. 2. 4.622. 3. 12. 4. \$1.41. 5. 87.035 ft. 6. \$393.80. 7. \$12.15. 8. \$4078.80. iii. 1. 7.968. 2. 6.032. 3. 140. 4. 0.064. 5. \$13.10. 6. 2.025 in. 7. 8.4525 in. 8. \$6.84. 9. \$150. 10. \$2200. iv. 1. \$12.05. 2. \$36.60. 3. \$26. 4. \$37.80. 5. \$36.60. 6. \$16.25. 7. 14¢. 8. \$7.60. 9. \$1.75. 10. 237.5 ft. v. 1. \$437. 2. \$986. 4. \$430.20; \$454.10; \$501.90. 5. \$57,200; \$93,340.

Page 193. 1. MDCCCCXXIV or MCMXXIV; MDCCCCXXX or MCMXXX; 69; 88. 2. 8708; 4354; 14,217,987; 3. 3. 21,987; 15,705; 59,195,286; 6. 4. 33,768; 25,326; 124,717,887; 7. 5. 36,234; 24,156; 182,347,605; 5. 6. 63,090; 49,070; 393,120,800; 8. 7. 35,505; 21,303; 201,696,804; 4. 8. 34,560; 27,648; 107,495,424; 9. 9. 28,301; 20,215; 98,075,094; 6. 10. 77,770; 62,216; 544,335,561; 9.
11. $1\frac{5}{12}$; $\frac{1}{12}$; $\frac{1}{2}$; $\frac{8}{8}$; $1\frac{1}{8}$. 12. $\frac{1}{24}$; $\frac{1}{24}$; $\frac{1}{8}$; $\frac{8}{8}$; $1\frac{1}{8}$. 13. $1\frac{7}{24}$; $\frac{1}{24}$; $\frac{5}{12}$; $1\frac{1}{15}$; $\frac{5}{16}$. 14. $\frac{12}{24}$; $\frac{18}{24}$; $\frac{1}{12}$; $5\frac{1}{3}$; $\frac{3}{16}$. 15. $\frac{7}{8}$; $\frac{6}{8}$; $\frac{3}{32}$; 6; $\frac{1}{8}$. 16. $1\frac{3}{8}$; $\frac{1}{8}$; $\frac{15}{32}$; $1\frac{1}{5}$; $\frac{6}{8}$. 17. $\frac{11}{16}$; $\frac{5}{16}$; $\frac{3}{32}$; $2\frac{3}{8}$; $\frac{3}{8}$. 18. $\frac{21}{48}$; $\frac{1}{48}$; $\frac{5}{48}$; $1\frac{1}{15}$; $\frac{1}{16}$. 19. 3.36. 20. 6.75. 21. 5.82. 22. 0.666. 23. 0.544. 24. 0.95. 25. 2.3. 26. 0.8. 27. 0.45. 28. 0.63. 29. 4 ft. 10 in. 30. 3 ft. 7 in. 31. 2 ft. $1\frac{1}{2}$ in. 32. 2 lb. 7 oz. 33. 2 bu. 1 qt. 34. \$155,701.75. 35. \$361,017.75. 36. \$257,085.13. 37. 17.82. 38. 19.66. 39. 9.133. 40. 153.239. 41. 247.544. 42. 861.764.

Page 194. 1. $6\frac{3}{8}$. 2. $7\frac{7}{8}$. 3. $8\frac{1}{8}$. 4. $6\frac{3}{4}$. 5. $9\frac{1}{4}$. 6. $5\frac{5}{8}$. 7. $7\frac{1}{8}$. 8. $7\frac{7}{12}$. 9. $8\frac{11}{12}$. 10. $13\frac{5}{12}$. 11. $5\frac{7}{10}$. 12. $7\frac{9}{10}$. 13. $8\frac{17}{20}$. 14. $11\frac{1}{3}$. 15. $11\frac{7}{15}$. 16. $12\frac{1}{2}$. 17. $13\frac{1}{8}$. 18. $11\frac{3}{4}$. 19. $12\frac{5}{8}$. 20. $6\frac{5}{8}$. 21. $4\frac{1}{8}$. 22. $5\frac{3}{8}$. 23. $2\frac{1}{4}$. 24. $2\frac{3}{4}$. 25. $4\frac{1}{8}$. 26. $2\frac{23}{24}$. 27. $5\frac{19}{24}$. 28. $3\frac{5}{12}$. 29. $5\frac{83}{24}$. 30. $2\frac{1}{24}$. 31. 42. 32. $55\frac{1}{4}$. 33. $87\frac{1}{16}$. 34. $114\frac{1}{16}$. 35. $96\frac{1}{16}$. 36. $66\frac{1}{2}$. 37. 76. 38. $35\frac{5}{12}$. 39. $44\frac{1}{12}$. 40. $53\frac{4}{5}$. 41. $67\frac{1}{3}$. 42. $53\frac{1}{16}$. 43. $42\frac{1}{8}$. 44. $76\frac{1}{20}$. 45. $56\frac{37}{64}$. 46. $6\frac{2}{3}$. 47. $13\frac{1}{3}$. 48. 2. 49. $\frac{1}{2}$. 50. 3. 51. $4\frac{1}{8}$. 52. $12\frac{3}{8}$. 53. $5\frac{1}{2}$. 54. 3. 55. $\frac{1}{3}$. 56. 2. 57. $\frac{1}{2}$. 58. 3. 59. $3\frac{3}{56}$. 60. $\frac{56}{171}$.

Page 197. 1. \$36.40. 2. \$25.25. 3. \$28.80. 4. \$8.10. 5. 28¢. 6. 32¢. 7. 5¢.

Page 198. 1. 11.25 lb. 2. 18. 3. 0.35 lb. 4. 3.54 lb. 5. \$101.20. 6. 12.8 T. 7. 321.16 lb. 8. 30.

Page 199. 1. $50. 2. 1. 3. $\frac{3}{8}$. 4. $1\frac{1}{8}$. 6. 1829. 7. $368.90. 8. $1\frac{3}{4}$; $\frac{3}{4}$. 9. $23.52. 10. $189. 11. $105.45. 12. $206.25. 13. $29.46.

Page 203. 7. 3; 7; 11. 8. 5; 35. 9. 1.12 T. 10. 11 ft. 11. 13. 12. 320; 480; 40; 60. 13. $225\frac{1}{2}$. 14. $78\frac{3}{5}$. 15. 27,800; 13.9 T. 16. 55,800.

Page 204. 1. 1500 sq. ft. 2. 1500 sq. yd. 3. 6600 sq. ft. 4. 4000 sq. yd. 5. 30,000 sq. ft. 6. 106,000 sq. ft. 7. 400,000 sq. ft. 8. 7700 sq. in.

Page 205. 1. $\frac{1}{10}$. 2. $\frac{1}{4}$. 3. $4. 4. No; $3\frac{3}{32}$ sq. rd. 5. 0.025 bu. 6. 121. 7. $9.43.

Page 206. 2. 210 sq. ft. 3. 3 ft. 9 in. by 6 ft. $10\frac{1}{2}$ in. 4. 2 ft. $9\frac{3}{4}$ in.; 2 ft. 6 in. 5. 300 sq. ft. 6. 90 sq. ft.; $152\frac{1}{4}$ sq. ft.

Page 207. 2. $\frac{1}{16}$.

Page 208. 8. 160 sq. ft. 9. 150 sq. ft. 10. 24 sq. in.

Page 209. 8. $13\frac{1}{2}$ in. by $16\frac{1}{2}$ in.

Page 211. 1. $33\frac{1}{3}$; $6\frac{2}{3}\frac{2}{3}$. 2. $7\frac{1}{2}$ yd. 3. $137\frac{1}{2}$ yd. 4. 338 ft. 5. 270.

Page 212. 2. 72 sq. in. 3. 168 sq. in. 4. 504 sq. ft. 5. 286 sq. yd. 6. 458.8 sq. ft. 7. $15\frac{5}{6}$ sq. in. 8. 171.13 sq. ft. 9. 1709.5 sq. ft. 10. 1207.42 sq. yd. 11. 1552.98 sq. mi.

Page 213. 1. 5.4. 2. 252. 3. 68.73 sq. in. 4. 903.5 sq. ft. 5. 462.68 sq. ft. 6. $18\frac{1}{3}$ sq. ft. 7. $22\frac{3}{11}$ sq. rd. 8. $9\frac{1}{4}$ sq. yd. 9. $65\frac{1}{8}$ sq. ft. 10. 101,460 sq. ft. 11. $33\frac{4}{4}\frac{1}{8}$ sq. ft. 12. $419\frac{3}{8}$ sq. yd. 13. 7 in. 14. 11 ft. 15. 12 ft. 16. 24 ft. 17. 80 ft. 18. 6 ft. 19. 6 ft. 20. 6 ft.

Page 215. 1. 58 ft. 9 in. 2. 39 ft. 2 in. 3. 85 ft. 4. 2. 5. 37 ft. 9 in. 6. $42\frac{1}{3}$ yd. 7. 29 ft. 5 in. 8. 27 ft. 7 in. 9. 10 ft. $6\frac{7}{8}$ in. 10. 15 yd. $1\frac{1}{4}$ ft. 11. 7 in. 12. 29 ft. 8 in. 13. 56 mi. 250 rd. 14. $33\frac{1}{3}$ ft. 15. 35.2 ft.; 35.2 ft.; 17.6 ft. 16. 8; 9; 7. 17. 30; 40; 50.

Page 217. 7. 698.75 sq. ft. 8. 260.87 sq. in. 9. 520.91 sq. in. 10. 244.44 sq. in. 11. $806\frac{9}{16}$ sq. ft. 12. $5\frac{5}{6}$ sq. ft. 13. $8\frac{1}{8}$ sq. ft. 14. 4.29 sq. ft. 15. 67.5 sq. ft. 16. 54.4 sq. rd.; 61.2 sq. in. 17. 600 sq. rd.; $3\frac{3}{4}$ A. 18. 3900.

Page 218. 1. 240 sq. rd. 2. $5450\frac{1}{2}$ sq. ft. 3. $167\frac{1}{16}$ sq. in. 4. 15.58 sq. in. 5. $16\frac{7}{8}$ sq. ft. 6. 336 sq. rd.

Page 220. 1. $1130\frac{1}{2}$. 2. 540. 3. $181\frac{1}{3}$. 4. $118.22. 5. $44.44; $55.56. 6. $10\frac{5}{8}$; 12. 7. $11,229\frac{1}{8}$. 8. 169,200 cu. ft. 9. 224; 600; 224. 10. $266.67. 11. 1904; 65,280 lb.

Page 223. 1. $134\frac{2}{5}$. 2. 108. 3. $7\frac{1}{8}$. 4. 900. 5. 6. 6. $437\frac{1}{2}$. 7. 40. 8. $56\frac{1}{4}$. 9. $921\frac{3}{5}$. 10. $1814\frac{2}{5}$. 11. 3072. 12. 1176. 13. 990. 14. $1520.64. 15. $1814.40. 16. $1094.40. 17. $1284.

Page 224. 1. 52. 2. 17.71. 3. 5.54. 4. 14.88. 5. $8\frac{1}{8}$. 6. $8\frac{1}{16}$. 7. 30. 8. 11,000. 9. $28\frac{1}{2}$. 10. 7.3. 11. $3.60. 12. 85.35. 13. 8663.

Page 225. 1. 4752. 2. 49.58 cu. in. 3. 159.58 cu. in., larger. 4. 489.58 cu. in., larger. 5. First; $370\frac{5}{2}$. 6. 1235¼ cu. in. 7. First; $50\frac{5}{8}\frac{5}{4}$ cu. in. 8. 869½; $427\frac{3}{3}\frac{1}{2}$; $1847\frac{1}{16}$; $1059\frac{5}{8}\frac{5}{4}$.

Page 227. 1. 12 sq. ft. 2. 6 ft. 3. 15 in. 4. 112½ sq. ft. 5. 43⅓ ft. 6. 12½ ft. 7. 27.5 ft. 8. 89 in. 9. 7.2 in. 10. 11 ft. 11. 12 ft. 12. 12 ft.; 12 ft.; 9½ ft.

Page 228. 1. 117.88. 2. 68.32 bu. 3. $1900.80. 4. 160; yes. 5. $4374. 6. $40.50. 7. 7.45. 8. 60. 9. 720 gal.

Page 229. 1. 80; 16; 22½. 2. 33⅓. 3. 28.57; 35.71. 4. 35; 28. 5. 62½. 6. 41⅔. 7. 33⅓. 8. 41⅔. 9. 36.36. 10. 44.44. 11. 1¾. 12. $\frac{1}{16}$; ¾.

Page 230. 1. 5 ft. 4 in.; 5⅓ ft.; 1 yd. 28 in. 2. 141 gal. 1 qt.; 141¼ gal. 3. 47 qt. 1 pt.; 47½ qt.; 11 gal. 7 pt.; 11⅞ gal. 4. 630. 5. 78⅛ lb. 6. 400. 7. 6 yd. 8. 54.03 T. 9. 216; 72. 10. 84; 252. 11. 21; 126. 12. 1988; 2000; 2004.

Page 231. 1. 280 lb. 2. $12. 3. $1.50. 4. $8.80. 5. 20 da. 6. $280,630.

Page 232. 1. $73.90. 2. 20. 3. $317.25. 4. 36. 5. $15.60. 6. $392.

Page 233. 1. 216. 2. $4. 3. 355. 4. $3.60. 5. 201,450. 6. $14,101.50. 7. 9¢; 88¢. 8. $\frac{1}{4}\frac{3}{1}\frac{3}{3}$.

Page 234. 1. 78,000,000. 2. 1,790,816 bu. 3. 111,926. 4. $3.04. 5. $30,400,000. 6. 102¾ lb. 7. $3,020,850,000. 8. 547,500,000; $153,300,000.

Page 236. 1. 96¢. 2. $1.44. 3. $1.47. 4. $1.45. 5. $6.16; $7.04. 6. $320; $576. 7. $2.24. 8. $178.50. 9. $76.50; $189. 10. $19.04; $32.64. 11. $24.64. 12. $2.72. 13. $1.44. 14. $16.25; $26. 15. $5.94; $14.52.

Page 238. 1. 250. 2. 10,304. 3. 17.32 mi. 4. 6 min. 25 sec. 5. 7. 6. 337½. 7. 8.8. 8. $3,000,000. 9. $12. 10. 14.67; 117.33; 10; 80.

Page 241. 1. 56,560; 565,600; 28,280; 141,400; 707,000. 2. 84,480; 844,800; 42,240; 211,200; 1,056,000. 3. 171,840; 1,718,400; 85,920; 429,600; 2,148,000. 4. 150,480; 1,504,800; 75,240; 376,200; 1,881,000. 5. 141,200; 1,412,000; 70,600; 353,000; 1,765,000. 6. 565,800; 282,900; 152,766; 186,714. 7. 2,135,000; 1,067,500; 576,450; 704,550. 8. 407,000; 203,500; 109,890; 134,310. 9. 486,700; 243,350; 131,409; 160,611. 10. 360,100; 180,050; 97,227; 118,833. 11. 5680; 2840; 1420. 12. 8408; 4204; 2102. 13. 41,932; 20,966; 10,483. 14. 198.28; 99.14; 49.57. 15. 38.884; 19.442; 9.721.

Page 244. 1. $3.50. 2. $92,570. 3. $82.50. 4. $79. 5. $47. 6. $587.50. 7. $850. 8. $7375. 9. $1,635,000. 10. $42,710. 11. $48,125. 12. $7050. 13. $18,000. 14. $23,125. 15. $55,000. 16. $965. 17. $2130. 18. $90,000. 19. $51,000. 20. $90,000. 21. $900,000. 22. $1,200,000. 23. 12,300. 24. 16,200. 25. 48,000. 26. $20,410. 27. 310,500. 28. $11,605. 29. 493,324. 30. $1,554,250. 31. $10,660,000. 32. $106,600. 33. 90,000,000. 34. $40. 35. $3000. 36. $4200.

37. $87.50. **38.** $460. **39.** $21. **40.** $16. **41.** $3600. **42.** $959,100. **43.** $900; $833.33; $1666.67. **44.** 300 lb.; 600 lb.

Page 247. **1.** 1749. **2.** 107. **3.** 698. **4.** 110.34. **5.** 35. **6.** 110.79. **7.** 407.34. **8.** 103.35. **9.** 714.32. **10.** 430.24. **11.** 299.16. **12.** 62.535. **13.** 236.16. **14.** 381. **15.** 716. **16.** 25; 137; $95; $7459; $195,053. **17.** 30; 33; $19; 2510; $15,560. **18.** $2001; $3003; $2212.20; $3621. **19.** 137; 113; 307; 370. **20.** 3,618,769; 18,093,845. **21.** 6; 14; $52; $10.08; $580. **22.** 2.61; 7.53; $21.75; $13.05; $65.25. **23.** 1.02; 5.79; $5.61; $6.42; $191.25. **24.** 2.94; 59.04; $44.28; $33.96; $829.50. **25.** 18. **26.** 17. **27.** 70. **28.** 190. **29.** 230.

Page 248. **1.** $168; $336. **2.** $405; $720. **3.** $148.50. **4.** $37.80; $39.90; $47.04. **5.** $180.98; $185.74; $218.12. **6.** $436.14; $466.79; $575.23. **7.** $557.81; $592.88; $749.06. **8.** $1109.99; $1061.38; $1243.33.

Page 250. **1.** $3.19. **2.** $53.63. **3.** $2520. **4.** $56.10. **5.** $44.10. **6.** $16.50. **7.** $27.50. **8.** $51.24. **9.** $17.25. **10.** $15. **11.** $88.20. **12.** $3.25; 39¢. **13.** $79.20. **14.** 74.

Page 252. **1.** $825.25. **2.** $30.10, profit. **3.** $53.95. **4.** $94.75, loss. **5.** 50¢, loss. **6.** $1.49. **7.** 43⅝¢; $80; 5¢. **8.** $13.05, loss.

Page 253. **1.** $33.95. **2.** $13.75; $1.25. **3.** 58¢. **4.** $5.25. **5.** $3.30. **6.** $80.75; $195.50. **7.** $131.60. **8.** 32¢. **9.** 8½.

Page 254. **1.** 3.41. **2.** 2.81. **3.** $362,700. **4.** 40,040. **5.** $13.20.

Page 255. **1.** $1704. **2.** $6.30. **3.** 213.6 mi. **4.** 122. **5.** 284. **6.** $960. **7.** 696 lb., ash; 22,815 lb., carbon. **8.** 394.4 lb., ash; 12,928.5 lb., carbon. **9.** 91.4 lb.; 91.4%. **10.** 275.

Page 256. **1.** $47.25. **2.** 34¢. **3.** $1.67. **4.** $27.59. **5.** $83.20; **2.** **6.** $7.94. **7.** 9; $30.

Page 257. **1.** $846,000,000. **2.** $35.25. **3.** $708.75. **4.** 30. **5.** 261. **6.** 0.73. **7.** 461,700,000 bu.

Page 258. i. **1.** 570 in. **2.** 58 qt. **3.** 170 qt. **4.** 200 min. **5.** $27,833,273.75. **6.** $13.19. **7.** $1,602,838.72. **8.** $7,562,406.25. ii. **1.** 1636.25 sq. ft. **2.** 81 sq. ft. **3.** $49\frac{1}{12}$ sq. ft. **4.** $17\frac{5}{24}$ sq. ft. **5.** 13 cu. ft. iii. **1.** $12{,}562\frac{1}{8}$ sq. ft. **2.** 308 sq. ft. **3.** 6734. **4.** $7500. **5.** $160; $480. iv. **1.** $65\frac{1}{6}$ in. **2.** $1\frac{5}{8}$ in. **3.** $83\frac{1}{8}$ in. **4.** $1\frac{5}{71}$. **5.** 2.3. **6.** $3.65. **7.** $4.20. **8.** 9 ft. **9.** $15\frac{1}{6}$ ft. **10.** $10\frac{1}{16}$ lb.

Page 259. **1.** 80,377,209. **2.** 92.044. **3.** $57.57. **4.** $38.84. **5.** $64.61. **6.** $54.05. **7.** $51.88. **8.** $63.98. **9.** 3.802968. **10.** $53.33. **11.** 5.46. **12.** $1\frac{1}{8}$. **13.** $\frac{3}{8}$. **14.** $1\frac{5}{8}$. **15.** $\frac{5}{8}$. **16.** $\frac{1}{6}$. **17.** $\frac{1}{16}$. **18.** $\frac{1}{16}$. **19.** $\frac{9}{32}$. **20.** $\frac{7}{8}$. **21.** $\frac{3}{4}$. **22.** $4\frac{3}{8}$. **23.** $5\frac{5}{6}$. **24.** $2\frac{3}{8}$. **25.** $5\frac{1}{8}$. **26.** $6\frac{1}{4}$. **27.** 0.84. **28.** 6.46. **29.** 0.21275. **30.** 4.5. **31.** 1.2.

Page 263. 1. \$10. 2. \$124.06. 3. 221. 4. 43¢. 5. 216. 6. \$103.68. 7. \$130.90. 8. 48. 9. \$129.27.

Page 264. 1. \$192.60. 2. \$117.40. 3. \$12.69. 4. \$11.91. 5. \$2.25. 6. \$75.80. 7. \$109. 8. \$48. 9. \$525. 10. \$100. 11. \$49.80.

Page 266. 1. \$355.63; \$368.84; \$18.75; \$0.68; \$3; \$0.68; \$3; \$24.25; \$10; \$6.50; \$7.98; \$5.76; \$108.89; \$259.95. 2. \$14.70.

Page 268. 1. \$13.25. 2. \$30. 3. \$39.75. 4. \$37. 5. \$12.10. 6. \$31.15; 144 bu.; \$224.28. 7. \$33.60. 8. \$63.10. 9. (1) $29\frac{1}{1}$ rd.; 29.88 sq. rd. (2) \$58.78. (3) \$11.05.

Page 270. 2. \$41.60. 3. John, \$7.80; William, Clarence, and Roger, \$5.20 each; Mary, Susan, Ella, and Ann, \$3.25 each; Henry and Frank, \$2.60 each. 4. \$67.60. 5. \$75.45. 6. \$58.91. 7. \$42.80. 8. \$36.77. 9. \$33.70. 10. \$314.73. 12. \$46.42. 13. \$43.60. 14. \$44.85. 15. \$48.10. 16. \$182.97. 17. 1st, Ann, \$1.92; 2d, William, \$1.89; 3d, Mary, \$1.86; 4th, Henry, \$1.84; 5th, Ella, \$1.79; 6th, Susan, \$1.74; 7th, Frank, \$1.69; 8th, Clarence, \$1.47; 9th, John, \$1.13; 10th, Roger, \$1.06. 18. \$497.70; \$248.85.

Page 273. 1. 216. 2. 96. 3. 176. 4. 252. 5. 4. 6. 4. 7. 35. 8. 2340.

Page 274. 1. 2437. 2. 277,666,667 cu. ft. 3. 1750 cu. ft. 4. 8. 5. 1.44 meters. 6. 3.92 ft. 7. \$2,169,720. 8. 9000; 288,000. 9. 1292. 10. \$152.80. 11. \$2412.

Page 276. 1. $6\frac{1}{2}\frac{3}{4}$; $14\frac{3}{8}$; $15\frac{5}{9}$; $32\frac{1}{3}$. 2. $2\frac{2}{4}\frac{3}{8}$; $5\frac{7}{4}\frac{7}{8}$; $\frac{2}{3}\frac{3}{4}$; $6\frac{1}{2}\frac{1}{4}$. 3. $15\frac{1}{3}$; $66\frac{3}{3}\frac{3}{3}$; $50\frac{5}{8}$; 2121; 2; 5; $1\frac{1}{3}$; $1\frac{2}{4}\frac{2}{7}$. 4. 800 rd.; $2\frac{3}{4}$ T.; 588 in.; 3 cd. 5. 211; 543; 276; 3780; 432; 371.91. 6. \$765; \$675; \$210; \$300; \$150; \$180. 7. \$168; \$812; \$880; \$1375; \$1620; \$25. 8. \$225; \$5400; \$3200. 9. Same. 10. 100.

Page 277. 2. \$115.41. 3. \$4938. 4. \$28,725. 5. \$307.20. 6. \$324. 9. 410. 10. \$242.

Page 278. 1. \$4.40. 2. 2.4. 3. 4320 bu. 4. 0.9. 5. 57. 6. \$675. 7. First. 8. \$175. 9. \$142.88. 10. 2 min.

Page 279. 1. 0.1; 0.4; 0.232; 0.723; 0.3. 2. 1.3; 15; 2.25; 3.007; 2.8. 3. 237.06; 17.78. 766.44; 40.78. 842.57; 182.85. 1059.06; 187.78. 1441.08; 165.72. 4. 104; 0.5; 111; 111; 11; 1.85. 5. 20. 6. 0.03125. 7. 146.61 ft. 8. 24,500. 9. 122,044 lb. 10. \$1656; \$442.40.

Page 280. 1. 7.46; 5.7; 62.4; 179.8; 272; 567; 97.812; 282.2. 2. 30.6; 27; 55.76; 67.76; 40.128; 65.794; 15.7768; 37.4376. 3. 19; 190; 0.25; 0.09; 0.13; 1.39; 0.32; 2143.13. 4. 0.541; 4.988; 14.071; 1377.097. 5. \$568,000. 6. 130,004,632 lb. 7. 174. 8. \$189,383,000. 9. 25¢. 10. 32.

Page 281. 1. \$68.32. 2. 15; \$3. 3. \$85.20. 4. \$2.97. 5. 9¢. 6. 0.1¢; 0.06¢; potato. 7. $\frac{7}{4}$. 8. $22\frac{1}{2}$. 9. $89\frac{5}{8}$. 10. 2,871,000; 71,775,000.

Page 282. 1. \$16.50. 2. \$41.85. 3. \$5.50. 4. \$37.50. 5. \$25.50. 6. \$6.75. 7. \$20.40. 8. \$5.25. 9. \$7.50. 10. 39; \$1.78.

Page 283. 1. \$115.50. 2. \$2.28. 3. \$45.50. 4. 547,500,000. 5. \$663.87. 6. \$610. 7. 14; \$2.86; \$72.86. 8. \$228.75; \$381.25. 9. \$198.25; \$259.25. 10. \$533.75; \$762.50.

Page 284. 1. $3\frac{7}{8}$; $1\frac{3}{5}\frac{1}{0}$; $82\frac{3}{3}\frac{1}{2}$. 2. $\frac{1}{4}\frac{7}{0}$. 3. $\frac{1}{4}\frac{7}{8}$. 4. 20. 5. $4\frac{3}{8}$ min. 6. $11\frac{5}{3}\frac{5}{2}$. 7. $16\frac{3}{5}\frac{3}{6}$ lb. 8. 100. 9. 9. 10. 15.

Page 285. 1. 110.85 A. 2. 142.37 mi. 3. 169.4 mi. 4. 137.7 mi. 5. 8.84. 6. \$18.22. 7. 2.78. 8. 1.5 ft.; 2.47 ft. 9. 0.1875; 0.15625; 0.109375; 0.075. 10. $\frac{1}{4}\frac{7}{0}$; $\frac{7}{8}$; $\frac{1}{3}$.

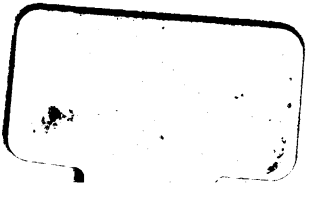
Page 286. 1. 198 in.; $5\frac{1}{2}$ yd. 2. $3\frac{1}{2}$ sq. yd.; 4536 sq. in. 3. $8\frac{1}{2}$ cu. yd.; $2551\frac{1}{2}$ cu. ft. 4. $14\frac{1}{4}$; 14. 5. 19,800 lb. 6. 57.75; 67.2. 7. 8. 8. 8640; 2160. 9. 288. 10. 1294 T.; 1670 T.

Page 287. 1. 127,050. 2. $3970\frac{5}{1\,6}$. 3. 129,318.75 T. 4. 1200 lb. 5. 1740. 6. 9.58 cu. in.; too large. 7. $21\frac{3}{4}$ lb.; 5568 oz.; 696,000 lb.; 4176 in.; 29 ft. 8. 406 oz.; $1\frac{7\,5}{1\,2\,8}$ lb.; 50,750 lb. 9. 6082.56. 10. 0.82.

Page 288. 1. \$7296. 2. \$5.75. 3. \$3.60. 4. \$49.35; \$87.42; \$109.98. 5. $16\frac{7}{8}$ da.; \$164.53. 6. 52.26. 7. 432 ft. 10 in. 8. 1863.0192. 9. $62\frac{1}{2}$. 10. $72\frac{3}{1\,0}$; $27\frac{3}{1\,0}$.

Page 289. 1. \$138; \$1587. 2. \$561; \$3961. 3. \$4.50. 4. \$300. 5. \$1360. 6. \$2.07. 7. \$220. 8. \$3430. 9. \$862.50. 10. 28.09 lb.

Page 290. 1. \$40.32. 2. \$540. 3. \$3686. 4. \$196.56. 5. 38¢. 6. \$97.65. 7. First; 60¢. 8. \$321.75. 9. \$29.25. 10. \$32.40.



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